

## MODELING WINDOWS IN ENERGYPLUS

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### ABSTRACT

We give an overview of how windows are modeled in the EnergyPlus whole-building energy simulation program. Important features include layer-by-layer input of custom glazing, ability to accept spectral or spectral-averaged glass optical properties, incidence angle-dependent solar and visible transmission and reflection, iterative heat balance solution to determine glass surface temperatures, calculation of frame and divider heat transfer, and modeling of movable interior or exterior shading devices with user-specified controls. Example results of EnergyPlus window calculations are shown.

### INTRODUCTION

It is estimated that window heat transfer produces, on average, 31% of the cooling load and 17% of the heating load in US commercial buildings [Huang and Brodrick 2000]. The corresponding numbers for residential buildings are 34% and 23%, respectively. It is, therefore, important to properly model window heat transfer mechanisms to determine the effect of improved window design on building energy use and occupant comfort. Mechanisms that need to be considered include conduction through glazing and framing elements, solar gain by transmission and absorption/reconduction, daylighting, natural ventilation, and infiltration.

EnergyPlus [Crawley et al. 2001] incorporates window calculations that use the best techniques available to model all of these mechanisms. The basic thermal and solar/optical model is based on procedures from the WINDOW 4 and WINDOW 5 programs [Arasteh et al. 1989, Arasteh et al. 1998].

Features of the EnergyPlus window calculation include:

- Layer-by-layer input of glass, gap and shade layers
- Exterior and interior windows
- Spectral or spectral-average glass optical properties

- Iterative heat balance solution to determine glass surface temperatures
- Simultaneous window calculation and room heat balance calculation
- Sub-hour time steps
- Modeling of frames and dividers
- Movable interior and exterior shading devices
- Electrochromic simulation
- Anisotropic sky model for calculation of incident sky diffuse solar radiation and shading of sky diffuse radiation
- Direct solar shading
- Sky long-wave shading
- Tracking where solar radiation from windows falls inside room
- Daylighting illuminance from windows
- Libraries of glass types, gas fills, shading devices and glazing systems

In the following we describe these features in more detail and give example results of EnergyPlus window calculations.

### OPTICAL CALCULATIONS

The solar radiation transmitted by a system of glass layers and the solar radiation absorbed in each layer depend on the solar transmittance, reflectance and absorptance properties of the individual layers. The absorbed solar radiation enters the glazing heat balance calculation that determines the inside surface temperature and, therefore, the heat gain to the room from the glazing. The transmitted solar radiation is absorbed by interior room surfaces and, therefore, contributes to the room heat balance. In addition, the visible transmittance of the glazing is an important factor in the calculation of interior daylight illuminance from the glazing.

In EnergyPlus the optical properties of individual glass layers are given by the following quantities<sup>1</sup> at normal incidence:

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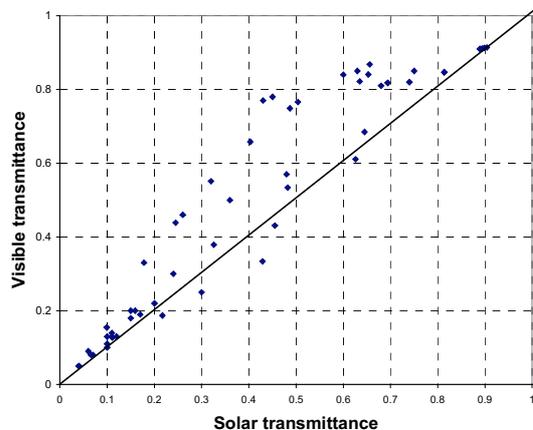
<sup>1</sup> It is straightforward to convert to this formulation from properties given alternatively in terms of thickness, index of refraction and extinction coefficient.

- Transmittance,  $T$
- Front reflectance,  $R_f$
- Back reflectance,  $R_b$

Here “front” refers to radiation incident on the side of the glass closest to the outside environment, and “back” refers to radiation incident on the side of the glass closest to the inside environment.

Transmittance and reflectance may be given as a function of wavelength, or as spectrally-averaged solar values ( $T_{sol}$ ,  $R_{sol}^f$ ,  $R_{sol}^b$ ) and visible values ( $T_{vis}$ ,  $R_{vis}^f$ ,  $R_{vis}^b$ ). Off-normal properties for uncoated glass are determined by using the Fresnel equations. For coated glass the angular dependence of uncoated clear glass is used if the transmittance of the coated glass is  $> 0.645$ ; the angular dependence of uncoated bronze glass is used if the transmittance of the coated glass is  $\leq 0.645$ .

EnergyPlus has a glass library with 59 entries covering glass types such as clear, tinted, reflective, low-E and spectrally selective. Figure 1 shows a plot of  $T_{vis}$  vs.  $T_{sol}$  for all of the entries. For daylighting, the entries above the diagonal line, which have  $T_{vis} > T_{sol}$ , are preferred in cooling dominated climates.



**Figure 1: Solar transmittance vs. visible transmittance for entries in the glass library.**

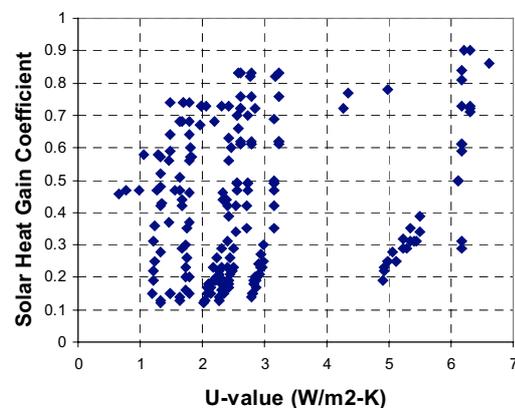
The overall transmittance and glass layer absorptance of a glazing system consisting of glass layers separated by gas layers are determined by solving recursion relations that account for multiple internal reflections within the system. The system properties are determined at angles of incidence,  $\theta$ , from  $0^\circ$  to  $90^\circ$  in  $10^\circ$  increments. The results are fit to a 5<sup>th</sup>-order polynomial in  $\cos \theta$  for use in the time-step calculation.

If the layer properties are given as a function of wavelength, the program calculates the spectral-average system properties by weighting by a solar spectral irradiance function to get the system solar

properties, and further weighting by the photopic response function of the eye to get the system visible properties.

EnergyPlus has a library of over 200 glazing systems built up of entries from the glass library. Included are single-, double-, triple- and quadruple-pane windows with different tints, coatings, glass thickness, gas fills, and gap widths. There is also a selection of experimental electrochromic glazings.

An overview of this library is given in Fig. 2, which shows a plot of the center-of-glass solar heat gain coefficient<sup>2</sup> (SHGC) at normal incidence for ASHRAE summer conditions vs. center-of-glass U-value for ASHRAE winter conditions for all of the entries except electrochromics<sup>3</sup>.



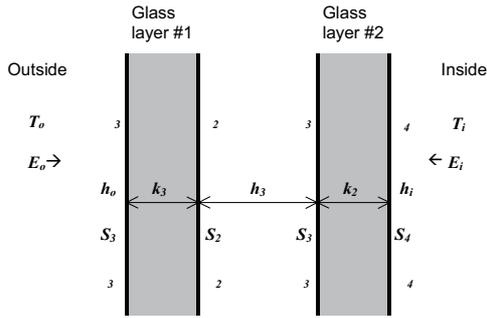
**Figure 2: Solar heat gain coefficient vs. U-value for entries in the glazing system library.**

## THERMAL CALCULATION

The window glass face temperatures are determined by solving heat balance equations on each face every time step. For a window with  $N$  glass layers there are  $2N$  faces and therefore  $2N$  equations to solve. Figure 3 shows the variables used for double glazing ( $N=2$ ).

<sup>2</sup> The solar heat gain coefficient is the solar gain through the glazing divided by the incident solar radiation under standard conditions.

<sup>3</sup> U-value and SHGC are as calculated by WINDOW 4; they are not used in EnergyPlus.



**Figure 3: Glazing system with two glass layers showing variables used in heat balance equations.**

The following assumptions are made in deriving the heat balance equations:

- The glass layers are thin enough (a few millimeters) that heat storage in the glass can be neglected; therefore, there are no heat capacity terms in the equations.
- The heat flow is perpendicular to the glass faces and is one dimensional. However, adjustments to the gap conduction in multi-pane glazing are made to account for 2-D conduction effects across the pane separators at the boundaries of the glazing.
- The glass layers are opaque to long-wave radiation. This is true for most glass products. For thin plastic suspended films this is not a good assumption, so the heat balance equations would have to be modified to handle this case.
- The glass faces are isothermal. This is generally a good assumption since glass conductivity is very high.
- The short-wave radiation absorbed in a glass layer can be apportioned equally to the two faces of the layer.

The four equations for double-glazing are as follows. (Equations for single glazing ( $N=1$ ) and for  $N=3$  and  $N=4$  are analogous and not shown.)

$$E_o \varepsilon_1 - \varepsilon_1 \sigma \theta_1^4 + k_1 (\theta_2 - \theta_1) + h_o (T_o - \theta_1) + S_1 = 0 \quad (1)$$

$$k_1 (\theta_1 - \theta_2) + h_1 (\theta_3 - \theta_2) + \sigma \frac{\varepsilon_2 \varepsilon_3}{1 - (1 - \varepsilon_2)(1 - \varepsilon_3)} (\theta_3^4 - \theta_2^4) + S_2 = 0 \quad (2)$$

$$h_1 (\theta_2 - \theta_3) + k_2 (\theta_4 - \theta_3) + \sigma \frac{\varepsilon_2 \varepsilon_3}{1 - (1 - \varepsilon_2)(1 - \varepsilon_3)} (\theta_2^4 - \theta_3^4) + S_3 = 0 \quad (3)$$

$$E_i \varepsilon_4 - \varepsilon_4 \sigma \theta_4^4 + k_2 (\theta_3 - \theta_4) + h_i (T_i - \theta_4) + S_4 = 0 \quad (4)$$

Here

- $E_o, E_i$  = Incident exterior and interior long-wave radiation ( $\text{W/m}^2$ )
- $h_o, h_i$  = Outside and inside air film convective conductance ( $\text{W/m}^2\text{-K}$ )
- $S_i$  = Radiation (short-wave and long-wave) from zone lights absorbed on face  $i$  ( $\text{W/m}^2$ )
- $T_o, T_i$  = Outside and inside air temperature (K)
- $\varepsilon_i$  = Long-wave emissivity of face  $i$
- $h_j$  = Conductance of gas in gap  $j$  ( $\text{W/m}^2\text{-K}$ )
- $\sigma$  = Stefan-Boltzmann constant

#### Absorbed Radiation

Short-wave radiation (solar and short-wave from lights) is assumed to be absorbed uniformly along a glass layer, so for the purposes of the heat balance calculation it is split equally between the two faces of a layer. Glass layers are assumed to be opaque to long-wave radiation so that the long-wave radiation from lights, people and equipment is assigned only to the inside (room-side) face of the inside glass layer. For  $N$  glass layers  $S_i$  is given by

$$S_{2j-1} = S_{2j} = \frac{1}{2} (I_{bm}^{ext} \cos \phi A_j^f(\phi) + I_{dif}^{ext} A_j^{f,dif} + I_{sw}^{int} A_j^{b,dif}), \quad j = 1 \text{ to } N$$

$$S_{2N} = S_{2N} + \varepsilon_{2N} I_{lw}^{int}$$

Here

- $I_{bm}^{ext}$  = exterior beam normal solar intensity ( $\text{W/m}^2$ )
- $I_{dif}^{ext}$  = exterior diffuse solar incident on glazing from outside ( $\text{W/m}^2$ )
- $I_{sw}^{int}$  = interior short-wave radiation (from lights and from reflected diffuse solar) incident on glazing from inside ( $\text{W/m}^2$ )
- $I_{lw}^{int}$  = long-wave radiation from lights and equipment incident on glazing from inside ( $\text{W/m}^2$ )
- $\varepsilon_{2N}$  = emissivity (long-wave absorptance) of the room-side face of the inside glass layer

## Solving the Glazing Heat Balance Equations

The equations are solved as follows:

1. Linearize the equations by defining  $h_{r,i} = \varepsilon_i \sigma \theta_i^3$ . For example, Eq. 1 becomes
 
$$\begin{aligned} E_o \varepsilon_I - h_{r,i} \theta_I + k_I (\theta_2 - \theta_I) + \\ h_o (T_o - \theta_I) + S_I = 0 \end{aligned}$$
2. Write the equations in the matrix form  $A\theta = B$ .
3. Use previous time step values of  $\theta_i$  to calculate starting values for the  $h_{r,i}$ .
4. Find the solution  $\theta = A^{-1}B$  by LU decomposition.
5. Re-evaluate the  $h_{r,i}$  using the new  $\theta_i$ .
6. Re-calculate  $\theta = A^{-1}B$  using the new  $h_{r,i}$ .
7. Repeat steps 4,5 and 6 until the difference,  $\Delta\theta_i$ , between values of the  $\theta_i$  in successive iterations is less than some tolerance value.

Currently, the test is

$$\frac{1}{2N} \sum_{i=1}^{2N} |\Delta\theta_i| < 0.01K$$

This method converges in 6-8 iterations. Convergence in 2-4 iterations is obtained by relaxation on  $h_{r,i}$ , i.e.,

$$(h_{r,i})_{new} \rightarrow 0.5[(h_{r,i})_{new} + (h_{r,i})_{previous}]$$

The value of the inside face temperature,  $\theta_{2N}$ , determined in this way participates in the room heat balance solution.

## INCIDENT SOLAR CALCULATION

EnergyPlus calculates the solar radiation incident on the outside of the window from the sun, sky and ground. Direct solar from the sun is determined from measured direct normal irradiance from the weather file and calculated incidence angle. Ground diffuse solar is determined from total solar incident on the ground, ground solar reflectance and view factor from window to ground.

Incident sky diffuse solar is determined from the Perez sky radiance distribution [Perez et al. 1987], which is a superposition of three components: circumsolar brightening, horizon brightening and isotropic dome. The proportion of each component in a given time step is determined by sky “brightness” and “clearness” parameters, which are

calculated from direct normal irradiance and total horizontal irradiance from the weather file. The sky diffuse irradiance on the window is then

$$I_{sky} = I_{horizon} + I_{dome} + I_{circumsolar}$$

where

$I_{horizon}$  = irradiance on window from sky horizon region

$I_{dome}$  = irradiance on window from sky dome

$I_{circumsolar}$  = irradiance on window from circumsolar region

## SHADOWING

For each window EnergyPlus calculates the shadowing of solar radiation caused by setback, overhangs, neighboring buildings and other obstructions<sup>4</sup>. An overlapping polygon method is used to calculate shadowing of direct solar [Walton 1985]. It results in a *sunlit fraction*,  $F_{sun}$ , which is the fraction of the area of the window that is illuminated by direct solar.  $F_{sun}$  is calculated for hourly sun positions on 14 representative solar paths at different times of the year. (The user can choose to have shadowing calculated for additional solar paths.)

Sky diffuse solar shadowing is calculated as follows. The sky is assumed to be a superposition of the three Perez sky components described above, except that the circumsolar brightening is taken to be concentrated at the solar disk, and the horizon brightening is taken to be concentrated in a line at the horizon. The following ratio is then calculated by dividing the horizon into a uniformly-spaced grid of points:

$$R_{horiz} = \frac{\text{Obstructed horizon irradiance}}{\text{Unobstructed horizon irradiance}} = \frac{\sum_i I_i F_i}{\sum_i I_i}$$

where  $I_i$  is the unobstructed irradiance on the window from the  $i^{\text{th}}$  horizon point,  $F_i$  is the sunlit fraction from radiation coming from the  $i^{\text{th}}$  horizon point, and the sums are over horizon points lying in front of the window.  $F_i$  is calculated using the Walton shadowing method as though the sun were located at the  $i^{\text{th}}$  horizon point.

The corresponding ratio for the isotropic sky dome is given by

<sup>4</sup> Currently, solar reflected from obstructions is not calculated.

$$R_{dome} = \frac{\text{Obstructed dome irradiance}}{\text{Unobstructed dome irradiance}} = \frac{\sum_{i,j} I_{ij} F_{ij}}{\sum_{i,j} I_{ij}}$$

where  $(i,j)$  is a grid of points in altitude and azimuth covering the sky dome,  $I_{ij}$  is the unobstructed irradiance on the window from the  $ij^{\text{th}}$  point,  $F_{ij}$  is the sunlit fraction for radiation coming from the  $ij^{\text{th}}$  point, and the sum is over points on the sky dome lying in front of the window.

Because the circumsolar region is assumed to be concentrated at the solar disk, the circumsolar ratio is

$$R_{circumsolar} = \frac{\text{Obstructed circumsolar irradiance}}{\text{Unobstructed circumsolar irradiance}} = F_i$$

The total sky diffuse irradiance on the window with shadowing is then

$$I'_{sky} = R_{horizon} I_{horizon} + R_{dome} I_{dome} + R_{circumsolar} I_{circumsolar}$$

In EnergyPlus,  $R_{horizon}$  and  $R_{dome}$  are calculated once for each window since they are independent of sun position.

## SHADING DEVICES

A window in EnergyPlus can have a shading device—such as a blind, pull-down shade, or drapes—which are considered to be perfect diffusers with optical properties that are independent of angle of incidence. Shades are entered as a separate interior or exterior layer characterized by solar and visible transmittance and front and back reflectance. If a shade is present the glazing system optical properties take into account inter-reflections between shade and glass layers. In the shade thermal calculation all short-wave radiation absorbed by interior shades is assumed to convect immediately into the room air.

Shades can be specified as insulating or non-insulating. If insulating, the shade is assumed to be in contact with the adjacent glass layer and the effect of the shade on the window conductance is determined as part of the glazing system heat balance calculation.

### Shade Control

Shades can be fixed or movable. Movable shades can be controlled by specifying a schedule and/or a trigger variable and a set point. The shade is deployed if the trigger variable exceeds the set point and the schedule is “on.” Allowed trigger variables include

- Solar radiation incident on the window

- Total horizontal solar
- Outside air temperature
- Previous time-step room air temperature or cooling load
- Daylight discomfort glare

## Switchable Glazing

EnergyPlus can simulate switchable glazing. An example is electrochromics, in which, when voltage is applied, the glazing switches from a clear state to a dark state to reduce solar gain or reduce glare. Switchable glazing can be controlled with the same schedule and trigger variable options as for movable shades. In addition, a control type is available for daylighting in which the transmittance of the glazing is adjusted so that the daylight illuminance is as close as possible to the illuminance set point. This gives just enough transmitted solar to meet the illuminance requirements and suppresses additional solar gain that might increase the cooling load.

## FRAMES AND DIVIDERS

Up to 10 to 20% of heat flow through a window can take place through its framing elements (frame and divider) (Fig. 4). EnergyPlus determines this heat flow from a heat balance calculation on the inside and outside surfaces of the framing elements. The calculation uses effective 2-D conductances, provided by WINDOW 5, for the framing elements. EnergyPlus calculates the direct and diffuse solar radiation absorbed by the outside and inside framing surfaces, and calculates the shadowing of the framing onto the glass. Also accounted for are the 2-D glass conduction effects caused by thermal bridging across between-pane spacers (see Fig. 4).

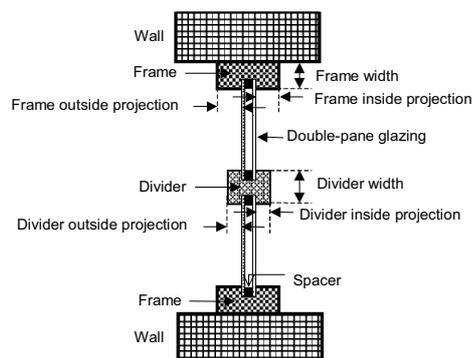


Figure 4: Cross section through a window showing frame and divider (exaggerated horizontally).

## INTERIOR SOLAR DISTRIBUTION

EnergyPlus determines how the solar radiation transmitted by a window is distributed among the interior surfaces of a room. Shadowing calculations, in which the window is a sending surface and the room surfaces are the receiving surfaces, are used to track where beam radiation falls inside the room. From this the beam solar absorbed by each surface illuminated by the window is determined. The reflected portion, and the entering diffuse solar, are assumed to be uniformly distributed among the surfaces. Calculating the interior solar distribution is important when the room contains elements, such as a thermally massive wall or floor, that can store the heat produced by absorbed solar radiation.

This solar-tracking method works well only if there are no internal obstructions in the room. An improved calculation that accounts for such obstructions is under development.

## DAYLIGHTING

Interior daylight illuminance from windows is calculated using methods from DOE-2 [Winkelmann and Selkowitz 1985]. The basic approach is to divide the window into small rectangular elements and find the daylight reaching the reference point directly from each element taking into account the luminance of the sky, the angle of incidence of light on the element, and the visible transmittance of the glazing at this angle. Summing over all of the elements gives the total direct illuminance at the reference point. Also found is the illuminance due to light that reaches the reference point after reflecting from room surfaces. The ratio of interior illuminance to exterior horizontal illuminance gives daylight factors for hourly sun positions along the same solar paths for which beam solar shadowing is calculated. The daylight factors are determined separately for four different sky types: clear, clear turbid, intermediate (partly cloudy) and overcast [Perez et al. 1990]<sup>5</sup>. In the time-step calculation the daylight factors are interpolated for the actual sun position, and then weighted according to the fraction of each sky type (determined from beam normal and total horizontal solar from the weather file) that is present at that time step [Perez et al. 1990].

## NATURAL VENTILATION

The COMIS multizone air flow program has been integrated into EnergyPlus [Huang et al. 1999],

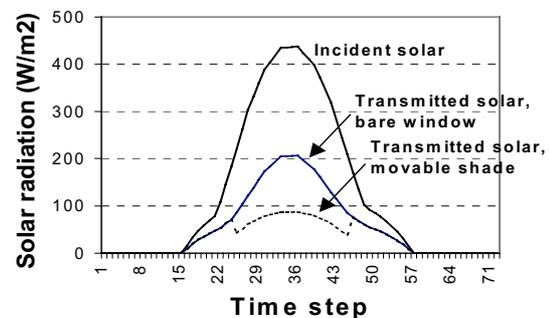
<sup>5</sup> Only two skies—clear and overcast—were used in DOE-2

allowing natural ventilation through open windows to be modeled. COMIS solves the pressure-flow equations associated with a building's network of openings of different types in exterior and interior surfaces. COMIS interacts with the EnergyPlus thermal calculation as follows: At the beginning of a time step COMIS calculates air flows that are used by the EnergyPlus room heat balance and HVAC calculation to get room air temperatures that step; these temperatures are then used by COMIS the next time step.

For natural ventilation some factors that are considered are opening area of one or more windows (as determined by a schedule or control algorithm), wind-induced pressure on the windows, and air temperature difference across the windows.

## EXAMPLE CALCULATION RESULTS

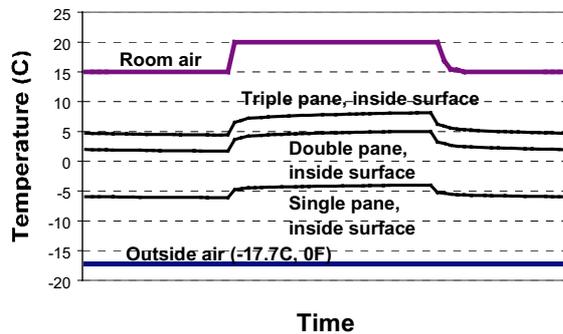
Example results of the EnergyPlus window calculation with a 20-minute time step are shown in Figs. 5 and 6. Figure 5 illustrates sun control with a movable exterior shade with a solar transmittance of 30% and reflectance of 50%. For south-facing clear double-glazing on a summer design day in Denver, this figure shows the direct plus diffuse solar radiation incident on the window, the transmitted solar with no shade, and the transmitted solar when the shade is deployed whenever the incident solar exceeds 200 W/m<sup>2</sup>.



**Figure 5: Transmitted solar radiation through clear double glazing with and without a movable shade; calculated by EnergyPlus for a Denver summer design day.**

Figure 6 shows the glass inside surface temperature for clear single-, double- and triple-glazing for a winter design day in Denver in which the outside temperature is -17.2C (0F) and the solar radiation is zero. The gap width for the multi-pane cases is 12.7 mm (0.5 in) and the gas fill is air. Also shown are the outside air temperature and the room air temperature, which reflects a thermostat setting of

15C at night and 20C during the day. As expected, the higher the number of panes, the higher the inside glass surface temperature.



**Figure 6: Inside glass temperature for single-, double- and triple-pane glazing calculated by EnergyPlus for a Denver winter design day with no solar radiation. The room air temperature and outside air temperature are shown for comparison.**

The glass temperature can be used to predict when condensation occurs; indeed, the single pane case is likely to be frosted at night in this example. Also, the glass temperature enters into the EnergyPlus thermal comfort calculation, where it contributes to the radiant temperature seen by occupants. Obviously, the higher number of panes the higher thermal comfort will be, especially if the occupant is located near the window.

## CONCLUSIONS

EnergyPlus makes available to analysts calculation capabilities that accurately determine—in a whole-building context—the performance of a wide range of window configurations for different climates and building types. The program makes it easy to assess not only the energy and peak load impacts of window choice, but it also allows related implications of windows to be analyzed such as thermal comfort, condensation, natural ventilation and daylighting.

Further work will be concentrated in the following areas:

- Developing and implementing a more accurate thermal model of shading devices that accounts for the buoyancy-driven air flow in the gap between the shade and adjacent glass, and that considers the long-wave radiation exchange between shade and glass. This will give a better estimate of the “inward-flowing fraction” of solar radiation absorbed by the shade and will give a more accurate determination of the window's radiant temperature—which is used in the thermal

comfort calculation—when the shade is in place.

- For both the daylighting and thermal calculations, extending the shade model to handle optically-complex devices, such as Venetian blinds, whose short-wave transmittance and absorptance, and distribution of transmitted radiation have strong dependence on angle of incidence and so should not be modeled as perfect diffusers.
- Allowing EnergyPlus to accept a window description file from WINDOW 5 so that exactly the same window calculated by WINDOW 5 can be exported to EnergyPlus for annual energy analysis.
- Calculating reflection of solar and visible radiation from exterior obstructions.

## ACKNOWLEDGMENTS

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