Stochastic Analysis for Design Space Exploration and Building Performance Optimisation

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ABSTRACT
This research presents a generic stochastic analysis workflow, which combines several types of sensitivity analysis, taking into account non-linearity as well as the need of practical feasibility in building design optimisation. A preliminary step divides the studied design variables into three categories according to their effect on building performances (no impact, linear impact, non-linear). Doing so results in a reduction of the parameter space as well as an overall reduction of computing time without loss of precision. Additionally, the workflow is enhanced by an uncertainty analysis that incorporates the effects of inherently stochastic variables acting as uncontrollable conditions on the building.

INTRODUCTION
Building design becomes an increasingly challenging task due to regularly rising expectations in international energy efficiency standards. Therefore, to ensure an efficient building design process, it is essential to understand and control how the future building energy performance is affected by various design decisions, which are reflected by design variables (DV). To quantify the building energy performance, key performance indicators (KPI), which are now widely accepted in the building simulation community can be computed (see e.g. Ugwu & Haupt 2007, Peng et al. 2012). This work focuses on KPIs expressed as numerical values, which are provided as outputs from simulation software like for example TRNSYS and EnergyPlus. The complexity in assessing the effect of DVs on KPIs resides in the fact that every single KPI is affected by many DVs, which are mostly of the time part of a complex analytical model relying on the laws of physics. This leads to a non-linear problem, which normally has to be explored via global sensitivity analysis (SA). Due to the very high dimension of the parameter space however, this would exceed a practically feasible computing time, especially given the fact that several design iterations have to be checked.

UNCERTAINTY IN BUILDING DESIGN AND LIFE CYCLE
In building design, the decisions with the biggest impact have to be made in early design stages when very little information is available. Therefore, to achieve the goals set by energy efficiency guidelines, awareness of involved uncertainties and their effect is vital from the early design phase. On the one hand, a designer needs to explore the effects of his decisions. On the other hand, he has to deal with stochastic factors, which lay beyond his control. This situation can be modelled using three types of uncertainty: (1) the uncertainty of decisions that are not yet made, (2) aleatoric uncertainty and (3) epistemic uncertainty. A discussion about the general influence of (2) and (3) in engineer modelling can be found in (Der Kiureghian & Ditlevsen 2009). (1) describes uncertainties that relate to the level of development of the design. Design decisions, represented by DVs that are not yet made are obviously uncertain in their outcome. Examples for DVs are the number of rooms per storey or the window-wall ratio. The entirety of possible decisions is called design space. One of the main goals of the presented work is to explore the impact possible design decisions have on energy and cost relevant KPIs. This can be achieved by performing SA. The different types of SA and their advantages as well as limitations will be introduced in the next chapter. (2) is also known as statistic uncertainty. These uncertainties are not directly controllable by the designer and have an important impact on the building performance. Examples are the climate, the fluctuation of energy prices as well as the occupancy and occupant behaviour during the building life cycle. We call this kind of variables stochastic regressors. Here we are interested not only in the regressor’s expected impact on energy relevant metrics, but also in the variance they introduce to these metrics, as both can be controlled to a certain extent by an ingenious design. To explore aleatoric uncertainties, an uncertainty analysis will be performed. The results describe the risk of performance deviations in a certain design due to its stochastic nature. As an example, uncertainty
analysis can provide a probability distribution of a KPI from which statistical indicators like among others its variance can be derived. Such indicators are introduced as key risk indicators (KRI) because together, they allow the designer or the decision maker an overview of the volatility of the KPIs. Further details about the introduction of the concept of KRI in performance-driven building design are provided in (Gnüchtel et al., 2015).

(3) is also called systematic uncertainty and describes uncertainty that is introduced by people not being able or willing to determine a value precisely. For example, the U-value of a wall by itself is not uncertain, but in practice, we use an expectation and thus an uncertain value. This last kind of uncertainty will not be part of this work, which focuses on (1) and (2).

The main ideas about uncertainty in the building design process and the preferred means of analysis are illustrated in Figure 1.

![Figure 1: Uncertainties and their analyses in the building design process](image)

**USAGE OF SENSITIVITY ANALYSIS FOR BUILDING DESIGN**

The idea of SA is to ‘study how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, to different sources of variation and of how the given model depends upon the information fed into it.’ (Saltelli et al. 2000) In our case, the model can be any kind of building simulation model, capable of producing the KPIs in which a designer is interested. The model input is the building design, driven by the designer’s various decisions, expressed by DVs. In the context of SA, these input variables are frequently called factors. The outputs are the values of the predefined KPIs, derived from the simulation output. This kind of analysis enables the designer to learn which DVs have which kind of impact on each of the KPIs of interest. There are many different types of SA available. A general overview of the existing methods in the context of building simulation is given in (Hopfe 2009) and (Tian 2013).

Two restricting factors have to be considered when discussing SA for building design optimisation. One is the computational cost, which in the context of SA depends predominantly on the number of model evaluations rather than the computation of the sensitivity indices, which is quite inexpensive in comparison. While the possibilities of grid or cloud computation softened this restriction, the goal of a design supporting workflow that has to be applied over several iterations demands for a close observation of computational costs. The second restriction is the complexity and the non-linearity of building simulation models. These two restrictions pose a conflict of objectives, as the sensitivity algorithms that can cope with non-linear models are the ones with the highest costs.

In the remainder of this work, we will distinguish between three kinds of SA, which will be introduced in the following.

The first category of SA methods is called screening. Screening methods are rather inexpensive in terms of computer resource usage and aim to identify the most impactful factors in a model with a huge number of inputs. This approach relies on the assumption that compared to the total number of inputs the number of these important factors is relatively small (see e.g. Morris 1987). Their drawback is that they give only qualitative information, which means they give a ranking of importance without quantifying the impact of each factor.

The second category consists of local methods. Local methods are popular in practice as they give quantitative information for low computing costs, using intuitive and easy calculations. Examples of local SA for energy efficient building design can be found in (Tavares & Martins 2007) and (Lam & Hui 1996). Local SA’s major drawback however is that it examines each factor individually at a single point in the design space, a so-called base scenario. This means that only linear models (or factors) should be analysed with local SA, and as complex models like a building model have to be considered non-linear until proven otherwise, the application of local SA cannot be recommended in the general case. A special case that suits the application of local SA is when the focus of the analysis is not on exploring the whole design space but on studying one special design, which can be used as base case. Then local SA answers the question how deviations from this design affect certain KPIs.

The third category is global SA. Global techniques aim to explore the complete design space by varying
all factors at once. This enables insights not only about each factor separately, but also about factor interactions. The obvious drawback is the high computational cost. The most popular approach in global SA are variance-based techniques. The principle of such methods is to allocate the model output variance to its inputs, as a factor that causes the majority of the model variance is considered particularly important. The central object of study is the so-called correlation ratio (McKay 1995), which is defined as follows:

Let \( x_1, ..., x_p \) be the inputs, \( x' \subseteq \{x_1, ..., x_p\} \) a subset of inputs and \( y = f(x_1, ..., x_p) \) the output of a model \( f \). The correlation ratio is defined by

\[
\frac{\mathbb{V}[\mathbb{E}(y|x')]}{\mathbb{V}(y)}
\]

and gives the share of \( \mathbb{V}(y) \) that is caused by \( x' \). E.g. for \( y = x_1 + x_2 \), with \( x_1 \) and \( x_2 \) following the same distribution, \( \frac{\mathbb{V}[\mathbb{E}(y|x_1)]}{\mathbb{V}(y)} = \frac{1}{2} \), as half the variance of \( y \) is caused by \( x_1 \).

From this definition, several sets of indices can be derived (Sobol 2001):

First-order indices: \( S_i = \frac{\mathbb{V}[\mathbb{E}(y|x_i)]}{\mathbb{V}(y)} \) for all \( i = 1, ..., p \)

Second-order indices: \( S_{i,j} = \frac{\mathbb{V}[\mathbb{E}(y|x_i x_j)]}{\mathbb{V}(y)} \) for all \( i \neq j \in \{1, ..., p\} \)

One can define indices analogously up to the order corresponding to the number of inputs.

All indices of all orders sum up to 1, so at last an especially important index, the total sensitivity index, can be defined:

Total Sensitivity Index: \( TS_i = 1 - S_{-i} \) for all \( i = 1, ..., p \)

\( S_{-i} \) stands for the sum of all indices NOT including \( i \), so \( TS_i \) gives the total effect of input \( x_i \), including all its interactions. For example, contemplating a model with only 3 inputs, the total sensitivity index of the first input would be defined as

\( TS_1 = 1 - S_2 - S_3 - S_{2,3} \).

To use SA in the building design process, the DVs need to be modelled as stochastic variables. Some short notes on this topic follow in the next chapter.

### STOCHASTIC MODELLING OF DECISION VARIABLES

For global SA, each DV needs to be represented by a distribution function. The question which distribution function to assign to a DV is not straightforward thereby. One possibility is to extract a distribution from any kind of historic data. For example, the distribution of the window-wall ratio could be extracted from existing buildings of similar use. While completely valid from the stochastic point of view, a content-related interpretation lacks persuasiveness. Is the probability of a designer’s decision really to be modelled by historic decisions in different contexts? The authors of this work are convinced that the freedom of choice is best represented by giving each possible decision equal probability. (Tian 2013) supports this conviction. This leads to the assignment of a uniform distribution for each DV, which needs two parameters: a minimum and a maximum value. Similarly, one could use a normal distribution for some DVs to state that the outer regions of the min-max interval are chosen less likely. A second facet of stochastic modelling is stochastic dependence. In colloquial terms, two stochastic variables are dependent when the realisation of one influences the probability distribution of the other. When extracting distributions from historical data, without a doubt one will also receive an interdependence scheme, represented by a covariance matrix. To induce a given covariance on a sample of DVs the method of Iman & Conover (1982) is well established and easy to implement. If, however, the past shall not influence our future decisions, then the modelling of a covariance structure becomes manual work. For every pair of DVs, we have to answer the question ‘Does a higher value of DV\( _i \), e.g. the façade’s U-value, alter the likelihood that the designer chooses a certain value of DV\( _j \), e.g. the window-wall ratio?’ This is obviously highly complicated and highly subjective work. The contrary approach is to assume the independence of all decisions, meaning that the decision for one DV does not affect the decision for another. This approach seems to be the most objective. In addition, it allows exploring the complete design space in an undistorted manner. Thus, the authors of this work support the independence assumption.

Summarising, there are several ways to model DVs and the decision between them seems to be of primary philosophical nature. The authors’ recommendation is the use of independent uniform distributions for all DVs, as this prevents the introduction of bias and thus allows for a holistic evaluation of all possibilities.
Figure 2: Building Design Optimisation Workflow

- Define KPIs and KRI
- Set up design alternatives (DA)
- Alternative Analysis (I)
  - Alternative of interest
    - Screening (II)
      - Non-important factors
      - Linear factors
      - Non-linear factors
        - Local SA (III)
        - Global SA (IV)
    - Create settings with optimised KPIs (V)
      - Optimised DA
        - Alternative Analysis
          - KPIs in targeted range?
            - yes
            - Uncertainty Analysis (VI)
              - KRIs OK?
                - yes
                - Final Solution(s)
                - no
            - no
          - no
WORKFLOW FOR BUILDING DESIGN OPTIMISATION

The aim of the workflow is to support the designer on his way to an intelligent, energy-efficient and risk-aware design. The overall workflow can be seen in Figure 2. The utilised symbols are explained in Figure 3.

Figure 3: Signification of Symbols in the Workflow

The workflow consists of three sequences. At first, a design for in-depth analysis is chosen, for example by running preliminary simulations for several design alternatives. In addition, the KPIs and KRIs of interest are selected and threshold values and/or target functions (most often min or max) are assigned. In a single iteration of the workflow, only KPIs that originate from a single simulation model can be processed. This means that all examined KPIs have to be derived from the same simulation tool, e.g. an energy simulation tool or a cost optimisation tool. The steps (II) to (V) of the workflow comprise the SA section, in which the relationship between the DVs in the upcoming simulation runs. After the sensitivity analysis sequence (UA) follows. Here the design’s robustness against inherently stochastic influences like climate and building usage is tested and KRIs are extracted for numerical verification.

The following subsections describe the three different and complementary workflow sequences in more detail.

Workflow Setup and Alternative Analysis

To set up the workflow, one or several KPIs and KRIs are defined (e.g. see Table 1) and threshold values for their acceptance are assigned. All KPIs have to originate from the same simulation model. Here it is important to note that the addition of KPIs from the same simulation model does not increase the computational cost of the SA itself, but complicates the screening’s classification.

Then several design alternatives are set up. Different basic variants can be tested here. In the following alternative analysis step (I), for each variant the KPIs are computed (e.g. by energy simulation) and studied to gain basic insight into the predicted performance of the building. Based on this information, a design is picked for profound analysis. While this workflow is designed to analyse one single variant from this point on, it is of course possible to repeat the workflow later with one of the other variants.

Sensitivity Analysis Sequence

Before the SA can start, one picks the DVs that will be varied and assigns probability distributions as discussed in the previous chapter. Then the SA starts with the screening (II). Here the method of choice is the Morris One At A Time Design (Morris 1991) in conjunction with the improvements introduced by Campolongo et al. (2007). The Morris method is the most popular and widely used screening method. Heiselberg et al. (2009) claim that ‘the Morris method (…) is evaluated as the most interesting for sensitivity analysis in sustainable building design’, as the information gain is very high, compared to the computational cost. There are two noteworthy drawbacks that force us to use other sensitivity methods in conjunction with the Morris Method: First, it returns only qualitative information of a DV’s impact on a KPI. This means a ranking for all DVs affecting one KPI can be created, but a comparison between different KPIs is not possible. Second, it says which DVs are part of interactions, but not which variables they interact with.

The Morris Method’s main concept is the computation of several so-called elementary effects (EEs) for each DV by varying one input at a time over a certain amount of fixed levels. To do so a range is required for every input that is to be examined. With this information, the method creates a sample plan, which prescribes the settings for all DVs in the upcoming simulation runs. After the simulations are executed, for each DV several EEs are computed and their mean μ, the mean of their absolute values μ* and their variance σ are studied. So for each KPI, this enables the classification of each DV into one of three categories (see Figure 4): (a) negligible (low μ*, low σ), (b) linear and additive (high μ*, low σ) or (c) non-linear or involved in interactions with other factors (high μ*, high σ) (see Figure 4).

Figure 4: Grouping of Screening Outcomes
If several KPIs from one simulation model are considered, the following simple rule has to be applied to ensure a correct overall classification of each DV:

- If a DV has a non-linear influence on at least one of the KPIs, it is considered non-linear.
- If a DV is not non-linear and has a linear influence on at least one of the KPIs, it is considered linear.
- The remaining DVs are considered negligible.

It must be noted that the categorisation into the three categories will not be unambiguous in most cases. The rule of thumb here is that the lower the threshold for $\mu^*$ and $\sigma$ to be considered low, the more exact, but also the more costly the following analysis steps will be.

Now the handling of the factors in each category will be explained:

All factors in category (a) are fixed at an arbitrary value, e.g. in the middle of their interval, or at a chosen baseline value. They are no longer of interest as their impact on the KPI(s) is deemed negligible. As the factors in category (b) are known to have mostly linear influence, their overall impact can be accessed via local SA. The easiest technique available would be to fix all factors but one at the middle of their intervals or at a chosen baseline value, and to perform one simulation with a low and one simulation with a high value of this factor. From these simulations, the slope of each KPI regarding the factor of interest can be computed. This slope gives the strength and the direction of the variable’s impact. For the DVs in category (c), a more sophisticated technique is needed. In principle, every global SA method is applicable in this step. Here we focus on two possible methods, which are well-known and well-studied. One is the Sobol method (Sobol 2001) which allows computing the indices up to an arbitrary number on the expense of simulation runs. The second method is the extended Fourier Amplitude Sensitivity Test, also known as eFAST (Saltelli et al. 1999), which gives the total indices for no additional cost when computing the first-order indices. This makes eFAST especially appealing when a holistic impression of each input’s impact is desired.

The procedure for both methods is similar. Inputs are the DVs with the distributions of their values. A sample plan is generated for which the simulations are executed. The simulation results, the KPIs, are used to compute the sensitivity indices.

When this step is completed, the designer has the information about the impact each DV has on each KPI. With this information, he can create one or several optimised designs (step (V) in Figure 2). To check if this design really fulfils the predefined KPI thresholds, or which is actually the best, another simulation run per design has to be performed. If the results are satisfying, the UA sequence is started. If not, the design has to be reworked.

**Uncertainty Analysis**

While the former SA analyses the designer’s possible choices, the upcoming UA analyses the influence of aleatoric uncertainties that can neither be avoided nor controlled completely. Some typical examples are the outdoor temperature and the climate change, the occupancy of the building and the occupants’ interaction with the building energy system, as well as energy prices and more. The impact of these stochastic regressors on the building performance is undeniable (see e.g. Richardson et al. 2008) and as it is impossible to predict them precisely, a statistical experiment is performed.

At first, the relevant stochastic regressors have to be identified and modelled appropriately. For this purpose, two aspects should be considered. First, it is important to identify the physical parameters properly that have real influence on building performance. Even if many parameters can be randomised, only some can significantly perturb building performance. In general, such parameters are well known by analysts who use building performance analysis models for computing KPIs. The examples mentioned previously give a short overview of such relevant kinds of parameter. Second, it is important to choose parameters for which reliable stochastic models can be derived. For this purpose, it is of interest to use parameters that dispose of lots of historical data. This historical data is generally provided by the continuous observation of the chosen stochastic regressors. Climate and cost parameters are best examples for observed values, as external temperature and market prices are monitored all the time at hour or even minute scale. For such information, many sources can be used by building analysts. For occupancy, there is also the possibility to rely on observed data. On the one hand, there already exist some available information like for example employee schedules in office building or results of household surveys (Ipsos-RSL, Office for National Statistics, 2003). On the other hand, it can be assumed that such information will be more and more available in the future thanks to sensor technologies integrated in BACS systems for identifying the presence and the activity of people in buildings. To every kind of stochastic regressor and related observed data an adapted stochastic model can be applied. Most of the time such stochastic models rely on existing models like for example the Wiener process, that can be applied for climate trend, or the auto-autoregressive integrated mean average (ARIMA) process for energy prices. Other kinds of stochastic models of interest are available in studies or surveys as provided, for example, by Möst & Keles (2010) for the energy market.
Once the relevant stochastic regressors are identified and modelled, they can be integrated as part of a building performance analysis model. In this context there are two possibilities: 1) the analysis model consists of stochastic variables, e.g. stochastic differential equations (SDE), or 2) the analysis model does not use stochastic variables directly but a deterministic paradigm. In the second case, the uncertainty analysis can rely on the statistical experiment by introducing a stochastic pre-processing and a post-processing that shall close the gaps of a deterministic model. As almost all building performance analysis tools available today rely on a deterministic model, this latter approach seems the most appropriate. Indeed, there is a lack of tools that provide building performance calculation based on stochastic models. Nevertheless, the pre-processing, after performing a stochastic sampling, can generate series of values or samples, e.g. a set of weather data for the building life cycle that can be processed in the deterministic model. In this context, the simulation algorithm of the used building performance analysis tool remains unchanged and several simulation runs are performed in order to simulate all samples. Then the post-processing can provide, after processing all results, a statistical distribution of values for each KPI.

KRIs can then be deduced based on the value distribution of a KPI. These can be statistical indicators derived from the distribution of a KPI. In this context, values such as standard deviation, mean value or mode (most probable value) can be used. Other interesting indicators would be the probability of being over or beneath a certain KPI target value, or being outside a certain targeted KPI value range. Such indicators are also easily deduced from the value distribution of a KPI. Some examples of KRIs that can be provided by the UA are shown in Table 1.

More than evaluating uncertainty in one certain building design, the KRIs support the evaluation of uncertainty in different design alternatives, preliminary selected in the alternative analysis. This enables comparing different design alternatives with regard to their volatility in building performance. This way the designer can select the design solution with the least exposure to uncertainty or decide to make changes in the building design, which reduce the level of uncertainty in building performance.

**CONCLUSION**

This work has presented a generic workflow that combines sensitivity analysis and uncertainty analysis with the aim of supporting an optimised building design. The combination of several SA methods takes into account the non-linearity of building simulation models while keeping computational cost in mind. The subsequent UA enables the study of the impact of different kinds of aleatoric uncertainty on building performances. The integration of both SA and UA as part of one building design workflow supports an optimised building design process. On the one hand, through SA the designer can get an idea of the most influent DVs and concentrate specifically on those for finding best design solutions. On the other hand, UA provides an overview of possible building performance deviations that can occur during the building life cycle because of its stochastic nature.

**SOURCES**


