EVALUATION OF THE DESIGN LENGTH OF VERTICAL GEOTHERMAL BOREHOLES USING ANNUAL SIMULATIONS COMBINED WITH GENOPT

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ABSTRACT
Software tools to determine the design length of vertical geothermal boreholes typically use a limited set of averaged ground thermal loads and are decoupled from building simulations. In the present study, multiannual building hourly loads are used to determine the required borehole lengths. This is accomplished within TRNSYS using GenOpt combined with the duct ground heat storage (DST) model for bore fields.

INTRODUCTION
The determination of the required total borehole length in a bore field is an important step in the design of vertical ground heat exchangers (GHE) used in ground-source heat pump (GSHP) systems. Undersized GHE may lead to system malfunction due to return fluid temperatures that may be outside the operating limits of the heat pumps. Oversized heat exchangers have high installation costs that may reduce the economic feasibility of GSHP systems.

Figure 1 illustrates schematically a typical GSHP system. It consists of eight boreholes and five heat pumps connected in parallel. Piping heat losses between the boreholes and the heat pumps are usually assumed to be negligible. Thus, the inlet temperature to the heat pumps, \( T_{\text{in,hp}} \), is equal to the outlet temperature from the bore field \( T_{\text{out}} \). Heat pumps can operate with \( T_{\text{in,hp}} \) as low as \( \approx -7 \, ^\circ\text{C} \) in heating and as high as \( \approx 45 \, ^\circ\text{C} \) in cooling. However, most designers use a safety margin and try to limit \( T_{\text{in,hp}} \) to a value of \( T_L \approx 0 \, ^\circ\text{C} \) in heating and \( T_H \approx 35 \, ^\circ\text{C} \) in cooling modes. Boreholes are typically connected in parallel and the inlet temperature to all boreholes, \( T_{\text{in}} \), is equal to the outlet temperature from the internal heat pump fluid loop, \( T_{\text{out,hp}} \).

The ground thermal conductivity, \( k_g \), thermal diffusivity, \( \alpha_g \), and the undisturbed ground temperature, \( T_g \), are usually evaluated (or estimated in the case of \( \alpha_g \)) from a thermal response test (TRT) performed prior to the determination of the design length. The borehole thermal resistance (from the fluid to the borehole wall), \( R_b \), can also be estimated from a TRT test or calculated from borehole heat transfer theory (Bennet et al., 1987).

As shown in Figure 1, the bore field geometry is characterized by the number of boreholes, \( N_b \), the borehole length, \( H \), the borehole spacing, \( B \), and the buried depth of the boreholes, \( D \). Each borehole has a radius \( r_b \) (not to be confused with \( R_b \), the borehole thermal resistance). As shown in section A-A, each borehole has two pipes with a radius \( r_p \) and a center-to-center distance equal to \( d_p \). For typical boreholes, \( H \) varies from 50 to 150 m. For such long boreholes, the value of \( D \) has minimal effects on borehole heat transfer. In this work, it is assumed that \( D = 1 \, \text{m} \) and it is not considered to be a factor to determine the design length.

Designing a GHE consists in finding the “optimum” combination of \( H, N_b \) and \( B \) such that the inlet temperature to the heat pumps doesn’t go below the minimum value of \( T_L \) or above the maximum value of \( T_H \). In this work, the optimum combination is the one leading to the smallest overall length \( L (= N_b \times H) \).

In the first part of the paper, the basic design methodologies used in typical software tools are examined and categorized into five levels of increasing complexity. Then, the DST model and GenOpt are briefly reviewed. This is followed by the proposed methodology to obtain the optimum design length. The objective function involves the length and number of boreholes and \( T_L \) and \( T_H \) are considered as constraints. Contrary to most sizing methods, the heat pump
Coefficient of Performance (COP) is considered variable and so the ground loads are determined iteratively. Finally, the proposed methodology is applied and compared to other design software tools in three test cases.

REVIEW OF DESIGN METHODOLOGIES

Spitler and Bernier (2016) categorized GHE sizing methodologies into five levels (0 to 4) of increasing complexity. The proposed methodology fits into the “level 4” category. These various levels will now be described with an emphasis on level 4.

Level 0 – Rules-of-Thumb

Rules-of-thumb relate the length of GHEs to the largest heating or cooling loads of the building or to the installed heat pump capacity. One popular rule-of-thumb in North America is to determine the length based on the simple formula: 150 feet of bore per ton of installed capacity (13 m of bore per kW of installed capacity). In the United Kingdom, look-up tables are used to obtain the maximum power that can be extracted per unit length for various ground conditions (Department of Energy and Climate Change, 2011). For example, for \( k_g = 2.5 \text{ W/m-K} \) and \( T_g = 12 \text{ °C} \), the recommended maximum power extraction is 50 W/m for 1200 hours of equivalent full load operating hours. The main problem with rules-of-thumb is that they only rely on peak loads and do not account for annual ground temperature increases (decreases) caused by load thermal imbalances.

Level 1 – Two ground load pulses

In level 1 methods, two lengths are calculated based on peak heating and cooling loads. Kavanaugh (1991) introduced a borehole thermal resistance in the analysis as well as an approximate factor to account for borehole-to-borehole thermal interference. Furthermore, the concept of temperature limits (\( T_P \) and \( T_H \) ) is introduced. Despite these improvements, level 1 methods suffer from the same problem as level 0 methods as they do not properly account for the effects of ground load thermal imbalances.

Level 2 – Two set of three ground load pulses

The three pulse methodology (3 pulses in heating and 3 in cooling) is introduced by Kavanaugh (1995) along with the concept of temporal superposition which leads to the development of Eq.1. In order to keep the analysis simple, the borehole thermal resistance is assumed to be negligible and so it has been eliminated from the equation (it will be reintroduced later). In this equation, \( L \) is the overall borehole length (= \( H \times N_g \) ), \( k \) is the ground thermal conductivity, \( T_f \) is the mean fluid temperature (= \( \frac{[T_{in,hp}+T_{out,hp}]/2} \) ) and \( T_g \) is the ground temperature. The temperature penalty, \( T_P \), accounts for the borehole-to-borehole thermal interference (Bernier et al., 2008). \( Q_1 \), \( Q_2 \), and \( Q_3 \) are three consecutive ground load “pulses” with time durations \( t_1 \), \( t_2 \), \( t_3 \). The values of \( t'_1 \), \( t'_2 \) and \( t'_3 \) are equal to \( t_1 + t_2 \), and \( t_1 + t_2 + t_3 \), respectively.

Finally, the function \( R_P \) is the thermal response of the ground which can be evaluated using several techniques. Figure 2 shows schematically three typical heat load pulses and their durations.

\[
L = \frac{1}{k} \frac{Q_1 \Gamma_2 (t'_2 - 0) + (Q_2 - Q_1) \Gamma_3 (t'_3 - t'_2) + (Q_3 - Q_2) \Gamma_4 (t'_4 - t'_3)}{T_f - (T_g + T_p)} \tag{1}
\]

In level 2 methods, the ground load pulses and their duration are typically assumed to be as follows: \( Q_1 \) is the annual amount of heat rejected (or collected) into the ground over a period \( t_1 \) (typically this period is 10 years). This is followed by a monthly ground load pulse \( Q_2 \) with a duration \( t_2 \) of one month. Finally, an “hourly” ground peak pulse \( Q_3 \) with a duration of \( t_3 = 6 \) hours is applied.

Building loads from energy simulation tools are generally not available in level 2 methods. Instead, peak building loads are calculated and the resulting peak ground loads, \( Q_3 \) are estimated. This estimation is based on a heat pump COP evaluated at peak conditions for \( T_{in,hp} = T_H \) in cooling and \( T_L \) in heating modes. Then, the average monthly ground load during the peak month, \( Q_2 \), is estimated assuming a run time fraction for the heat pump during that month. Finally, the average annual ground load, \( Q_1 \), is estimated. This can be done using the concept of equivalent full load heating and cooling hours as described by Spitler and Bernier (2016). It is to be noted that \( Q_1 \) and \( Q_2 \) are based on assumed constant COP values because the evolution of \( T_{in,hp} \) is unknown in level 2 methods. These calculations are performed for both heating and cooling modes giving two sets of three ground load pulses. The end result of these calculations is illustrated in Figure 3a for a case where the building is mainly in cooling mode. In this example, \( Q_1 \), \( Q_2 \), and \( Q_3 \) have the
following values: 19.2, 41.9, and 139.7 kW. These values are used in a test case to be examined shortly.

Figure 3: Typical ground loads related to level 2 sizing methods and the variation of $T_{out}$ related to these loads

Eq. 1 forms the basis of the ASHRAE sizing method (ASHRAE, 2015) where $r_G$ is based on the analytical solution to ground heat transfer referred to as the infinite cylindrical heat source or ICS (Bernier, 2000). Approximate values of $T_p$ are tabulated for typical cases in the ASHRAE handbook (ASHRAE, 2015). Since the ICS and the tabulated values of $T_p$ are independent of $H$, then $L$ can be determined directly without iterations. The resulting value of $L$ is the length required for the outlet temperature from the borehole, $T_{out}$, to remain below $T_H$ (or above $T_L$ for a heating case). This is illustrated in Figure 3b. This figure shows the evolution of $T_{out}$ for the three ground load pulses of Figure 3a and shows that $T_{out}$ reaches $T_H$ at $t_3$.

If more precise values of $T_p$ are required in Eq. 1, then the approach suggested by Bernier et al. (2008) should be used. However, in this case $T_p$ depends on $H$, and iterations are required to determine $L$. Typically, 3-4 iterations are necessary before convergence. This can be done for rectangular bore fields automatically and rapidly with the Excel spreadsheet developed by Philippe et al. (2010).

As suggested by Ahmadfard and Bernier (2014), the thermal response $r_G$ in Eq. 1 can also be based on Eskilson’s $g$-function (Eskilson, 1987). With this approach the temperature penalty, $T_p$, is no longer needed and Eq. 1 takes the following form where $r_G$ is determined by the $g$-function of the bore field:

$$L = \frac{1}{2\pi k} \frac{Q_1 r_G(t_1' - 0) + (Q_2 - Q_1) r_G(t_2' - t_1') + (Q_3 - Q_2) r_G(t_3' - t_2')} {t_f - t_g}$$

(2)

Since $g$-functions depend on $H$, an iterative process is required. This process can be computationally intensive if $g$-functions need to be evaluated during the process. Pre-calculated $g$-functions can be used to reduce computational time.

If average and peak ground heat loads are available for each month, Eqs. 1 or 2 can be modified to size the ground heat exchangers based on these pulses. In such cases, as suggested by Monzó et al. (2016), the monthly average and peak heat pulses can be rearranged into a format similar to Eqs. 1 and 2 as shown in Eqs. 3 and 4 where the overall length is evaluated for each month $j$:

$$L_j = \frac{1}{k} \frac{Q_1 r_G(t_1' - 0) + (Q_2 - Q_1) r_G(t_2' - t_1') + (Q_3 - Q_2) r_G(t_3' - t_2')} {t_f - t_g}$$

(3)

$$L_j = \frac{1}{2\pi k} \frac{Q_1 r_G(t_1' - 0) + (Q_2 - Q_1) r_G(t_2' - t_1') + (Q_3 - Q_2) r_G(t_3' - t_2')} {t_f - t_g}$$

(4)

where $Q_{1,j}$ is the average of monthly heat loads, from the 1st month to the $j-1$ month, $Q_{2,j}$ is the monthly average heat load of month $j$, and the $Q_{3,j}$ is the cooling or heating peak of month $j$. $t_1' = t_1$, $t_2' = t_1 + t_2$, $t_3' = t_1 + t_2 + t_3$ and $t_4' = t_1 + t_2 + t_3 + t_4$ where $t_{1,j} = \sum_{m=1}^{j-1} t_{m,j}$, $t_{2,j} = t_{h,j} - t_{m,j}$ is the duration of the month $i$ and $t_{h,j}$ is the duration of the cooling or heating peak for month $i$.

Similar to previous cases, the effective borehole thermal resistance is neglected in these equations. Also, the duration of the peak heat loads, $t_{h,j}$, is usually considered to be 6 hours regardless of their actual duration. Since $T_{p,j}$ in Eq. 3 and the ground thermal response in Eq. 4 depend on $H_j (=L_j/N_p)$ which is unknown, $L_j$ is determined iteratively. The maximum value of $L_j$ would be selected as the overall length of the ground heat exchangers.

Figure 4 illustrates schematically this concept with six monthly loads and six peak heat load pulses and their durations (bottom portion of the graph). They are summarized into three heat load pulses $Q_{1,j}$, $Q_{2,j}$ and $Q_{3,j}$ (top portion of the graph).

Figure 4: Evaluation of three ground heat load pulses and their durations for month $j$

Level 3 – Monthly average and peak heat pulses

Unlike level 2 methods which use three heat load pulses for the determination of borehole length, level 3 methods use a succession of monthly loads each
followed by a peak load at the end of the month. This results in $24n$ terms where $n$ is the number of years covered by the analysis. In level 2 methods, a fixed value of $T_p$ is used to obtain $L$ directly (or with some iterations if improved values of $T_p$ are used). However, in level 3 methods, $L$ is fixed and $T_p$ is evaluated after each of the $24n$ ground heat pulses.

Eqs. 5 and 6 show the governing equations for level 3 methods. In this case, $T_p$ is evaluated at the end of a 10 year analysis. Therefore, the numerator consists of a summation of 240 terms. Eq. 5 is used when the ground thermal response is given by the ICS while Eq. 6 is used when g-functions are used.

$$T_f = (T_p + T_0) + \frac{Q_0r d(t_{240} - t_0) + (Q_2 - Q_0)c(t_{240} - t_0) + (Q_2 - Q_0)r d(t_{240} - t_0) + \ldots + (Q_{239} - Q_{239})r d(t_{240} - t_0) + (Q_{239} - Q_0)(t_{240} - t_0)}{2.4kL}$$

$$T_f = T_p + \frac{Q_0r d(t_{240} - t_0) + (Q_2 - Q_0)c(t_{240} - t_0) + (Q_2 - Q_0)r d(t_{240} - t_0) + \ldots + (Q_{239} - Q_{239})r d(t_{240} - t_0) + (Q_{239} - Q_0)(t_{240} - t_0)}{2.4kL}$$

In Eqs. 5 and 6, $t_n' = \sum_{i=1}^n t_i$ and $t_i$ is the duration of load $Q_t$.

Figure 5 presents schematically a sub-set of Eq. 5 and 6 with six monthly loads and six peak heat load pulses and their durations. The values of $t_n'$ are also shown.

Eqs. 5 and 6 form the basis of some software design tools such as EED (Hellström and Sanner, 1997 and BLOCOM, 2015) and GLHEPRO (Spitler, 2000). Both of these tools use g-functions to calculate the ground thermal response factors. In such tools, ground loads can either be calculated separately or iteratively within Eqs. 5 or 6. In the former case, approximate values of the heat pump COP are used to calculate ground loads based on building loads. In most cases, two constant COP values (heating and cooling) are used. When ground loads are calculated iteratively within Eqs. 5 or 6, it is possible to use time-varying COPs as values of $T_{in, hp}$ are available during the calculation process.

Figure 6 shows an example of the various steps involved in a level 3 method. Figure 6a shows the building load variations used in the example for the 1st year of operation. It consists of 12 monthly average and 12 peak heat loads of a building with a cooling dominated load. These loads can be obtained by post-processing hourly loads obtained from a building simulation software tool or by simple approximations based on peak load calculations. Figure 6b shows this load over the 10 year period of the analysis. Figure 6c and 6d present, respectively, the heating and cooling COP values used to convert the building loads into ground loads. In this particular case, constant values are used. Figure 6e shows the resulting series of ground loads to be used in Eqs 5 and 6.

Eqs. 5 and 6 cannot be solved directly as $T_p$ (in the case of Eq. 5) or g-functions (for Eq. 6) depend on $L$. Instead, a value of $L$ is guessed and Eqs. 5 or 6 are solved to obtain $T_{out}$ at the end of each individual time period (there are 240 evaluations of $T_{out}$ in the case of a 10 year analysis). If $T_{out}$ is outside the bounds fixed by $T_H$ and $T_L$ then a new value of $L$ is chosen. This process is repeated until $T_{out}$ reaches either $T_H$ or $T_L$. For the case presented in Fig. 6, the evolution of $T_{out}$ for the final iteration is presented in Fig. 6f where it is shown that $T_H$ is reached in the summer of the 10th year.

If COPs are assumed to vary with $T_{in, hp}$ then there is a second iterative process that goes through steps illustrated in Fig. 6b to 6f for each of the $24n$ ground heat pulses until COP values converge.
Level 4 – Hourly simulations

Level 4 methods push the level of granularity to hourly values. Thus, in essence, level 4 methods use equations similar to Eqs. 5 or 6, but with 87600 terms (in the case of a 10 year analysis). Level 4 methods should be more accurate but are also more computationally intensive. As in level 3 methods, an initial value of \( L \) is guessed and equations similar to Eqs. 5 or 6 are solved to obtain \( T_{out} \) at the end of each hour of the analysis period. If \( T_{out} \) is outside the bounds fixed by \( T_H \) and \( T_L \), then a new value of \( L \) is selected and a new iteration is initiated. This process is repeated until \( T_{out} \) reaches either \( T_H \) or \( T_L \) during the analysis period.

The solution process in a level 4 method is shown schematically in Figure 7 for the final iteration. Fig. 7a shows the hourly building loads for the first year. The same building load is repeated for 10 years in Fig. 7b. Fig. 7c and 7d show the calculated COP values used to obtain the ground loads which are shown in Fig. 7e. Finally, Fig. 7f shows the hourly variations of \( T_{out} \) (=\( T_{in,bp} \)) for the final iteration. This figure will also be used later in conjunction with test case 2.

Level 4 methods have been developed in recent years. For example, Nagano et al. (2006) have developed a sizing tool which applies the ICS to determine the hourly changes of temperature in the ground and in the heat carrier fluid. The model can also evaluate hourly variations of the ground load and COP. Henault et al. (2016) have suggested a method which predicts and optimizes the performance of hybrid GSP systems on an hourly basis. This method uses spectral-based simulation tool (Pasquier et al., 2013) that relies on g-functions and is applicable to variable heat pump COPs. The method gives the optimal number and location of vertical boreholes, the optimal number of heat pumps and their operating temperature limits as well as the optimal energy savings based on the financial performance of the system.

In a level 4 method, it is necessary to evaluate bore field heat transfer for each hour. One of the most widely used tools for such a task is the DST (Duct ground heat storage) model (Hellström, 1989), which has been implemented in the TRNSYS simulation environment. The DST model is not a GHE sizing program. However, it could be used repeatedly by changing borehole lengths “manually” until the temperature limits are reached. The objective of this paper is to show how to automate this process in combination with GenOpt.

Before addressing the proposed methodology, the DST model and GenOpt are briefly presented.

DST MODEL IN TRNSYS

The vertical U-tube ground heat exchanger model in TRNSYS is type 557 and is based on Hellström’s DST model (Hellström, 1989). It was originally developed to simulate seasonal thermal storage of densely packed boreholes in an axisymmetric cylindrical configuration with a given bore field volume, \( V_{BTES} \). Chapuis and Bernier (2009) have provided a description of the calculations involved in the DST model. In summary, the DST model divides the ground formation into two parts: the local region around a single borehole and the global region located between the boundary of the storage volume and the far-field radius. The model uses a one dimensional numerical model to solve for the ground temperature in the local region and a two-dimensional finite difference model to simulate the ground temperature in the global region.

One of the very important limitations of the DST model has to do with borehole locations. The user selects \( H \), \( N_b \) and one of the following two parameters: \( V_{BTES} \), or \( B \). The remaining fourth parameter is calculated internally by the DST model using Eq. 7.

\[
V_{BTES} = \pi \times 0.525^2 \times B^2 \times H \times N_b \tag{7}
\]

Figure 8 shows the borehole layout for a 37 borehole configuration and the origin of the value of 0.525 in Eq. 7. In this study, the borehole locations are evaluated based of this configuration.
**TRNOPT AND GENOPT TOOLS**

GenOpt (Wetter, 2011) is a generic optimization program that was developed to address the class of optimization problems generally faced by Building Performance Simulation (BPS) users: a scalar cost function (e.g. the energy cost) must be minimized, the cost function (i.e. the BPS tool) is computationally intensive, and its derivatives are not known. The program offers a wealth of optimization algorithms capable of dealing with continuous and discrete variables, and uses multithreading when the algorithms allow it. Simple constraints (bounds) can be placed on optimization parameters, and more complex constraints can be implemented through penalty functions. The tool interfaces with BPS tools through text files, which makes it compatible with virtually all (batch-mode capable) BPS tools. GenOpt is also configured through text files. The “initialization file” defines the optimization problem at a high level (e.g. which simulation tool is used to evaluate the cost function, where to find the results, what to save, etc.). The “configuration file” provides specific information to interact with the BPS tool (e.g. the batch command to launch the program), and the “command file” provides detailed information on the optimization problem (variables, bounds, stopping criteria, and optimization algorithm). A “template” input file for the BPS tool must also be provided; where optimization parameters are replaced with special codes (e.g. “L” would replace the borehole length). GenOpt provides optimization results in a log file and also through a simple user interface that displays a graph of the current optimization parameters and cost function.

Configuring a GenOpt problem requires editing the 3 text files mentioned above, which can be intimidating and error-prone for non-expert users. To simplify this task, a dedicated GenOpt interface was created for TRNSYS. The interface, known as TRNOPT (TESS, 2014), automatically generates the text files required by GenOpt and also edits the TRNSYS input file (known as the deck file) to insert the special codes recognized by GenOpt to designate the optimization parameters. The interface also makes it easier to define the nature of the optimization variables (continuous or discrete, bounds and initial values, etc.) and the selected algorithm. Finally, TRNOPT comes with a dedicated “printer” component for TRNSYS that writes the value of the cost function in a text file easily interpreted by GenOpt.

The strength of GenOpt lies in its generic nature, which makes it capable of reading and creating text files, and in the powerful optimization algorithms implemented in the program. However, selecting and configuring an algorithm for a specific problem requires some expertise and/or trial and error to avoid or minimize the impact of convergence issues, e.g. local minima or truncation of digits in the cost function. TRNOPT helps with the mechanics of setting up a GenOpt optimization problem with TRNSYS, but it does not reduce the difficulty of finding meaningful results when addressing a complex, multivariable optimization problem in the context of Building Performance Simulation.

**PROPOSED METHODOLOGY**

A level 4 sizing method is proposed in this section. The objective is to find the required borehole length iteratively using the DST model combined with GenOpt in the TRNSYS environment.

Equation 8 shows the objective function that is applied in this work. This objective function can be used for minimizing only the height $H$ for a given value of $N_b$ or for minimizing the product of $H \times N_b$. It forces the optimization to find cases that satisfy the design temperature limits $T_L < T_{out} < T_H$. In Eq. 8, $T_{out,min}$ and $T_{out,max}$ are, respectively, the minimum and maximum outlet fluid temperatures from the bore field occurring during the selected period of the optimization.

$$\begin{align*}
\min \text{Obj} &= H \cdot N_b + C \times (P_1 + P_2) \\
P_1 &= [(T_L - T_{out,min})^T(\frac{1}{T_{out,min}} T_L)] \\
P_2 &= [(T_{out,max} - T_H)^T(\frac{1}{T_{out,max}} T_H)]
\end{align*}$$

When $T_{out}$ satisfies the temperature limits, this function reduces to $H \cdot N_b$. However, if the maximum fluid temperature, $T_{out,max}$, goes above $T_H$ or the minimum fluid temperature, $T_{out,min}$, is below $T_L$ then penalties equal to $C \times (T_{out,max} - T_H)$ or $C \times (T_L - T_{out,min})$ are applied to the objective function. $C$ is a large coefficient assumed equal to $10^9$ in the present work. The penalty is thus proportional to the temperature differences $(T_{out,max} - T_H)$ or $(T_L - T_{out,min})$. In this way, simulations that lead to
\( T_{\text{out,max}} \gg T_H \) (or \( T_{\text{out,min}} < T_L \) ) will be heavily penalized.

In Eq. 8, the expressions \( LT(T_{\text{out,min}},T_L) \) and \( GT(T_{\text{out,max}},T_H) \) are “lower than” and “upper than” functions and are equal to 1 respectively when \( T_{\text{out,min}} < T_L \) and \( T_{\text{out,max}} > T_H \). For other values these functions are zero. It should be noted that only one of these two conditions may occur as the system can be either be sized in heating or cooling.

**IMPLEMENTATION IN TRNSYS**

Figure 9 presents schematically the optimization procedure implemented in TRNSYS as well as some of the important scripts. This procedure can be used for optimizing one or all three design parameters \((H, N_b, \text{and } B)\). Numerical values used in the script refer to test cases presented in the next section. There are five blocks \((0 \text{ to } 4)\) of models and each will now be briefly described.

In blocks 0 to 2, the input parameters \( H, N_b, \text{and } B \) are specified as well as the temperature design limits and the heat pump COPs. A parameter \((COP_{\text{cond}})\) is also defined to indicate whether a constant or a temperature-dependent COP is used. The storage volume, \( V_{\text{BTES}} \), is also evaluated (see Eq. 7). Pre-calculated hourly building loads, \( q_{gb} \), are read by a standard data reader (Type 9a). A building model (Type 56) could also be used to determine the building loads. Next, the building loads are converted into ground loads, \( q_{gb} \), based on \( COP_{\text{cond}} \). Note that if temperature-dependent COPs are used, they are evaluated each hour based on a performance map giving COPs as a function of \( T_{\text{out}} \) (see \( COP_{H-2} \) and \( COP_{C-2} \) in Figure 9a).

In block 3, ground heat transfer is evaluated using the DST model. The inlet and outlet fluid temperatures of the bore field are evaluated. If the COPs are temperature dependent, then TRNSYS will iterate at each time step until converged values of the fluid temperature, COPs, and ground loads are obtained.

The objective function is evaluated in block 4. At each time step, the outlet fluid temperature from the bore field is checked against the recorded values (Type 55) of \( T_{\text{out,min}} \) and \( T_{\text{out,max}} \) obtained up to that time step. If \( T_{\text{out}} \) is lower than \( T_{\text{out,min}} \) or higher than \( T_{\text{out,max}} \) then these parameters are updated with the value of \( T_{\text{out}} \) at the current time step. At each time step, the objective function is also calculated. The values of \( T_{\text{out,min}} \) and \( T_{\text{out,max}} \) are compared to \( T_L \) and \( T_H \), respectively, and a penalty is applied according to Eq. 8. Then, the calculated objective function is compared with its recorded values. If it is higher than its recorded values it is updated.

At the end of the simulation period, the maximum recorded value of the objective function is sent to GenOpt, which analyzes the value of the objective function and updates the value of the design input parameters. Then, a new multi-year simulation is initiated. This process is repeated until the objective function converges to the user-specified tolerance or the iteration reaches its maximum number.
APPLICATIONS

Three sizing test cases are considered in this section to show the applicability of the proposed method and to compare results with other sizing methods. The reported calculation times are for a computer equipped with an Intel core i7 processor (2.80 GHz) and 4 GB of RAM.

Test case #1

The first test case is somewhat academic as it involves a perfectly balanced ground load. This case involves the optimization (i.e. minimization) of $H$ for a 37 borehole configuration. The load profile is shown in Figure 10. This load is obtained using the methodology proposed by Bernier et al. (2004) with the following parameters: $A=75000$, $B=2190$, $C=80$, $D=2$, $E=0.01$, $F=0$ and $G=0.95$. On a yearly basis, the amount of heat collected in the summer is exactly equal to the amount of heat collected in the winter. This implies that the temperature penalty, $T_p$ - due to borehole-to-borehole interference is zero. In addition, the ground temperature is chosen to be exactly at the midpoint between the values of $T_L$ and $T_H$. So, in theory, the required borehole length should be the same for heating and cooling.

![Figure 10: Ground load used for test case #1](image)

Since ground loads are given directly, COPs do not need to be calculated. Therefore, block 2 (in Figure 9) is omitted.

Table 1 gives the value of the design parameters for test case #1. The coordinate search optimization method was selected based on a trial and error procedure which indicated that it required the smallest number of iterations. Table 2 shows the corresponding GenOpt parameters. The extent of the search domain is selected to be between 30 to 200 m with initial steps of 10 m.

| Table 1: Design parameters used for test case #1 |
| Parameter | Value |
| Borehole radius ($r_b$) | 75 mm |
| Borehole spacing ($B$) | 7 m |
| Pipe nominal radius ($r_p$) | 25 mm |
| Number of pipes in each borehole | 2 |
| Ground thermal conductivity ($k_g$) | 1.9 W/m.K |
| Undisturbed ground temperature ($T_0$) | 17.5°C |
| Minimum temperature limit ($T_L$) | 0°C |
| Maximum temperature limit ($T_H$) | 35°C |
| Flow rate ($m$) | 5.55 kg/s |
| Fluid specific heat ($C$) | 4 kJ/kg.k |

| Table 2: Parameters used in optimization method for test case #1 |
| Parameter | Value |
| Mesh size divider | 2 |
| Initial mesh size exponent | 0 |
| Mesh size exponent increment | 1 |
| Maximum number of step reductions | 4 |

Table 3 presents the evolution of $H$ calculated by GenOpt as well as the corresponding values of the objective function. The iterative process starts with an initial length of 40 m and converges to a final borehole length of 67.5 m and an overall bore field length of 2497.5 m. This is done in 13 iterations (i.e. 13 10-year simulations) and the calculation time is 3.5 minutes. For iterations 1, 2, 3, 7, 10, and 12, it can be seen that the guess values for $H$ leads to high values of the objective function caused by values of $T_{out}$ going beyond $T_H$ or $T_L$.

Further optimization tests were performed with a smaller step size (2 m) and with two starting points ($H=40$ or $H=200$ m). As shown in Figure 11 the end result is the same whether $H=40$ or $H=200$ m are used as starting points. This graph shows a global minimum of 2471.6 at 66.6 m, a value close to the one obtained above for a coarser step size. However, this required 85 iterations.

| Table 3: Borehole lengths determined by GenOpt and the corresponding objective function for test case #1 |
| No. | H (m) | Obj. |
| No. | H (m) | Obj. |
| 1 | 40 | 2.82E+10 |
| 2 | 50 | 1.42E+10 |
| 3 | 60 | 4.91E+09 |
| 4 | 70 | 2590 |
| 5 | 80 | 2960 |
| 6 | 75 | 2775 |
| 7 | 65 | 1.33E+09 |

![Figure 11: Plot of the objective function for test case #1](image)
profile for $T_{\text{out}}$ of the 1st year (and also for the 10 year duration) is equally close to both $T_L$ and $T_H$. This shows that both constraints are satisfied and, as expected with balanced ground loads and a ground temperature exactly equal to the mean of $T_L$ and $T_H$, the borehole length required is exactly the same in heating and cooling. Thus, the proposed method is able to handle perfectly balanced loads.

This problem is solved using level 2 and 3 methods as well as the proposed level 4 methodology. For levels 2 and 3, the building load is converted into ground loads with two constant COPs of 3.86 and 4.03 for cooling and heating, respectively. The ground loads for the proposed methodology are evaluated iteratively using temperature dependent COPs which are evaluated based on the performance map given in Table 4.

### Test case #2

For the second test case, the proposed methodology is used to determine the required borehole length of a GSHP system for a cooling dominated 1500 m² building located in Atlanta (Bernier, 2006). Figure 7a shows the evolution of hourly building load for the 1st year. It is assumed that the operation starts on January 1st and that the same yearly load is repeated over a ten year period as shown in Figure 7b.

The design parameters are similar to the ones that are used for test case #1 and reported in Table 1. The only differences are the undisturbed ground and the ambient (surface) temperatures which are both set at 15 °C and $T_H$ which is set equal to 38 °C.

This test case is solved using level 2 and 3 methods as well as the proposed level 4 methodology. For levels 2 and 3, the building load is converted into ground loads with two constant COPs of 3.86 and 4.03 for cooling and heating, respectively. The ground loads for the proposed methodology are evaluated iteratively using temperature dependent COPs which are evaluated based on the performance map given in Table 4.

### Table 4. Performance map for the heat pump used in test case #2

<table>
<thead>
<tr>
<th>$T_{\text{in,HP}}$ (°C)</th>
<th>COP$_H$</th>
<th>$T_{\text{in,HP}}$ (°C)</th>
<th>COP$_H$</th>
<th>$T_{\text{in,HP}}$ (°C)</th>
<th>COP$_C$</th>
<th>$T_{\text{in,HP}}$ (°C)</th>
<th>COP$_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.57</td>
<td>4.42</td>
<td>26.67</td>
<td>6.04</td>
<td>15.56</td>
<td>4.35</td>
<td>8.16</td>
<td>3.96</td>
</tr>
<tr>
<td>4.44</td>
<td>4.4</td>
<td>29.44</td>
<td>6.2</td>
<td>21.11</td>
<td>6.51</td>
<td>43.33</td>
<td>3.31</td>
</tr>
<tr>
<td>2.00</td>
<td>4.85</td>
<td>32.22</td>
<td>6.35</td>
<td>26.67</td>
<td>6.89</td>
<td>48.89</td>
<td>2.75</td>
</tr>
<tr>
<td>15.56</td>
<td>5.29</td>
<td>29.44</td>
<td>5.38</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Level 2 results

The building load shown in Figure 7a has a cooling peak $Q_p=111$ kW during the month of July. With a COP of 3.86 in cooling this gives a peak ground load $Q_3$ of 139.7 kW. One common assumption is to assume that the average monthly ground load is equivalent to 30% of the peak load during that month. Thus, $Q_3 = 0.3 \times Q_3 = 0.3 \times 139.7 = 41.9$ kW. To obtain $Q_1$, the annual average ground load, the concept of equivalent full load hours is used (Carlson and Thornton, 2002). For this type of building located in Atlanta, the equivalent full load hours of operation in heating and cooling are approximately, $EFLH_H = 500$ h and $EFLH_C = 1500$ h, respectively. This gives a value for $Q_1$ of 19.2 kW. The duration of $Q_1$ is considered to be 6 hours. This problem can be solved by using the spreadsheet developed by Philippe et al. (2010) with an appropriate value of $T_p$ for the axisymmetric geometry (Figure 8). The required length is calculated using Eq. 1:

$$ L = \frac{19.2 r_c(88350 – 0) + (41.9 – 19.2) r_c(88350 – 87600)}{139.7 r_c} \times \frac{1}{1.9} $$

It should be noted that in this problem the borehole resistance is not zero, so a $Q_3 R_p$ term is added in Eq. 9 (with $R_p = 0.2$ mK/W). The only unknown in this equation is $T_p$. It depends on the borehole length and it needs to be evaluated iteratively. Using the methodology suggested by Bernier et al. (2008), it can be shown that $T_p$ is equal to $+6.1$ °C for a 10 year period. Solving Eq. 9, gives an overall length of 3185.8 m and a borehole length of 86.1 m. This calculation involves 6 iterations (to calculate $T_p$) with a convergence criteria on $L$ set at 0.1%. This problem is solved in 35 seconds.

### Level 3 results

The hourly building load profile given in Figure 7a is converted to 12 average monthly loads, $Q_m$, and $2 \times 12$ hourly cooling, $Q_{h,c}$, and heating, $Q_{h,H}$, peak heat loads. These loads are listed in Table 5. These building loads are further converted into ground loads assuming constant COPs for heating and cooling. This procedure is also illustrated in Figs 6a to 6e. Then, for each month, two lengths, one based on cooling peaks and another based on heating peaks, are evaluated using Eq. 3. Then, the maximum of the 240 lengths is selected as the final required length.
The results show that the required overall bore field length is 2848 m with individual borehole lengths equal to 77.0 m. The temperature penalty is also equal 6.9°C. The calculation time is approximately 2.5 minutes using the same convergence criteria and the same computer used in the level 2 results.

Level 4 results (proposed methodology)

Results are obtained using the same optimization method as in test case #1. COPs are evaluated each hour based on the calculated value of $T_{\text{out}}$ for the corresponding hour. This implies some iterations within TRNSYS at each time step. Similar to test case #1, the length of each borehole is obtained in the search domain (from 30 to 200 m) with initial steps of 10 m. Results are presented in Table 6. The iteration process starts with a length of 40 m and converges to the borehole length of 75 m in 14 iterations with a calculation time of 5 minutes. The optimization process was also run with a smaller step size (2 m) with two starting points ($H=40$ or $H=200$ m). Figure 13 shows the shape of the objective function for various borehole lengths. This graph shows a global minimum for an objective function of 2765.8 at a corresponding borehole length of 74.8 m. This process required 82 iterations.

![Figure 13](image1.png)

**Figure 13.** The objective function of the second test case.

Table 5. Monthly average and peak building heating and cooling loads used in level 3 for test case #2

<table>
<thead>
<tr>
<th>Period</th>
<th>$Q_{\text{in}}$ (kW)</th>
<th>$Q_{\text{out,max}}$ (kW)</th>
<th>$Q_{\text{out,min}}$ (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan.</td>
<td>-10.6</td>
<td>28.4</td>
<td>-86.4</td>
</tr>
<tr>
<td>Feb.</td>
<td>-5.0</td>
<td>42.5</td>
<td>-80.8</td>
</tr>
<tr>
<td>Mar.</td>
<td>6.4</td>
<td>68.0</td>
<td>-57.9</td>
</tr>
<tr>
<td>Apr.</td>
<td>16.8</td>
<td>74.3</td>
<td>-50.3</td>
</tr>
<tr>
<td>May.</td>
<td>27.8</td>
<td>95.9</td>
<td>0</td>
</tr>
<tr>
<td>Jun.</td>
<td>34.7</td>
<td>104.0</td>
<td>0</td>
</tr>
<tr>
<td>Jul.</td>
<td>37.3</td>
<td>111.0</td>
<td>0</td>
</tr>
<tr>
<td>Aug.</td>
<td>35.3</td>
<td>104.7</td>
<td>0</td>
</tr>
<tr>
<td>Sep.</td>
<td>27.5</td>
<td>88.8</td>
<td>0</td>
</tr>
<tr>
<td>Oct.</td>
<td>14.8</td>
<td>77.3</td>
<td>-7.6</td>
</tr>
<tr>
<td>Nov.</td>
<td>2.4</td>
<td>42.0</td>
<td>-86.3</td>
</tr>
<tr>
<td>Dec.</td>
<td>-7.8</td>
<td>27.2</td>
<td>-12.4</td>
</tr>
</tbody>
</table>

Table 6. Borehole lengths determined by the proposed methodology and the corresponding objective function for test case #2.

<table>
<thead>
<tr>
<th>No.</th>
<th>H (m)</th>
<th>Obj.</th>
<th>No.</th>
<th>H (m)</th>
<th>Obj.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
<td>2.28E+10</td>
<td>9</td>
<td>72.5</td>
<td>7.89E+08</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>1.36E+10</td>
<td>10</td>
<td>77.5</td>
<td>2867.5</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
<td>6.64E+09</td>
<td>11</td>
<td>73.8</td>
<td>3.19E+09</td>
</tr>
<tr>
<td>4</td>
<td>70</td>
<td>1.76E+09</td>
<td>12</td>
<td>76.3</td>
<td>2821.3</td>
</tr>
<tr>
<td>5</td>
<td>80</td>
<td>2960</td>
<td>13</td>
<td>74.4</td>
<td>1.05E+08</td>
</tr>
<tr>
<td>6</td>
<td>90</td>
<td>3330</td>
<td>14</td>
<td>75.6</td>
<td>2798.1</td>
</tr>
<tr>
<td>7</td>
<td>85</td>
<td>3145</td>
<td>final</td>
<td>75</td>
<td>2775</td>
</tr>
<tr>
<td>8</td>
<td>75</td>
<td>2775</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Figure 14](image2.png)

**Figure 14.** Variation of $T_{\text{out,max}}$ over 10 years for each iteration.

Table 7 summarizes the results obtained with the three methods. As it can be seen, there are differences among the methods with the proposed method giving the shortest length.

**Table 7:** The results of the three sizing levels for test case 2

<table>
<thead>
<tr>
<th>Sizing method</th>
<th>H (m)</th>
<th>L (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 2</td>
<td>86.1</td>
<td>3185.8</td>
</tr>
<tr>
<td>Level 3</td>
<td>77</td>
<td>2848</td>
</tr>
<tr>
<td>Proposed method (Level 4)</td>
<td>75</td>
<td>2775</td>
</tr>
</tbody>
</table>

**Test case #3**

In the third test case, the number of boreholes is optimized for a fixed value of $H$. This case is important as the design variable, unlike the two previous test
In this case the particle swarm optimization with inertia weight method is used. Table 8 illustrates the values of the parameters that are used in this optimization method. The parameter “number of particle” which is defined in Table 8 specifies the number of simulations that are run simultaneously. It is important for this number to be sufficiently large so as to search the whole domain for the design variable. Here, this number is defined as 5 and so 5 simulations with different guess values of borehole numbers are run simultaneously. Based on the results of these 5 simulations the optimization starts the next generation of simulations. As specified in Table 8 the “number of generations” is selected as 6. Therefore, the optimization should find the global minimum in 5x6 simulations. If these values are increased, the chance of finding the optimum is increased, but the number of simulations is also increased. If the optimization makes a guess that has already been analyzed it uses its history and does not simulate it again. When the optimization finds an “optimum” and if the “seed” number is greater than 1, then the algorithm starts a new set of optimization and uses this optimum as the initial guess value.

Table 8: Values of various parameters used in the optimization of test case #3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neighborhood Topology</td>
<td>g best</td>
<td>Social Acceleration</td>
<td>0.5</td>
</tr>
<tr>
<td>Neighborhood Size</td>
<td>1</td>
<td>Max Velocity Gain</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Continuous</td>
<td></td>
</tr>
<tr>
<td>Number Of Particle</td>
<td>5</td>
<td>Max Velocity Discrete</td>
<td>0.5</td>
</tr>
<tr>
<td>Number Of Generation</td>
<td>6</td>
<td>Initial Inertia Weight</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>Final Inertia Weight</td>
<td>0.5</td>
</tr>
<tr>
<td>Cognitive Acceleration</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For this case, the optimization has found the optimum of 28 boreholes with 17 iterations with a calculation time of 2 minutes. The initial guess for this problem was set at 40 boreholes.

**CONCLUSIONS**

In this work, the determination of the borehole length in ground-coupled heat pump systems with successive “manual” iterations using the duct ground heat storage (DST) model in TRNSYS is automated using GenOpt.

In the first part of the paper, the various design methods are reviewed starting with level 0 methods (rules-of-thumb based on peak loads) up to level 4 methods based on hourly simulations. The proposed methodology fits into the level 4 category.

Then, the paper explains how the objective function and related constraints are implemented in a TRNSYS/GenOpt configuration. The objective is to find the smallest overall length \( L = N_p \times H \) while satisfying two temperature constraints, the minimum and maximum allowable inlet temperature to the heat pumps. The method is adaptable, through a judicious choice of the optimization algorithm in GenOpt, to discrete or continuous design variables. The coordinate search method is used for determination of the borehole length (continuous variable) and the particle swarm optimization method is used for the determination of the number of boreholes (discrete variable).

Furthermore, the proposed method is able to handle cases where the heat pump COPs varies, as a function of the fluid inlet temperature, throughout the simulation.

To show the applicability of the method, three test cases are presented. In the first case, 37 boreholes are sized for perfectly balanced annual ground loads with a ground temperature chosen to be exactly at the midpoint between the minimum and maximum allowable temperature limits of the heat pumps. The proposed method was able to find, as it should in this symmetrical case, identical borehole lengths (67.5 m) in both heating and cooling. This was obtained after 13 iterations, i.e. 13 10-year simulations, in 3.5 minutes.

For the second and third test cases, the required overall length of a bore field for a cooling dominated 1500 m² building located in Atlanta is determined for 10 years of operation. For the second test case, the number of boreholes is fixed (37) and the method is used to find the minimum borehole length. The optimization process found that the required length is 75 m. A total of 14 iterations are required with a calculation time of five minutes. This closely corresponds to the values of 86.1 m and 77.0 m obtained using two standard sizing,
lower levels methods. In the third test case, the borehole length is fixed at 100 m and the number of boreholes is optimized. The proposed method evaluated the optimum boreholes number to be 28. This was done in 17 iterations and it required two minutes of calculation time.

The proposed methodology is relatively easy to use and applicable to cases where either continuous or discrete variables are optimized. However, more inter-model comparisons and perhaps validation cases with good experimental data are required to perform further checks of the proposed method.

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REFERENCES


