Using Multiple Linear Regression to Estimate Building Retrofit Energy Reductions

Wesley Bowley1, Paul Westermann1, Ralph Evins1

1 Department of Civil Engineering, University of Victoria, Victoria, Canada
*corresponding author

Abstract: This work applies multiple linear regression to a building energy retrofit database of the City of Victoria in order to determine the energy reductions associated with different retrofit measures. The results of the regression are then used to construct marginal abatement cost curves for retrofit options. A comparison between continuous and binary variables is performed to examine their effect on accuracy. It was found that the accuracy is comparable ($R^2$ for binary: 0.81, $R^2$ for continuous: 0.76). The regression results estimated that building envelope retrofits could reduce energy use by 40%, and heating system retrofits can reduce energy use by up to 30%. Switching to electric heat pumps could reduce emissions by an estimated 80%.

Keywords: retrofit, building stock, multiple linear regression

INTRODUCTION

Retrofitting residential buildings has great potential to reduce carbon emissions through both improvements to the building envelope and by upgrading the heating systems. In British Columbia, the low carbon content of grid electricity makes converting to electrically-driven heating systems an excellent way to decarbonise the building stock. Retrofitting can also reduce energy bills for occupants. However, retrofitting measures incur significant up-front costs, which must be balanced against the possible benefits.

There are numerous ways to analyze the cost effectiveness of retrofit actions as well as how much each particular retrofit action reduces energy use. Physical modeling software can estimate the energy use of a building given many parameters and environmental conditions. However it is time consuming and impractical to model every building in a municipal building stock, and the required data is often not available.

One way around this is to collect data by surveying building characteristics as was done by Dall’O’ et al. (2012). Another option is to use aggregate data from a national level and assume that this is representative of the local building stock as in Constantinos (2007), which may not be accurate.

Another option is to create building archetypes that are representative of the buildings in the stock, so that detailed simulation can be performed on a smaller number of archetypes rather than on all the buildings in the stock, while still being representative. Linear regression is sometimes combined with archetypal analysis as in Chidiac (2011), however this study only covered office buildings.

Martinez et al. (2018) use multivariate linear regression to assess the energy use reduction of retrofits that include and exclude building envelope upgrades. They found that upgrading building envelopes increase the energy savings. However the dataset is somewhat limited in size, in addition to no consideration to specific components of the retrofits (e.g. Insulation, windows, etc.).

Walter and Sohn Walter (2016) use a multivariate linear regression model to predict energy use intensity with variables representing building parameters such as climate zone, heating system type, etc. The model quantifies the contributions of each characteristic to the overall energy use, then the energy saving from modifying or retrofitting that particular characteristic is inferred. The analysis is limited however in that it uses only pre retrofit data and isn’t validated using pre and post retrofit energy use data.

This work aims to use multiple linear regression (MLR) to derive the statistical impact of each retrofit measure on the total percentage energy reduction. This has also been extended to carbon emissions and energy bills by making assumptions about the breakdown of energy use. Our method is similar to that used in Walter (2016) , however the key differences are that we performed the regression on the percentage energy reduction between the pre and post retrofit energy use as opposed to just on the pre retrofit energy use. The accuracy of the regression is discussed as well as potential ways to improve it.

The results of this analysis were then used to construct marginal abatement cost (MAC) curves, which quantify the cost and benefit of each possible retrofit measure. MAC curves provide a simple way of expressing this relationship. They are simplified representations of the underlying problem in that they rely on the assumption of linearity, i.e.
that separate measures can be recombined in any manner, and that the total impact will be the linear sum of their individual impacts.

The study is based on a dataset of several thousand building retrofit evaluations in the City of Victoria compiled by National Resources Canada (NRCan). This gives the retrofit actions that were recommended and performed across 50 categories alongside the pre- and post-retrofit energy use as estimated using software called HOT2000.

**Methodology**

There are several steps to the analysis. First the available NRCan data on the energy use reduction of building retrofits has been cleaned and processed. The cost data associated with each measure has also been collated. Next a multiple linear regression process has been used to approximate the contribution of each individual measure to the total reduction. These coefficients are used to generate MAC curves, which are analysed and then scaled to the whole building stock. Please note that due to space constraints, we are limited in the amount of data that can be shown. This includes many building parameters such as pre and post retrofit heating system efficiency and retrofit measure costing.

**Database analysis**

The database is created from pre and post retrofit energy audits where parameters are recorded such as wall, foundation and ceiling insulation, number of energy star windows, and information about the heating system type (various types of gas or oil furnace, ASHP, electric base boards, etc.) and fuel type (oil, natural gas, electricity, wood). These parameters were used to create HOT2000 models of the buildings and the pre and post retrofit energy use was estimated. It is the difference between these values, i.e. the change in energy use, which is used for our calculations.

Ideally it would be better to obtain energy use from direct measurements or from simulation using a more advanced tool such as EnergyPlus. However, pre and post retrofit measurements are rarely available, nor are the many parameters needed for more detailed simulation. This paper describes a methodology that can be used on other building energy databases that could perhaps have direct energy use measurements, or are for different cities.

Before the dataset could be used, it was organized and cleaned. Building entries that did not perform post retrofit energy audits were removed since they provided no way of assessing improvements due to retrofits. Building entries were grouped based on different parameters, and erroneous values were removed.

**Multiple linear regression analysis**

Multiple linear regression models are an extension of the standard linear regression approach that can be used to quantify the impact of multiple inputs on one output. They are a class of statistical model that generate aggregated statistical insights from many individual observations. In this study it is used to analyse retrofit measures on city level using data on building level.

Multiple linear regression generates very useful results: unlike other methods, the fitted coefficients relate directly to the variables of interest, in our case the different retrofit measures. The weakness of the method is that it assumes all relationships between the inputs and the output to be linear and independent, i.e. that there are no non-linear relationships and no interactions between variables so that the total impact will be the linear sum of the individual impacts. Since this is also an assumption of the MAC curves that the outputs will be used to construct, this is not particularly detrimental.

In this study, we use linear regression methods to quantify the impact of different building retrofit measures (e.g. wall insulation improvement, replacement of heating system, etc.) on the reduction in the annual energy consumption, carbon emissions and energy costs of a building. The model is fitted using 7000 data entries relating to retrofitted buildings within the City of Victoria. The impact of each retrofit measure is captured by the regression coefficients $p_i$ of the fitted model as shown by the mathematical formulation of the regression model:

$$\Delta E = p_{air}X_{air} + p_{window}X_{window} + p_{ASHP}X_{ASHP} + ... = \sum p_i X_i,$$

where $X_i \in [0,1], p_i \in \mathbb{R}, i = \text{measure index}.$ The output variable $\Delta E$ represents the percentage reduction in energy consumption per unit floor area. Each coefficient $p_i$ is multiplied by a binary variable $X_i$ which indicates whether the respective retrofitting measure $i$ was performed ($X_i=1$) or not ($X_i=0$). The method provides the values of $p_i$, which here can be interpreted as the percentage by which the energy consumption is lowered if each of the different retrofit options is implemented independently. The larger $p_i$, the larger the impact of retrofitting measure $i$. The output variable $\Delta E$ is the difference between the pre- and post-retrofit annual energy use as estimated in the HOT2000 simulation on building level divided by the building area, in units of GJ/m$^2$/a.

As an example, we consider a simple case where there are three possible measures: windows can be retrofitted, an air...
source heat pump can be installed, and wall insulation can be improved. Fitting the model to lots of different observations on buildings having conducted these measures will give the coefficients $p_{\text{window}}$, $p_{\text{ASHP}}$, and $p_{\text{wall}}$, and the linear regression model estimates the reduction in energy consumption $\Delta E$ to be:

\[\Delta E = p_{\text{window}}X_{\text{window}} + p_{\text{ASHP}}X_{\text{ASHP}} + p_{\text{wall}}X_{\text{wall}}\]  (2)

For a specific building in which the windows and walls are upgraded but no heat pump is added, the percentage reduction in energy consumption is predicted to be:

\[\Delta E = p_{\text{window}} \times 1 + p_{\text{ASHP}} \times 0 + p_{\text{wall}} \times 1\]  (3)

i.e. the sum of the coefficients for the measures that were implemented. The full model is an extension of this to include all 17 measures, and hence has 17 coefficients.

**Model fitting**

The coefficients of the model are determined using ordinary least squares (OLS) methods. The model fitting and all related computations were programmed using the Python SKLearn Toolbox. To guarantee a statistically robust and accurate model, multiple steps were undertaken:

- The physics of the building heat balance show that the actual reduction due to building envelope and heating system retrofits are interlinked. For example, improving the insulation of a building with a low efficiency heating system is much more influential than of a building with a highly efficient heating system. To remove this link, the model was fitted to the percentage reduction in energy, emission or energy cost of a building. This modification eliminates the need to generate multiple models for each heating system type.
- The data set was scanned for outliers and 18 data points were removed.
- The coefficients resulting from the OLS fit were tested for statistical significance using the p-value score. All variables that are not statistically significant (i.e. whose p-value is larger than 0.005) are rejected from the model. The associated samples in which the associated measure is present are also removed, to reduce the variation in the remaining data.
- To verify the accuracy, the model was fitted to 90% of the data and its performance validated on the other 10% of the data. The samples for the validation set were chosen randomly.

**MAC CURVES**

Marginal abatement cost curves are used to compare the cost effectiveness of all retrofit measures in reducing carbon emissions. MAC curves integrate the previous findings on the impact of different retrofits on building energy consumption and the respective costs. The major advantage of MAC curves is the way they incorporate cost and emissions goals into one graph and display the most economical pathway of actions to reach a specific target.

First the energy consumption reductions must be converted into carbon emissions reductions by multiplying the reduction by the carbon factor associated with that of the heating system and fuel type. The carbon factors for each fuel type was obtained from the BC Ministry of Environment (2016). Efficiencies of the heating systems were also accounted for.

MAC curves represent each retrofit measure according to the following metrics:

- **Annual kgCO₂ savings (per m² floor area), horizontal axis:** This number uses the coefficients of the multiple linear regression model as shown in the previous section. The percentage reduction value of each measure is multiplied by the total average pre-retrofit emissions in kgCO₂/m².

- **Annual cost per kgCO₂ savings ($ per m² floor area), vertical axis:** The value above is divided by the cost of the measure. We compute the *equivalent annual cost* (EAC) to compare assets with different lifetimes, as determined for different building retrofit measures. EAC also considers the cost of capital by integrating current interest rates and inflation rates in Canada; a value of 1.16% was used Bank of Canada (2017).

MAC curves also have an advantage when paired with linear regression that they make the same assumptions regarding linearity and independence. This means that the assumptions of one method do not limit the ability or accuracy of the other method.

Energy consumption reductions are also converted into energy bill reductions by obtaining fuel cost data for Victoria, and then multiplying these factors by the energy reductions according to the fuel types (BC Hydro (2016), NRCAN (2015), FortisBC (2017)). All three metrics are examined in the results section.

**Table 1:** Variables used in the multiple linear regression.

| $R_{SI}$ insulation have units of m²*K/W |
result and discussion

In this section we first present the results of the model fitting, followed by an analysis of model accuracy, and finally the MAC curves derived from the model results.

Multiple linear regression results

The coefficients $p_i$ of the multiple linear regression analysis give the average percentage reduction in energy use associated with each retrofit measure. The measure indexes $i$ are given in Table 1. The results are shown in Figure 1; the numbers in brackets beside each retrofit option give the number of associated entries present in the data. The error bars display the standard error associated with each regression coefficient $p_i$. This is equivalent to the standard deviation of the model error, and therefore if the error is assumed to be normally distributed, then 68% of values will have an error less than or equal to the standard error.

Energy consumption

Energy consumption is lowered most effectively by installing more efficient heating systems, ideally an air source heat pump. The model suggests that a change from an electric furnace to an ASHP lowers the total energy consumption by 24%, a change from a gas furnace to an ASHP by 29% and a change from an oil boiler leads to a reduction of 37%. Installing new furnaces (especially gas or electric furnaces) leads to significant reductions in energy demand of between 10% and 17%. The reduction potential of ground source heat pumps is estimated to be 30%, but unfortunately since the dataset only features a very low number of samples (12), this value may not be accurate, and a detailed analysis of their impact is not possible.

Improving the building envelope also helps to lower energy consumption. Installing a highly effective wall insulation ($R_{SI}$-value > 0.75 m²K/W) cuts energy consumption by 16%; major improvements in the floor insulation lower the energy consumption by around 10%. Improving the ceiling insulation, replacing the windows or increasing air tightness have a smaller impact. However, it should be highlighted that the building envelope retrofits can be combined, and accumulate such that they may have a similar impact to a heating system upgrade. If all possibly combinable building envelope improvements (Air tightness, window replacement, ceiling $R_{SI}$-Value > 4 m²K/W, wall $R_{SI}$-value > 0.75 m²K/W and foundation $R_{SI}$-Value > 1 m²K/W,) are conducted a total energy consumption reduction of 41% is predicted.

Variable | Description
--- | ---
thermostat | Addition of a thermostat
e2G | Upgrade of an electric heating system to a newer electric heating system
E2G | Change from electric to gas fired heating system
E2O | Change from electric to oil fired heating system
G2E | Change from gas fired to electric heating system
G2G | Renewal of gas fired heating system
G2O | Change from gas to oil fired heating system
O2E | Change from oil fired to electric heating system
O2G | Change from oil to gas fired heating system
O2O | Renewal of oil fired heating system
GSHP | Change from any system to a ground source heat pump
e2ASHP | Change from electric furnace to air source heat pump
G2ASHP | Change from gas furnace to air source heat pump
O2ASHP | Change from oil furnace to air source heat pump
Upgrade | Renewal of air source heat pump
Air | Increasing air tightness of building, e.g. by fitting draft excluders.
Window | Replacing windows
CRSI 0-4 | Improving the ceiling insulation by an $R_{SI}$ value between 0 and 4
CRSI 4+ | Improving the ceiling insulation by an $R_{SI}$ value of more than 4
FRSI 0-1 | Improving the foundation insulation by an $R_{SI}$ value between 0 and 1
FRSI 1-2 | Improving the foundation insulation by an $R_{SI}$ value of more than 1
WRSI 0-0.75 | Improving the wall insulation by an $R_{W}$ value between 0 and 0.75
WRSI 0.75+ | Improving the wall insulation by an $R_{W}$ value of more than 0.75
that the carbon factor if British Columbia’s electricity grid is very low due to abundant hydro power, and these findings may not be the same for grids with a higher carbon factor.

**Reduction in energy costs**

The fundamental driver of energy costs are current fuel prices in Victoria as well as the effectiveness of the envelope and the efficiency of the heating system. Natural gas currently has the lowest cost and heating oil the highest cost per kWh; heat pumps have the highest efficiency of all heating systems. Based on this, the analysis of the results in the plot below are straight-forward. Changes from any system to a natural gas-fired system are estimated to reduce energy bills by at least 40% (electricity to gas) to 50% (oil to gas). The model suggests that installing a heat pump lowers bills by 24% (electric furnace to ASHP) to 38% (oil to ASHP). Two buildings which removed a gas system and installed an electric furnace instead suffered an increased energy bill of 61%. The reduction in bills by building envelope improvements are similar to the ones found for the reductions in energy demand.

**Retrofit sequence effects**

The order in which retrofits are applied to buildings can have an effect of the cost effectiveness of retrofits. The most obvious case is increasing envelope insulation and changing heating system type. If a building has poor insulation, it is going to require more heat through the year changing heating system type. If a building has poor insulation were added first, it would decrease demand, and reduce the fuel costs, and lowering the cost effectiveness of a heating system upgrade.

The effect is more complex when emissions are considered. Switching from a fossil fuel heating system to an electric based one could be much more cost effective in terms of emissions than upgrading insulation or windows once electrifying the heating system has taken place. This is mainly due to the carbon factor of electricity being very low, so the reduction in emissions due to envelope upgrades after heating system electrification is almost negligible. Energy reductions obtained through envelope upgrades are still desirable however.

**Model accuracy and prediction performance**

The quality of the fitted model may be assessed by its ability to predict the energy reduction of the 10% of buildings that were not included in the fitting process (see Methodology section). The mean absolute error (MAE) and the standard deviation (SD) are given in Table 2. These indicate how much the model prediction of the annual reduction (in energy, emission or cost) deviates from the actual annual reduction. For example, for predicting the energy reduction we obtained a mean absolute error of 6.3% +/- 5.0%. Hence, in 68% of the cases (assuming normally distributed errors) the absolute prediction error is between 1.3% and 11.4%.

The prediction performance of the model was significantly improved over the course of this study, predominantly by converting the values to be estimated to percentage changes, adding further variables (e2ASHP, g2ASHP, o2ASHP) and eliminating outliers from the data.

The MAE and SD remain reasonably similar between the fitting data (90% of samples) and the testing data (10% of samples). This implies that the model is not ‘over-fitted’ to reproduce the fitting data as well as possible but then failing to accurately predict new testing data. The similarity implies that this is the limit of how well a linear model of this nature can represent the data available. Improving on this would either require more data (a greater number of samples), or better data (giving more details on the nature of the buildings or the actions performed). The latter is likely to give the best improvements, since the standard error values are reasonable.

**Table 2: Model fitting results showing mean absolute error (MAE) and standard deviation (SD) for fitting and validation data for energy, emissions and cost models.**

<table>
<thead>
<tr>
<th></th>
<th>Fitting Data</th>
<th>Val. Data</th>
<th>Fitting Data</th>
<th>Val. Data</th>
<th>Fitting Data</th>
<th>Val. Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emissions reduction</td>
<td>4.98</td>
<td>5.01</td>
<td>6.33</td>
<td>5.42</td>
<td>6.02</td>
<td>5.51</td>
</tr>
<tr>
<td>Cost reduction</td>
<td>6.02</td>
<td>6.51</td>
<td>6.02</td>
<td>5.51</td>
<td>6.02</td>
<td>5.51</td>
</tr>
</tbody>
</table>

**Linear vs continuous variables**

The regression analysis was performed using binary variables as opposed to continuous variables for several reasons. Firstly the retrofit measures that were recorded were a mix between continuous and binary with the majority being binary. For example, heating system upgrade was binary whereas insulation R value was continuous. The continuous values were separated into levels (e.g. wall R value increased by 0 to 2 m²K/W, or 2 to 4 m²K/W or by more than 4 m²K/W); a binary variable was assigned to each level and the appropriate binary activated depending on the R value change that each entry performed.
Figure 1: Results of the multiple linear regression for energy consumption, cost of fuel and carbon emissions. Each column shows the percentage reduction due to that variable. Variable descriptions are given in Table 1.
Secondly, having the different levels of binaries for continuous retrofit measures also made it easier to determine if there were diminishing returns associated with different levels of that variable, whereas it could be more difficult to determine that with continuous variables due to the $p_i$ coefficient needing to be constant over the whole range. Effectively the use of binaries is capturing high-level non-linearities in the system at the expense of low-level precision.

Thirdly, the binary values may more accurately represent retrofit measures as they would be performed in reality. Wall R-value would not typically increase by 1.37 for example, but rather would be increased in discrete intervals determined by the way the construction materials are sold and installed. The discrete levels could represent separate consecutive applications of spray foam or layers of fiberglass batting. This could have practical advantages in applying this method and its results to creating municipal policy for retrofit incentives as it is simpler to communicate the requirements to residents or contractors. Interpreting the discrete variables is as simple as reading the number from the plot, whereas with a continuous variable it is necessary to account for the units of the factors before multiplying them by the result.

A comparison between using continuous variables to represent the continuous data and binary variables, as opposed to entirely discrete variables was performed on the retrofit data, to determine its effect on accuracy. Continuous variables were used for insulation R values for foundations, walls, and ceilings, as well as furnace efficiency, while the rest of the variables were left as binaries since the data only indicated if they were performed or not.

A comparison of the binary and continuous fitting results showed that there was little change in the accuracy of the MLR, with the $R^2$ value decreasing slightly when continuous variables were used (0.81 for binary, 0.77 for continuous). One potential reason for the similar accuracy is that although we used binary variables, we had previously discretized continuous data into brackets that were each represented with a binary variable. If a single binary was used to represent an entire continuous range of data then this would likely give much poorer accuracy.

**MAC curves**

The results of the multiple linear regression have been combined with cost data and scaled by the city building stock to produce MAC curves, which we present in the following two sections.

**Envelope and heating curves**

First, we give separate results for building envelope retrofits and for HVAC retrofits. These are presented separately because the HVAC options are dependent on both the initial heating system type and on the preferences of the building owner (e.g., in prioritizing cost reductions over emissions savings).

Figure 3 shows that nearly all the retrofits that can be performed on the building envelope have negative annual cost over their lifetimes, meaning that they will pay back in energy bill savings over this period. Figure 4 shows the MAC curve for heating systems. It shows that switching oil furnaces to electric or ASHP are the most cost effective carbon reduction options. The negative cost indicates that owners would save money by switching from oil to any other heating system. Likewise, switching from gas to electricity provides large carbon reductions, however due to the low price of gas there is a positive cost over the lifetime.

**Whole building stock results**

The MAC curves were then used to assess the cost effectiveness of different heating system retrofits applied to the City of Victoria residential building stock. This was done by estimating the proportions of residential buildings that had gas, oil, and electric heating systems according to utility connection data, BC. Ministry of Environment (2012).
The retrofit measures were then applied in these proportions to the total residential stock area. It is assumed that all building envelope items that have a negative cost will be implemented. Regarding the heating system retrofit, two different approaches are studied:

1. *Green* approach: Based on the results above the most emissions can be avoided if gas and oil furnaces are replaced by energy efficient air source heat pumps (expected emissions reductions of 78% and 79%). This scenario represents the CO$_2$ emissions that can be avoided if all carbon-intensive furnaces in Victoria are replaced by air source heat pumps.

2. *Cost-effective approach*: In this scenario those heating system retrofits are considered which offer the lowest abatement cost per kg CO$_2$ while providing significant CO$_2$ reductions. All gas furnaces and electric furnaces are replaced by air source heat pumps, while oil furnaces are changed to low cost gas fired heating systems. Note, that the only difference between the green and cost-effective approach is the change of oil furnaces to ASHPs instead of a change to gas furnaces.

The results for these scenarios are given in Figure 5. Total carbon emissions, equivalent annual costs and the initial investments are shown. Equivalent costs include the annualized initial investment using the current Canadian interest and inflation rates over 20 years, as well as savings from the lowering of energy bills.

The initial investment in heating system upgrades is expected to be 72M$ for the *cost-effective* approach (gas furnaces and air source heat pumps) and 90M$ for the *green* approach. The building envelope upgrades have an initial investment cost of 166M$. However, it has to be noted that the building envelope cost can be reduced if fewer measures (e.g. only wall insulation and air tightness upgrades, no ceiling or foundation insulation upgrades) are conducted. This is not possible for heating system upgrades as a full system must be purchased. This gives total initial costs of between 238 and 256M$. The estimated total annual emissions savings when the building envelope upgrades are combined with the *green* option for heating system upgrade is around 49,000 t CO$_2$. The equivalent annual costs are all negative which indicates a long-term cost saving by performing the retrofit scenarios through reduced energy bills.

The CO$_2$ abatement cost calculated in this study was compared to other MAC curves from nearby studies. The abatement costs range from $-14 to $-250 CAD$/tCO$_2$ compared to our value of $-210 (Municipality of North Cowichan (2013), Canadian Association of Petroleum Producers (2015), City of Toronto (2017), McKinsey & Company (2007)). The negative values indicate that money is saved. It is worth noting that those studies are performed for different spatial scales and specific retrofit measures performed were not well defined.

**LIMITATIONS AND FUTURE WORK.**

A limitation of this work is the assumption of linearity in retrofit measures and their effects. Namely that the effect of two retrofit measures together do not necessarily equal the sum of effects if they were implemented individually. We recognize that assuming linearity is not entirely accurate representation of reality. However in the absence of detailed building dimensions for creating physical models, the only other option is to do more complex machine learning and non-linear modeling methods, which become more and more “black box” with complexity. We want to use a simple method that is as “white box” as possible so that it can be understood and adopted by municipalities as a tool for meeting their emissions targets.

Another limitation is that the database that was used calculates primary energy use based on the output of a HOT2000 simulation of a model with the recorded building parameters. A database that uses has directly measured energy use values pre and post retrofit would be ideal.

Future work could include moving to a non-linear model or machine learning algorithm to analyze the effects of retrofit measures, to get around the assumption of linearity that is made for this analysis. It would be interesting to then compare the results.

**CONCLUSION**

In this paper a novel methodology for estimating stock-level energy use reductions for building retrofits is applied to a dataset for residential buildings in the City of Victoria.

The method uses multiple linear regression to estimate the amount of energy that each retrofit measure can save when applied to a building. The results of the MLR analysis are used to construct marginal abatement cost curves indicating the most cost effective and carbon saving measures. The MAC curves were then scaled by the residential building stock of Victoria to get an idea of the citywide potential for carbon reductions and the associated costs.

MLR is a relatively simple yet powerful tool that can be applied to datasets created from actual measurements from energy audits or simple simulations based on building surveys. The model was formulated using binary variables, with discrete intervals used to represent continuous data such as insulation R values. This resulted in a relatively quick set up and gives results that are simple to understand and use without post processing. A comparison was performed using the same dataset but with continuous variables where possible, and the results showed that there
Figure 3: MAC curve for building envelope retrofits.

Figure 4: MAC curve for heating system retrofits. The different types overlap since only one can be performed at a time, so it is not a true MAC curve, but the comparison between options is still useful.

Figure 5: Equivalent annual cost, initial investment and carbon emissions savings of retrofit scenarios 1 and 2.
was little change in accuracy, and even a slight decrease for the analysis using continuous variables. The results of the MLR analysis are then used to create MAC curves, one for building envelope retrofits, and another for heating system upgrades. These are then scaled by the number of residential buildings in the City of Victoria to get an estimate of the magnitude of energy and emissions savings that could be achieved if these measures were applied. If all combinable building envelope retrofits are performed, energy use could be reduced by as much as 40%. Switching heating system types from oil and/or gas to electric, preferably with an ASHP, can give significant reductions in emissions. If all gas and oil heating systems were changed to ASHP then emissions could potentially be reduced by up to 80%. Part of this is due to the efficiency of ASHPs, but it is also due to the low carbon intensity of grid electricity in BC. Even if oil and gas were converted to electric resistance heating, reductions of up to 60% are estimated.

This paper has demonstrated that multiple linear regression using binary variables is a powerful tool. It is relatively simple to use and produces results which are easy to interpret. It can be combined with MAC curves since both methods have the same assumptions. These methods can be very useful for practical applications such as municipal policy and planning.

REFERENCES


FortisBC, 2017., Vancouver Island Rates, Available: https://www.fortisbc.com/NaturalGas/Homes/Rates/VancouverIsland/Pages/default.aspx


