Quantifying the scalability of reduced-order grey-box energy models for commercial building stock modeling

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Abstract

Grey-box models are extensively employed in building energy simulations. However, the grey-box approach often leads to application and stakeholder specific models, for instance, the design approach of grey-box modeling for commercial buildings differs on a case by case basis. Often, the network order limits the scalability of these networks. Reduced-order grey-box modeling approaches counter these limitations by achieving a trade off between model complexity and desired accuracy. This study, therefore, formulates a generalized methodology to quantify scalability associated with reduced-order grey-box models for heat demand modeling of commercial buildings. The devised methodology assesses model scalability through (1) scalability feature-definition, (2) model identification, (3) multi-level modeling and (4) KPI identification procedures. This study formulates a test-case of 10 buildings (on university campus) with varied operations to implement the devised methodology. Results indicate that model scalability directly associates with the nature of building operation. Furthermore, similar zone variables can effectively represent an entire building provided that the considered zone pre-dominantly occupies majority of the building's indoor space.

Introduction

Retrofitting existing buildings can significantly reduce the building sector energy consumption and associated CO_2 emissions as these buildings predominantly consume a major proportion of the overall building sector energy (Robinson et al., 2017). It has been estimated that 80% of the buildings are over 10 years old and account for 98% of the total floor area in comparison to new buildings constructed every year (International Energy Agency, 2017). To meet the required targets, average building energy use per person globally needs to fall by at least 10% (less than 4.5 MWh) by 2025. Concerted global effort is needed to rapidly expand, strengthen and enforce building energy policies across all countries to avoid inefficient building investments.

Simulation based design of buildings can enable the identification of cost-effective retrofit strategies. Over the past few decades, new building energy modeling tools and methodologies have been devised to support building professionals in their effort to optimize the design and thereby, enhance energy performance. Building Energy Performance Simulation (BEPS) tools take into account the multitude of complex inter dependencies, internal and external inputs as well as performance objectives (Choi, 2017). These tools can effectively evaluate retrofit design strategies at the individual building scale or at a district scale (Yang et al., 2018). However, the existing BEPS tools produce non-scalable and non-generalizable models (Wang et al., 2013). The development of detailed building models (using BEPS tools) often requires detailed geometric and non-geometric data. As each individual building differs in terms of structural parameters and nature of operation, developing a detailed model for each building is often impractical.

It is of paramount importance to devise generalizable and scalable modeling approaches that represent the actual behavior of the building under consideration. Grey-box modeling is one such approach, which delivers the advantages of data-driven and physical modeling approaches. This approach entails the accuracy levels of physical modeling approaches and the computational efficiency of data-driven modeling approaches (Coakley et al., 2014). However, the grey-box approach often leads to application and stakeholder specific models, for instance, the design approach of grey-box modeling for commercial buildings differs on a case by case basis. Often, the network order (defines the level of model complexity and is equivalent to the number of network capacitances) limits the scalability of these networks. Reduced-order grey-box modeling approaches counter these limitations by achieving a trade off between network order and desired accuracy. There is a need for a generalizable and scalable framework to address the limitations associated with grey-box models.

It is evident that modeling a building stock implies the consideration of different subsystems, and hence, the size of the problem can significantly grow depending on the analyzed system. Scalability is an important issue when modeling the building stock and assesses whether a similar network order could represent different zones inside a building and eventually trace the dynamics of the entire building. Different variants of the definition of scalability exist in the building simulation domain. A study by Arteconi (2018) defined scalability for large scale integrated energy systems as an attribute that could represent the presence of different interacting sub-systems. The scalability could further be associated with achieving a trade off between the necessary level of detail and the computability of the model. Another definition of scalability focuses on the integration between building information modeling and simulation-based design in terms of the size and complexity of the models that supply building simulation with site-specific information, the computational requirement of simulations involving multiple building domains, and the feasibility of integrating whole building simulation in the design process (Wang et al., 2013). For grey-box networks, scalability signifies the ability of a system to represent extended system variables (or added system states) (Bacher and Madsen, 2011).

A study by Heidarinejad et al. (2017) devised urban-scale reduced-order building energy models using highly influential variables in the heat transfer process. Although the framework employed a reduced number of parameters, the study implemented white-box models and mainly focused on the exterior representation of the building to account for the external inputs. Kontokosta and Tull (2017) developed a data-driven prediction model for energy use in buildings at urban scale. Although the results provide an effective overview of urban building energy systems, the study does not address building level dynamics.

The chief contribution of this research is a novel framework to address the scalability of grey-box networks at the zonal, multi-zone, whole building and building stock levels. Addressing the scalability at different modeling levels effectively propagates building level dynamics at each level. This study proposes a zonal approach to propagate the building level dynamics at various modelling scales. The approach assesses zone model scalability relative to existing multiple zones. The methodology addresses the generalisation issues of grey-box networks through a network formulation procedure based on building physical and operational parameters. The paper establishes the significance of grey-box model features through data analytics and experimental simulations.

The paper is organized as follows: Section II outlines the devised methodology to address the scalability of grey-box models. Section III demonstrates a case study to implement the proposed methodology. Section IV details the analysis of results and subsequent discussions. Conclusions and future work are presented in Section V.

Methodology

Scalability issues often associate with building energy performance models irrespective of the implemented modeling technique (Arteconi, 2018). Previous frameworks have either dealt with this issue at the building zone level or at the urban level (Heidarinejad et al., 2017). At the zone level, model dynamics are often not able to reciprocate with a cluster of buildings. While at the urban level, models usually lose the associated building dynamics due to oversimplification of building characteristics.

This paper aims to develop a framework for assessing the scalability potential of grey-box models. The developed framework evaluates how efficiently a single grey-box model can represent different zones inside a building. Additionally, the framework also evaluates the extension of a zone grey-box model to represent the heat dynamics at the whole building and building stock level. It is worthwhile to mention that the dynamics closely relate to the heat demand patterns for any building stock. Furthermore, these patterns depend on the local climate, heat transfer through the building envelope, daily operations and occupancy schedules. Hence, the framework uses these patterns as a foundation to build the grey-box models upon.

The devised methodology follows a five step process to assess the scalability of grey-box models, which include scalability definition, collection of data inputs, grey-box model formulation, demonstration test case development and Key Performance Indicators (KPI) specification (Figure 1).

1. Scalability Definition: This process defines the scalability feature using previous literature, technical reports, expert views and model requirements. After an in-depth review of the published scalability studies both in the building simulation and other modeling domains, scalability of the grey-box model relates to a model's ability to represent extended system variables, for instance, when modeling urban building stocks (from individual building to urban stock level). Scalability measures the ability of a grey-box model to retain the original network order while introducing new state spaces (temperature). Therefore, this study examines whether a single network can represent multiple zones, individual buildings and the building stock. Furthermore, this study also evaluates the associated scalability potential using certain KPIs.

2. Data Inputs: This process involves data collection pertinent to building stock modeling. Two sets of data are of paramount importance when formulating grey-box models. The first set mainly includes weather variables, namely, ambient temperature, global solar irradiation and wind speed. The second set involves building related variables specific

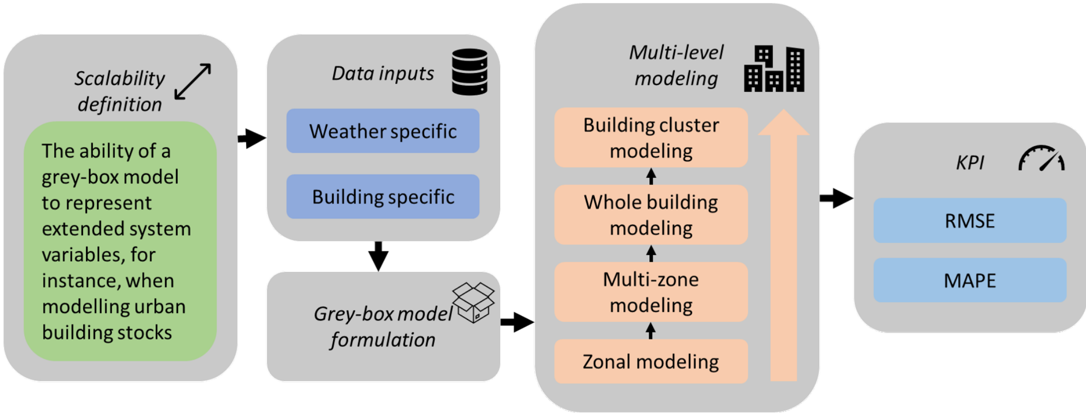

Figure 1: Devised methodology to assess the scalability potential of grey-box models.

to grey-box model formulation. This set comprises of measured building heat demand patterns, internal temperature profiles, grey-box model parameters (if already known), building zoning/space usage, occupancy schedules and building function.

This process further implements an initial statistical analysis with building heat demand data to decipher the existing variations in respective profiles. This study implements an ANalysis Of VAriance (ANOVA) technique to analyze the demand variations mainly because ANOVA can establish the statistical significance of demand variations with time. ANOVA is a statistical technique used to check if the means of two or more groups are significantly different from each other (Crawley, 2007).

In some instances, ANOVA procedures can be enhanced by considering additional information that is available and is known to be correlated with one or more of the factors under investigation. F-test statistic (ratio of two quantities that are expected to be roughly equal under the null hypothesis) compares the results from ANOVA. A large value of F-test statistic signifies that the variability of group means is large relative to the within group variability (required for rejecting the null hypotheses). Employed in combination with p-values, the F-test statistic tests the significance of null hypotheses.

3. Grey-box Model Formulation: This process involves the formulation of grey-box models (RC network models) based on the network order. A typical thermal resistance and capacitance (RC) network models the heat transfer and thermodynamics of the building control studies. Such networks have been widely implemented in the optimization of building control systems (Bacher and Madsen, 2011; Weber and Jóhannesson, 2005). The RC network model assumes steady state heat transfer through the building envelope.

The model formulation process usually implements a parameterization procedure (calibration) to determine the parameters of the grey-box model when the individual parameters are not available. This can be integrated with a network order identification procedure, which determines the optimal number of states required to represent the building under consideration. The order identification procedure outlines the underlying structure of the grey-box network and considers thermal zoning of spaces inside the building where each thermal zone possesses similar characteristics and can be represented using a similar RC network. This process uses a generalised methodology devised by the authors in a previous publication (Shamsi et al., 2019). The order identification procedure uses the results of ANOVA analysis to check for individual building heat demand patterns to identify whether a lumped parameter model is sufficient to represent the building dynamics. Furthermore, buildings with relatively high values of normalised heat demand are assigned separate RC network branches for exterior walls and interior mass. In case of any recurring heat demand fluctuation in hourly profiles, separate thermal zones are assigned to the building.

4. Multi-level Modeling: Grey-box models differ in their respective structures based on the underlying building dynamics. The structure is dependent on the number of existing zones inside the building. The multi-level modeling procedure assesses the relevance of individual zone models in representing various zones inside a building (Figure 1). This process defines the type of testing required to evaluate the scalability feature.

To assess the scalability of grey-box networks, this study implements a bottom-up energy modeling approach that includes:

(a) Zonal Modeling: A grey-box RC network (considered as the baseline model) represents a single zone inside the building. The zone space corresponds to the highest percentage of space usage for the considered building.

(b) Multi-zone Modeling: The identified zonal grey-box network models multiple zones existing inside the same building. This process updates the network parameters depending on the heat dynamics of respective zones.

(c) Whole Building Modeling: The process further updates the zone model to represent the heat dynamics at the whole building level.

(d) Building Cluster Modeling: This involves the identification of the cluster of buildings that represent a similar variation in heat demand profiles. The ANOVA technique analyzes the significance of variations as described above. The whole building grey-box network represents and simulates the cluster of buildings.

5. Key Performance Indicators (KPIs): It is crucial to associate performance measures to models identified at each level. This study implements two KPIs, namely, Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE), to assess the performance of the zonal, multi-zone, whole building and building cluster grey-box models.

(a) Root Mean Square Error (RMSE): This KPI measures the differences between values predicted by a hypothetical model and the observed values (Chai and Draxler, 2014). Furthermore, RMSE values also indicate the quality of fit between the actual data and the predicted data. These values are useful when large errors are particularly undesirable (for instance, when predicting internal temperatures).

$$RMSE = \sqrt{\sum_{i=1}^{n} \frac{(\hat{y}_i - y_i)^2}{n}} \qquad (1)$$

where $(\hat{y}_1, \hat{y}_2, ..., \hat{y}_n)$ are the predicted values, $(y_1, y_2, ...y_n)$ are the measured values and n is the number of observations.

(b) Mean Percentage Absolute Error (MAPE): This KPI measures the size of error in percentage terms. It is the average of the absolute percentage errors of forecasts (Chai and Draxler, 2014).

$$MAPE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{(\hat{y}_i - y_i)}{y_i} \right| * 100 \qquad (2)$$

where $(\hat{y}_1, \hat{y}_2, ..., \hat{y}_n)$ are the predicted values, $(y_1, y_2, ...y_n)$ are the measured values and n is the number of observations. This measure is a bit easy to interpret and is independent of scale.

Case Study

To demonstrate the application of the devised framework, this study analyzes the buildings connected to the district heating network at UCD to test the grey-box model scalability. An initial analysis of the buildings using ANOVA test characterizes the nature of individual building operation. This test compares whether the fluctuations in the heat demand profiles are significant or not (Table 1). The tested groups for each building include the daily heat demand profiles over a two week testing period from 15/01/2020 to 24/01/2020. Over the two week period, the building has full occupancy and hence, provides a complete representation of system dynamics.

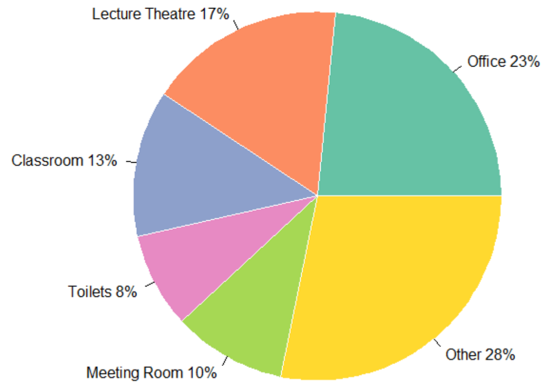

Figure 2: The proportion of space usage by different zones in the Sutherland School of Law.

The considered set of buildings represent varied nature of operation and use of available space. Corresponding to a significance level of $\alpha = 0.10$ (based on the sample size), the daily variations for Tierney building, Sutherland School of Law, James Joyce Library, Newman Building, Richview and School of Agriculture and Food Science are found to be insignificant (Table 1). Also, significant variations exist in the daily heat demand profiles for rest of the buildings, namely, Restaurant, Student Centre, O'Brien Centre for Science and Sports Centre.

This study demonstrates the scalable modeling process for Sutherland School of Law. Detailed information regarding the building features (detailed plans, measured consumption and internal temperature profiles) is available for this building. Furthermore, the building is relatively new and constitutes several zones. To demonstrate the applicability for building stock modeling, the study extends the modeling process to include the Tierney building.

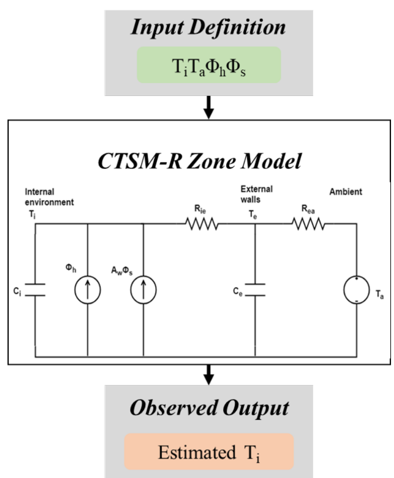

Figure 3: Grey-box modeling using CTSM-R to predict the internal temperature profile.

Table 1: Different types of case study buildings with associated significance level in daily variations. The ANOVA analysis classifies these buildings on the basis of existing variations in the daily heat demand profile of each reference building. The ANOVA test establishes whether the daily variations in heat demand are statistically significant or not.

Building Name	**Building Function**	**ANOVA Test p-values (Daily Variations,** $\alpha = 0.10$)
Tierney	Administrative office spaces	0.462, Not significant
Sutherland School of Law	Offices, Lecture theatres, Classrooms and Meeting rooms	0.291, Not significant
Restaurant	Main cafeteria and retail shops	0.088, Significant
Student Centre	Offices, auditorium, pharmacy and cafe	0.056, Significant
James Joyce Library	Big open spaces, study spaces and retail shops	0.482, Not Significant
Newman	Offices, Lecture theatres,Classrooms and Meeting rooms	0.351, Not significant
Richview	Offices, Classrooms and Meeting rooms	0.679, Not significant
O'Brien Centre for Science	Offices, Lecture theatres, Classrooms, Meeting rooms, Laboratories and restaurant	0.045, Significant
School of Agriculture & Food Science	Offices, Lecture theatres,Classrooms and Meeting rooms	0.565, Not Significant
Sports Centre	Court areas, Gym and Swimming pool	0.076, Significant

Scalability Definition: A grey-box model's ability to represent extended system variables, for instance, when modelling urban building stocks (from individual building to urban stock level).

Data Inputs:
T_a Outside Dry Bulb Temperature in $°C$
G_s Global Solar Irradiation in kW/m^2
T_i Internal Temperature Profiles in $°C$
P_h Measured space heat demand in kW

Test Case Building Zones: The Sutherland School of Law largely consists of office spaces, lecture theatres, classrooms, meeting rooms, toilets and other spaces (Figure 2). The offices constitute a major portion of the overall space usage followed by lecture theatres, classrooms, meeting rooms and toilets. The spaces categorized as other include storage spaces, corridors, photocopy rooms and stairs. Lecture theatre represent large spaces that can accommodate more than 200 students. Classrooms are relatively smaller and can accommodate a maximum of 50 students. As the building has numerous temperature data sensors, temperature data is gathered from one sensor per zone to identify the internal temperature profiles.

Grey-box Modeling: The modeling process implements grey-box models to predict the internal temperature profiles of different zones inside the building. Corresponding to a significance level of 0.10, the variations are found to be non-existent in the heat demand profile of the building (p-value = 0.291, Table 1). Hence, the order identification step suggests a lumped parameter model would be sufficient to represent the small-office building dynamics. Corresponding to a floor area of 3300 m^2, normalising the demand with respect to the floor area yields high demand levels per m^2 of space use. Henceforth, separate RC network branches are assigned to exterior walls and interior mass. As the demand fluctuations are deemed insignificant by the ANOVA test, a second order grey-box model is sufficient to represent the building dynamics. The process uses CTSM-R to estimate the parameters of the grey-box network and predict the internal temperature profiles (Figure 3). By using a continuous time formulation of the dynamics and discrete time measurements, the CTSM-R tool bridges the gap between physical and statistical modeling. The testing period starts from 15/01/2020 and ends on 24/01/2020.

Results and Discussion

The test case building comprises several zones; each of which is simulated individually to obtain the internal temperature profiles. The simulation process considers the office grey-box network as the baseline model to compare against other zone grey-box model predictions (Figure 4). The process implements RMSE

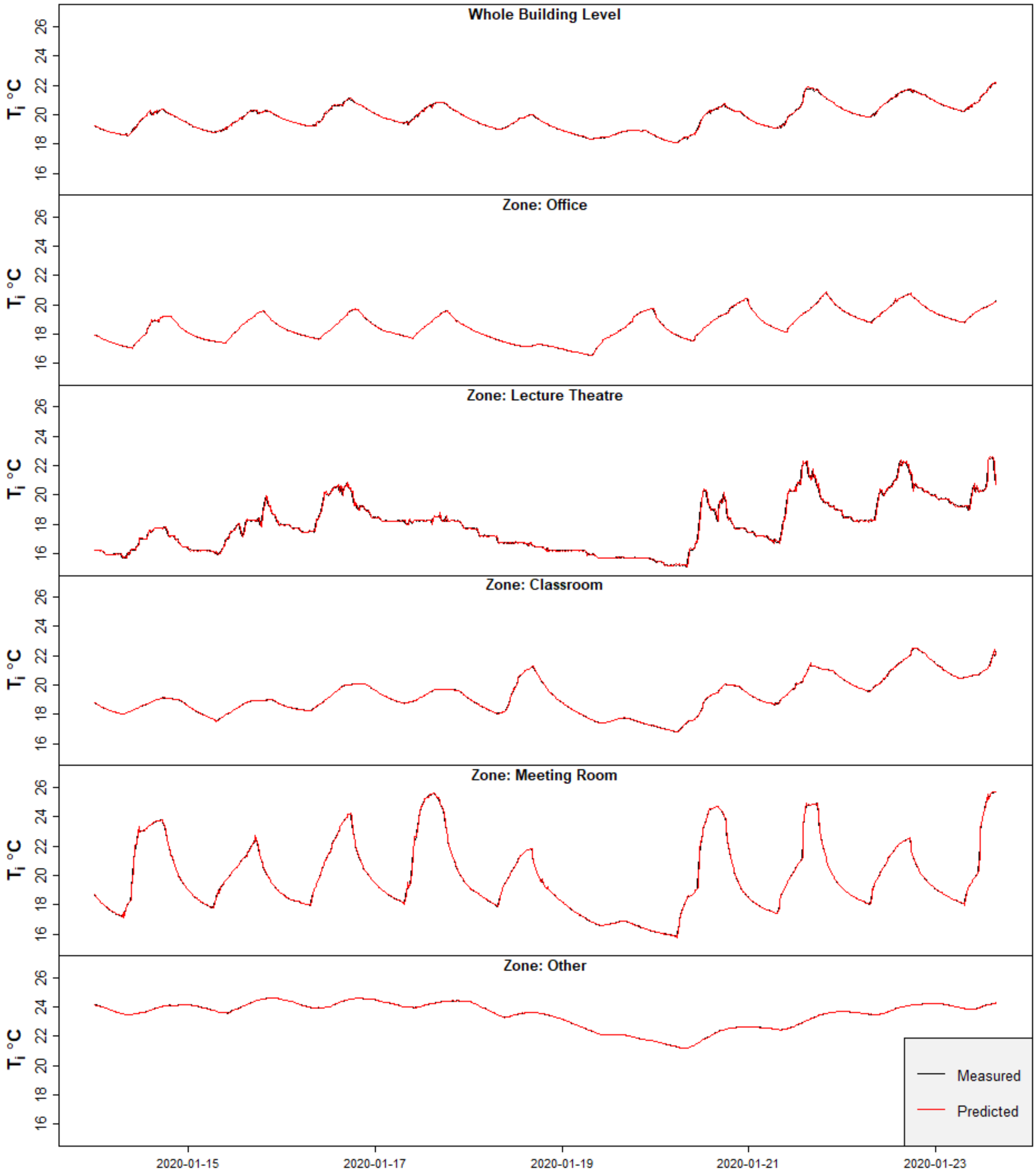

Figure 4: Measured and predicted internal temperature profiles at the whole building level and respective zone level for Sutherland School of Law over a two week period.

Table 2: RMSE and MAPE values for different zones in the considered building

Zone	RMSE (^{0}C)	MAPE (%)
Whole Building	0.049	0.149
Office	**0.023**	**0.069**
Lecture Theatre	0.151	0.462
Classroom	0.032	0.083
Meeting Room	0.107	0.224
Other	0.015	0.059

and MAPE values to analyze the individual prediction results (Table 2). It should be noted that individual zones use the second order network (with updated parameters) for simulation. The simulation of different zones can be described as:

Office: This zone represents a major portion of space usage inside this building. Furthermore, as evident from the measured and predicted temperature profiles, the variations are almost non-existent throughout the two week period (Figure 4). The second order model is able to trace the dynamics with an RMSE of 0.023°C and a MAPE of 0.069. The test case implementation uses these values as the baseline to compare the other zone predictions.

Lecture Theatre: When a similar office grey-box model with updated parameters predicts the internal temperature profile of the lecture theatre, the RMSE and MAPE values shoot up to 0.151°C and 0.462. These error values are the highest amongst all the zones, which is probably due to the temperature variations existing in this zone. It can be concluded that a third order network would be able to accurately trace the dynamics and thereby, significantly reduce the error in prediction. It is worthwhile to note that the theatre usually accommodates more than 200 students at once, which is a direct indication of the observed variations in internal temperature profiles.

Classroom: The RMSE (0.032°C) and MAPE (0.462) values for the classroom are relatively lower than the lecture theatre but more than the office. Therefore, a second order model is able to trace the dynamics of this zone. Although the classroom accommodates a maximum of 50 students (quite less than the lecture theatre), this zone has a varied nature of operation (space is heated only during lectures). When comparing the measured and predicted internal temperature profiles, it is quite evident the strength of variations lies in between the variations for the office and lecture theatre.

Meeting Room: The observed prediction error is much higher for this zone when compared to the office (RMSE = 0.107°C and MAPE = 0.224). Similarly, internal temperature profile variations are quite evident for this zone 4. This can be attributed to the fact that the meeting room zone has a varied schedule of operation, which is often random. Furthermore, the meeting rooms have different sizes and hence, can accommodate varying number of people. Therefore, a third order model would significantly reduce the error and increase the accuracy of prediction.

Other: This zone represents spaces such as corridors, storage spaces, photocopy rooms and stairs. The simulation implements the average internal temperature values for parameter estimation and subsequent predictions. This zone observes the lowest RMSE and MAPE values of 0.015°C and 0.059, which is mainly due to two reasons. Firstly, temperature variations are almost non-existent in these spaces and secondly, any plausible temperature profile variations are averaged out when considering the overall average temperature.

The simulation process further implements the office grey-box model at the whole building level. To obtain the internal temperature profile at the building level, the simulation process averages the data from all temperature sensors. This process then performs simulations to predict the internal temperature profiles at the whole building level (considering the entire building as one zone).

Whole Building: The RMSE and MAPE values for internal temperature predictions are found to be 0.049°C and 0.149, which are quite close to the values obtained for the office zone. This clearly indicates the suitability of implementing a zone level model for forecasting predictions at the whole building level. Furthermore, a third order network would reduce the prediction errors but can be deemed insignificant when compared to the existing errors in other zones.

It would be interesting to note that although the building loses a few dynamics when considering the temperature averages, these dynamics can be revived by adding an additional state variable to the existing grey-box network. Furthermore, another interesting fact relates to the building function. As offices occupy a major portion of the space usage of the test case building, implementing a zonal office model at the building level yields accurate results. This might not be the case for buildings with laboratories or restaurants constituting a major portion of the space usage.

Building Stock Modeling: To demonstrate the applicability of the methodology for building stock modeling, this study uses the Tierney building from the network of ten buildings mentioned earlier. This building consists mainly of offices with approximately 60% of the space being dedicated to administrative offices. The head demand variations for this building are smooth and decade towards the end of the day. We conducted an ANOVA test to verify if the heat demand profiles of Sutherland School of Law and Tierney building belong to the same group. The test statistics (F-value = 1.67, p-value = 0.196) corresponding to a significance level of 0.10 indicate that the null hypotheses stands true and hence, building demand profiles belong to the same group. The simulation process implements the similar second order grey-box network to predict the internal temperature profiles for the Tierney building over the same two week period. The RMSE and MAPE values of 0.056°C and 0.167 indicate that the second order network is able to trace the dynamics of the second test case building.

Conclusions

With a drive towards achieving an integrated energy system, there is a need for holistic and scalable building modeling approaches to model the commercial building stock. This study proposes a framework to evaluate the scalability associated with grey-box models through the assessment of the building energy model accuracy and complexity. The resulting model accuracy indicates the suitability of reduced-order grey-box models when modeling building clusters. Building's indoor floor space is indicative of the scalability of zone thermal networks. The developed framework aids the identification of scalable models and thus facilitates simplified creation of reduced-order grey-box energy models at the building stock level. The results of this study could support the current need for the assessment of consumption patterns of commercial building stock. The framework could further be implemented to study the post-retrofit heat consumption patterns at the individual building as well as the district scale.

It should be noted that another important issue when representing building energy systems is related with uncertainties due to uncertain design parameters (network parameters) or inherent uncertain parameters (weather). For grey-box models, the uncertainties can be addressed using inverse uncertainty quantification technique, which associates confidence intervals with network parameters. Integration of an uncertainty framework would enhance the value of this research.

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