NUMERICAL CALCULATION OF THE DIRECT SOUND TRANSMISSION USING A COMBINED FINITE ELEMENT - STATISTICAL ENERGY ANALYSIS APPROACH

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ABSTRACT

The finite element method and the statistical energy analysis are commonly used to calculate sound transmission in building acoustics. At lower frequencies the finite element method allows an accurate prediction of the modal behaviour of the analysed constructions. Investigations at higher frequencies require a reduction of the maximal element sizes to allow a correct representation of the corresponding wavelengths. As a result the computation times increase and the practical usage of finite element method is limited. Alternatively the statistical energy analysis allows efficient acoustic calculations especially at mid and high frequencies. For this purpose requirements like a sufficient modal density and modal overlap within the investigated frequency bands have to be fulfilled. At low frequencies these requirements cannot be met due to the modal behaviour of the single components of the construction. Taking into account all these aspects, only a combination of both methods leads to calculations covering an extended frequency range at the scope of building acoustics. In this paper, an approach for the combination of these two methods is proposed using a specialized numerical computing environment. The frequency ranges, where the single methods produce reliable results are determined by an advanced analysis of the vibration behaviour of the components. The obtained results are compared to measurements with focus on a comprehensible construction. The investigations show that an appropriate combination of the two calculation methods can lead to efficient and more accurate results. Therefore the applicability of calculations at building acoustics is increased especially for commonly used constructions.

INTRODUCTION

Numerical calculation models can save both time and financial resources if they are used at an early design phase to predict the sound transmission of innovative building constructions. Currently, a single suitable calculation approach that allows the prediction of sound transmission over the whole frequency range of building acoustics is not known. While the finite element method (FEM) allows an accurate prediction especially of the modal behavior at low and very low frequencies, the computation time and the required computing power grow rapidly with increasing frequencies. Furthermore, the growing sensitivity to structural and constructional variations at higher frequencies requires the use of statistical approaches. For this purpose the statistical energy analysis (SEA) uses a different approach by calculating the exchange of sound energy between structures using energy as the state variable (Lyon & DeJong, 1995). The method allows fast calculations up to very high frequencies even of complex structures. But at lower frequencies, requirements like equipartition of modal energy in each subsystem or a weak coupling between subsystems are no longer met, leading to unreliable calculation results (Hopkins, 2007). A combination of SEA and FEM approaches seems to be promising to cover the whole frequency range of building acoustics. Special attention has to be paid to the medium frequency area where neither SEA nor FEM provide satisfying results. This so called mid-frequency gap is currently an active field of research and several approaches of closing this gap are proposed, for example by (Kouyoumji, et al., 2014), (Reynders, et al., 2014), (Buchschmid & Müller, 2009) and (Gagliardini, et al., 2005). In this paper an approach for the combination of these two methods is proposed using a specialized numerical computing environment. In a first step the sound reduction indices of a lightweight test-specimen were measured in one-third-octave-bands. An experimental modal analysis (Möser, 2010) of the test-specimen was carried out to determine the frequency range were reliable results from the different calculation approaches can be expected. Since a three-dimensional FEM model of the entire testing facility would lead to additional uncertainties and, in practice not suitable, high computation times, a reduced two-dimensional model was developed to calculate the sound transmission at lower frequencies. The testing facility was modeled using a commercial SEA software and the sound transmission was calculated within a wide frequency range. Finally the sound transmission indices calculated by FEM and SEA were combined and compared with equivalent measurement results. Special attention was paid to lower frequencies due to the growing uncertainties of the measurement results according to EN ISO 12999-1 (Austrian Standards Institute, 2015-03-15).
MATERIALS AND METHODS

FEM

The FEM-based calculations were carried out using the software COMSOL Multiphysics Version 5.2 (COMSOL, 2016). Theoretically, it is possible to calculate the three-dimensional sound field of the whole testing facility (Figure 1) even up to high frequencies.

But the usability of such a complex model is limited due to its high computing time and its high requirements on the computational power. Additionally, with increasing frequencies and thus decreasing wavelengths, especially the vibroacoustic response becomes very sensitive to geometrical and physical parameters like fabrication and assembly tolerances or variations in material properties and joint behaviour (Atalla & Sgard, 2015). Besides the practical limitations, this leads to additional restrictions caused by the deterministic approach of FEM-based calculations.

The number of degrees of freedom (DOF) that have to be calculated can be used to indicate the computational requirements and therefore the usability of a model. With a growing number of DOF, also the computing time will increase. Equation 1 gives a practical estimation of the minimum number of degrees of freedom (DOF) that should be used in a three-dimensional model (Petritsch, 2011):

\[ D_{\text{min}} = 1728 \cdot \frac{f_{\text{max}}^3 \cdot V_D}{c^3} \]  

\[ D_{\text{min}} = \text{minimum number of DOF}; \ f_{\text{max}} = \text{maximum frequency}; \ V_D = \text{Volume of the acoustic domain}; \ c = \text{speed of air}. \]

As an example, a maximum frequency of 1 kHz and the combined volume of the receiving and the source room of approx. 138 m³ would lead to a minimum number of 5,9e+06 DOF. In combination with a high frequency resolution this large number of DOF would again result in high requirements of the computational hardware and impractically high calculation times. To reduce the number of DOF a reduced two-dimensional model was developed. Figure 2 shows the three main parts of the reduced model: the test-specimen (1), the acoustic domain (2) and the perfectly matched layer (PML) (3). A diffuse pressure field was implemented to excite the specimen, which splits the acoustic domain into two parts. The test-specimen was modeled in real-size with a width of 1.48m and a thickness of 12.5 mm.

The orthotropic material properties of the gypsum fiber board were derived from an FEM-based optimization approach. During this optimization process the calculated modes were fitted to the measured mode frequencies and mode shapes by using an linear-elastic orthotropic material model. Table 1 shows the material properties as a result of the optimization approach and the material properties as stated in the product data sheet. The orthotropic material properties were also used in the SEA model.

<table>
<thead>
<tr>
<th>Table 1: Material properties of the test-specimen</th>
<th>DATA SHEET</th>
<th>FEM STUDY</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho ) [kg/m³]</td>
<td>1150</td>
<td>1120</td>
</tr>
<tr>
<td>( E_x ) [Pa]</td>
<td>3.8e9</td>
<td>4.65e9</td>
</tr>
<tr>
<td>( E_y ) [Pa]</td>
<td>-</td>
<td>2.92e9</td>
</tr>
<tr>
<td>( G_{xy} ) [Pa]</td>
<td>1.6e9</td>
<td>1.51e9</td>
</tr>
<tr>
<td>( \nu ) [-]</td>
<td>0.1875</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Within the PML, waves are damped continuously. Therefore they are not reflected at the interface or the outer boundaries, so the model simulates a free field wave propagation. As a consequence there is no need for modelling rooms to enable a diffuse excitation of the specimen, which results in a further decrease of calculation time. An adaptive, user controlled mesh with second order elements was used to discretize the geometry. The maximum element size was limited to \( \lambda_{\text{min}}/10 \), where \( \lambda_{\text{min}} \) is the wavelength of the highest considered frequency. Rectangular elements where used for the PML, while the acoustic domain and the test-specimen were discretized using triangular elements (see Figure 3). Quadratic shape functions were used.
A multifrontal massively parallel sparse direct solver (MUMPS) was implemented as a numerical solver. The reduced model offers the possibility of estimating the incident and radiated sound power (of the test specimen) without modelling the entire testing facility, avoiding common known problems and uncertainties arising from an implementation of the geometric description of the testing rooms. The sound reduction indices were derived by evaluating the incident sound power at the side of the excitation and the sound power radiated by the test-specimen at the opposite side in one-third-octave-bands according to EN ISO 10140-2 (Austrian Standards Institute, 2010-11-15).

\[
R = 10 \log \frac{W_1}{W_2}
\]  

\( R \) = sound reduction index in dB; \( W_1 \) = incident power in Watt; \( W_2 \) = radiated power in Watt.

Since the reduced model is capable of predicting the incident and radiated sound power of the test-specimen without modeling the entire testing facility, common known problems and uncertainties resulting from the geometry of the testing rooms do not influence the calculation results.

**SEA**

All calculations based on SEA were carried out using the software SEAWOOD that is a special version of SEA+ for wood building industry (InterAC, 2014-5). Since SEA allows fast calculations of complex structures up to high frequencies, the entire testing facility was modeled (see Figure 4).

The SEA-model was calibrated using data calculated from several measurements. The absorption coefficients of the source- and the receiving room were calculated using reverberation time measurements. The total loss factors of the test-specimen were calculated from structure-borne reverberation time measurements.

The sound reduction indices were derived with Equation 3 in one-third-octave-bands according to EN ISO 10140-2 by using the calculated sound pressure levels of the source- and the receiving room. The values for the area of the test opening and for the equivalent absorption area of the receiving room were taken from measurements.

\[
R = L_1 - L_2 + 10 \log \frac{S}{A}
\]  

\( R \) = sound reduction index in dB; \( L_1 \) = sound pressure level of the source room in dB; \( L_2 \) = sound pressure level of the receiving room in dB; \( S \) = area of the test opening in m\(^2\); \( A \) = equivalent absorption area of the receiving room in m\(^2\).

**Measurements**

All measurements were carried out in a testing facility that complies with EN ISO 10140 standard series. As a test-specimen one layer of a gypsum fiber board was used (length/width/height: 1,48 m/1,00 m/0,0125 m). The test-specimen was mounted flush with the wall of the source room and was supported by two pieces of elastomer under the left and right corner to reduce the transmission of sound energy via the bearing. A construction with a significant high sound insulation was used to reduce the opening of the testing facility (Figure 5).
Institute, 2014-09-15). As shown in Figure 6 a layer of mineral wool was used to reduce the coupling between specimen and wooden frame as well as between specimen and testing facility.

![Figure 6: Cross section of the mounting of the test-specimen. gypsum fiber board (1), wooden frame (2), linseed oil putty (3), mineral wool (4)](image)

**Experimental modal analysis**

An experimental modal analysis (EMA) of the test-specimen was carried out in order to verify above which frequency the requirements of SEA are met. The EMA was carried out by using an impact hammer to excite the gypsum fiber board and measuring the resulting acceleration on the surface. As shown in Figure 7 the accelerometer was placed in the middle of the lower right quarter of the plate. The accelerometer remained at the fixed position while the position of the excitation was varied to reduce the measuring effort. This procedure was possible since the test-specimen can be considered as a linear time-invariant system. Therefore the principle of reciprocity can be assumed (Möser, 2010). Three measurement paths were chosen were the transfer functions (inertance) were determined at measurement positions with a distance of 2 cm to detect bending modes in several directions. As shown in Figure 7 the horizontal path was measured from left to right, the vertical path from bottom to top and the diagonal path from the lower left to the upper right corner.

![Figure 7: Experimental modal analysis of the test specimen: horizontal (1), vertical (2) and diagonal (3) measurement path; accelerometer (a)](image)

**Combination of FEM and SEA**

To combine and visualize the sound reduction indices calculated by FEM and SEA and to calculate the modes per band of the test-specimen, the numerical computing software MATLAB (MathWorks, 2016) was used.

**RESULTS AND DISCUSSION**

The first step was to determine the frequency ranges where each calculation approach can be applied. Since the upper frequency limit of FEM can not be clearly specified, the SEA requirement of a sufficient number of modes per band was used to determine the frequency band above which results from SEA with low variance can be expected for the specimen. As a consequence, this frequency band was used as an initial upper limit for the FEM-based calculations. In general, the number of modes per band increases with higher frequencies. As shown by (Fahy & Mohammed, 1992) there should be at least five modes per frequency band to give estimates with low variance when using SEA. In a first step the frequency response function (FRF) of each measurement point of the EMA was plotted consecutively on the y-axis to visualize the modal behavior of the plate from 20 to 1000 Hz (Figure 8).

![Figure 8: Visualization of the modal behavior of the specimen based on the experimental modal analysis: horizontal (upper), vertical (middle) and diagonal (lower) measurement path](image)
The mean FRF of all measurement points of the horizontal measurement path (Figure 9) was then used to calculate the modes per one-third-octave band of the test-specimen from 50 to 500 Hz. The modes detected by the algorithm are marked with red circles. Figure 8 and Figure 9 show that between 50 and 160 Hz distinct modes can be clearly determined. With higher frequencies the structure-borne sound field is getting more and more diffuse.

Figure 9: Mean frequency response function of the horizontal measurement path and detected modes (red circles)

Figure 10 shows the modes per one-third-octave band that were calculated from measurements and the modes per band that were calculated by the SEA software.

Figure 10: Comparison between modes per band calculated by SEA and calculated from measurement

The results from the EMA as well as the calculation results from SEAWOOD show, that more than five modes per band occur above 315 Hz. As a result, the 315 Hz band is the lowest frequency band above which SEA results with low variance can be expected.

As a next step the reduced FEM model was used to calculate the sound reduction indices from 50 to 500 Hz. The measured sound reduction indices and the sound reduction indices calculated by FEM are shown in Figure 11 in one-third-octave-bands. Since the testing facility was designed for measurements above 100 Hz, measurement results below 100 Hz do not represent the actual sound reduction of the specimen. A dashed line was used to indicate the increasing uncertainty of the measurement results at lower frequencies.

Figure 11: Sound reduction indices from measurement and calculation results based on FEM

When comparing measurements with calculations it should be kept in mind that, especially with lower frequencies, the uncertainty of the measurement results increases because of the decreasing modal density and modal overlapping of the airborne sound field of the testing facility. As a result, the equal distribution of acoustic energy is no longer ensured especially for low frequency bands. Depending on the spatial distribution of the distinct modes of the source and the receiving room, the geometry of the testing facility can influence measurement results to a greater or lesser extend. Additional uncertainties can result for example from the mounting conditions of the specimen and from the calibration of the measurement equipment which leads to increasing standard deviations. Figure 12 shows the standard deviation \( \sigma \) and the standard deviation with a coverage probability of 95%, \( \sigma_{95} \) according to EN ISO 12999 (Austrian Standards Institute, 2015-03-15).

Figure 12: Measured sound reduction indices and standard deviations according to EN ISO 12999-1

In a next step SEA was used to determine the sound reduction indices with focus on higher frequencies. Although the previous investigations showed that reliable results can only be expected above 315 Hz, calculations were carried out in an extended frequency range from 50 to 5000 Hz. The measured sound reduction indices and the sound reduction indices calculated by SEA are shown in Figure 13 in one-third-octave-bands. From 315 to 5000 Hz minor...
deviations between measurement and calculation results were achieved. As expected, the deviations increase below 315 Hz since the requirements of the specimen for SEA calculations are no longer met, showing a maximum deviation of 11.7 dB at 50 Hz.

Figure 13: Sound reduction indices from measurement and calculation results based on SEA

Figure 14 shows the FEM-based results from 50 to 500 Hz, the SEA-based results from 200 to 5000 Hz and the measured sound reduction indices from 50 to 5000 Hz in one-third-octave bands. Since SEA results with low variance can be expected above 315 Hz and FEM-based calculations were carried out up to 500 Hz, the two calculation approaches can be connected in a frequency range from 315 to 500 Hz. Because of the minor deviations between measurement results and SEA-based results above 315 Hz, the sound reduction indices calculated by SEA were used between 315 and 5000 Hz.

Figure 14: Results of FEM- and SEA-based calculations compared with measurement

Finally an extended range of building acoustics from 50 to 5000 Hz is covered by combining both calculation approaches, shown in Figure 15. From 50 to 250 Hz the results from FEM-based calculations were used, from 315 to 5000 Hz the SEA-based calculation results were used. The combined calculation results lead to a single number value for the sound reduction index of \( R_{w,\text{sim}} = 34.3 \) dB. The measured sound reduction indices lead to a single number value for the sound reduction index of \( R_{w,\text{meas}} = 34.4 \) dB.

Figure 15: Calculated sound transmission loss and measurement (FEM: 50 – 250 Hz; SEA: 315 – 5 kHz)

CONCLUSION

A hybrid calculation approach is proposed that combines FEM and SEA based calculations of the sound reduction index of a comprehensible construction to cover the whole range of building acoustics. The calculation results were compared to measurements carried out in a testing facility that complies with standard series EN ISO 10140. Special attention was paid on the measurement setup to get reliable results that can be used as reference values. Especially at lower frequencies the uncertainty of the measurement results increases due to the modal behaviour of the testing rooms and the test-specimen. The combination of FEM-based results at lower frequencies and of SEA-based results at higher frequencies can lead to an overlapping area were the results of both calculation approaches are not reliable. At lower frequencies, some of the requirements for the use of SEA, like a sufficient modal density of the test-specimen, are no longer met. The lowest frequency band where reliable SEA results can be expected was determined by carrying out an experimental modal analysis. At higher frequencies the practicability of FEM is limited by its sensibility to small structural changes and its increasing calculation time. To extend the usability of the FEM approach to a higher frequency range, a reduced model was developed to calculate the sound reduction indices. This model reduction is a promising approach to reduce the calculation times significantly while reliable results can still be achieved at lower frequencies. Future investigations of the applicability of the proposed methods are planned, dealing with more complex structures used at timber buildings, like multilayer constructions and walls made of cross-laminated timber.

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