# Study of Dynamic Thermal Performance of Active Pipe-embedded Building Envelopes Based on Frequency-Domain Finite Difference Method

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**ABSTRACT:** Active pipe-embedded building envelope is a building external wall or roof in which pipes are embedded in the wall slab or roof slab for allow water circulating in these pipes for heat transfer. This structure may utilize low-grade energy sources due to the enlarged heat transfer surface between the slab mass and water in the pipe allowing for substantial heat flow for relatively small temperature difference. It may actively intercept heat to reduce the heat transfer for the ambient air to indoor space in summer and coolth to reduce the heat loss of the indoor air in winter season. This paper presents the thermal performance of this structure under various disturbances in frequency domain by using Frequency-Domain Finite Difference (FDFD) method since frequency characteristics may well represent the dynamic characteristics of dynamic systems. In the meantime, the frequency characteristics of this structure are also compared with other numerical results which are obtained in time domain.

**Keyword:** Active pipe-embedded building envelope, Dynamic thermal performance, FDFD, Low-grade energy source

## 1. Introduction

Active pipe-embedded building envelope is a new building envelope structure, which is an external wall or roof with pipes embedded in it and utilize the circulating water in the pipe to transfer heat/coolth inside the structure directly. This structure may significantly enlarge heat transfer surface between the structure mass and the water in the pipe for allowing substantial heat flows even for relatively small temperature differences between the mass and water (Xu et al. 2010). It may sufficiently utilize the low-grade energy source to weaken the influence of outdoor climate on the indoor environment, reduce the external heat transfer, and improve indoor thermal comfort. For this system, the equipment consuming high-grade energy is the pump/fan.

Due to the embedded pipes, the heat transfer of the opaque building envelopes becomes complicated, and it is also not easy to develop the heat transfer model for heat flow calculation. And there have some studies on the model of the pipe-embedded building envelope. Xie et al. (2012) developed two-dimensional steady-state model of active pipe-embedded building envelope. Schnurr and Rogers (1970) proposed a two-dimensional steady-state heat transfer model of pipe-embedded structure for snow melting system, and finite difference method is employed to solve the temperature distribution for optimizing system design. Zhang and Pate (1986a, 1986b, and 1987)

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presented both two-dimensional steady and dynamic models for pipe-embedded structure as ceiling for hot water heating. Antonopoulos et al (1992) developed one-dimensional steady-state analytical model and a two-dimensional steady-state model for temperature field analysis in the panel. Although many models of pipe-embedded structure were developed, but the most model is simple, and the frequency characteristic of this structure has not been analyzed by the researchers since the frequency characteristics represent exactly the dynamical thermal behaviors of the real system of this structure.

This paper proposed a two-dimensional model for frequency characteristic analysis of this structure by using FDFD method (i.e., FDFD model). A concrete wall of 240mm thickness is used for pipes embedded as the active pipe-embedded building envelope prototype. FDFD method can be used to calculate the frequency thermal performance of this structure under various disturbances with different frequency ω. CFD model is also used to simulate the thermal characteristics of this structure under a certain frequency for reference. The comparison shows that FDFD model and CFD model can predict the frequency characteristic of this structure well. However, FDFD model can predict it directly and quickly while CFD model does it indirectly requiring more computation. The frequency thermal performance of this structure by using FDFD model is represented as amplitude and phase angle. The results show that the amplitude and phase angle of the heat flux of different surface are also same in low frequency region while they change as the frequency in high frequency region.

## 2. Description of the pipe-embedded building envelope

A concrete wall of 240mm thickness is used for pipes embedded as the active pipe-embedded building envelope prototype in this study. The schematic of the cross section of the pipe -embedded building envelope is shown as Figure 1. The pipe is in the middle of the wall. Polybutylene tube with the diameter of 20mm is used in this study, and the thickness of pipe is neglected for investigating the thermal performance of this structure conveniently. The density of the concrete is $800 kg/m^3$, the thermal conductive 1.5W/m ·k, and the specific heat 1050 J/kg ·℃.

The section *abcd* is used for model development, and some simplifications are assumed. It assumes the faces (i.e., the upper face and the lower face of cross section) between two adjacent embedded pipes is adiabatic due to the symmetry (i.e., the pipe spacing is uniformly arranged). The left and right faces is the external wall surface and internal wall surface respectively. The model is considered as two-dimensional when the heat transfer along the length direction of the pipe is neglected. The circular temperature of the fluid in the pipe is uniform. In practical applications, the embedded-pipe building envelope may have different pipe spacing such as 100mm, 150mm, 200mm, 250mm and 300mm etc. In this study, the pipe spacing is taken as 200mm.

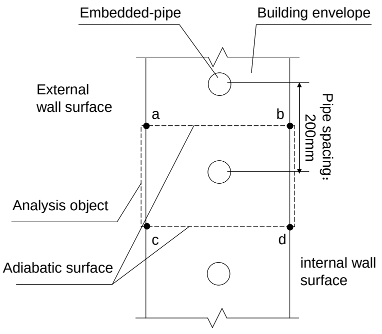
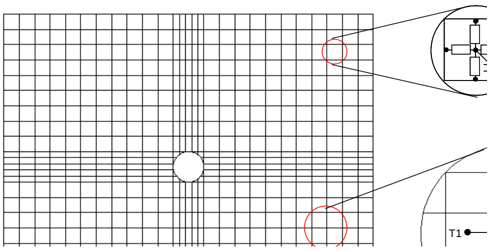


**Figure 1** *Schematic of the cross section of the active embedded-pipe building envelope*



*active-embedded building envelope*

***Figure 2*** *Discrete grid model of the*

## 3. Modeling of this structure using FDFD method and algorithm

Frequency-domain finite-difference (FDFD) method is a numerical solution method for problems usually in electromagnetism and sometimes in acoustics, based on finite-difference approximations of the derivative operators in the differential equation being solved (Zhao et al. 2010). In this study, FDFD method is used for the frequency characteristic analysis of the pipe-embedded building envelope. Finite-difference time-domain method is widely used for complicated problem solution. The big difference between FDFD method and FDTD method is that FDFD method solves problem in frequency domain while FDTD solves problem in time domain although finite-difference method for model discretization may be the same or be similar.

Finite-difference method is used for discretizing the model as a series of cells as shown in Figure 2 for modeling the thermal performance of the structure by using FDFD method. Each cell in this model is represented by using four resistances and one capacitance, and the temperature of the capacitance represents the temperature of the cell. The temperature of each cell with frequency ω may be represented in complex domain, and can be expanded as real component and image component as Equation (1). For each cell or boundary node, the thermal equilibrium equation with the adjacent cells or nodes can be described in terms of the heat flow in frequency domain. Equation (2) shows the thermal equilibrium equation of one cell as shown in Figure 2 with the adjacent four cells by using the center finite difference method. By expanding each temperature as real component and image component, Equation (3) can be obtained to represent thermal equilibrium of the cell in matrix form. When the thermal equilibrium in matrix form of each cells or boundary nodes are described, linear simultaneous equations as Equation (4) can be obtained to present the FDFD model of the active pipe-embedded building envelope structure.

$$\theta_{\mathrm{i}} = \bar{\theta}\exp(i(\omega t + \Phi)) = \bar{\theta}\exp(i\Phi)\exp(i\omega t) = (u + iv)\exp(i\omega t) \tag{1}$$

$$s_1(\theta_1 - \theta_0) + s_2(\theta_2 - \theta_0) + s_3(\theta_3 - \theta_0) + s_4(\theta_4 - \theta_0) = dxdy\rho c\omega\frac{\partial\theta_0}{\partial t} \tag{2}$$

$$\begin{bmatrix} s_1 + s_2 + s_3 + s_4 & -dxdy\rho c\omega \\ dxdy\rho c\omega & s_1 + s_2 + s_3 + s_4 \end{bmatrix}\begin{bmatrix} u_0 \\ v_0 \end{bmatrix} = \begin{bmatrix} s_1u_1 + s_2u_2 + s_3u_3 + s_4u_4 \\ s_1v_1 + s_2v_2 + s_3v_3 + s_4v_4 \end{bmatrix} \tag{3}$$

$$\begin{bmatrix}
\sum\limits_{j\neq 1}^{n} s_j & -dxdy\rho c\omega & 0 & 0 & .\;.\;.\;. & 0 & 0 & 0 & 0 \\
dxdy\rho c\omega & \sum\limits_{j\neq 1}^{n} s_j & 0 & 0 & .\;.\;.\;. & 0 & 0 & 0 & 0 \\
0 & 0 & \sum\limits_{j\neq 2}^{n} s_j & -dxdy\rho c\omega & .\;.\;.\;. & 0 & 0 & 0 & 0 \\
0 & 0 & dxdy\rho c\omega & \sum\limits_{j\neq 2}^{n} s_j & .\;.\;.\;. & 0 & 0 & 0 & 0 \\
. & . & . & . & .\;.\;.\;. & . & . & . & . \\
. & . & . & . & .\;.\;.\;. & . & . & . & . \\
. & . & . & . & .\;.\;.\;. & . & . & . & . \\
. & . & . & . & .\;.\;.\;. & . & . & . & . \\
0 & 0 & 0 & 0 & .\;.\;.\;. & \sum\limits_{j\neq N-1}^{n} s_j & -dxdy\rho c\omega & 0 & 0 \\
0 & 0 & 0 & 0 & .\;.\;.\;. & dxdy\rho c\omega & \sum\limits_{j\neq N-1}^{n} s_j & 0 & 0 \\
0 & 0 & 0 & 0 & .\;.\;.\;. & 0 & 0 & \sum\limits_{j\neq N}^{n} s_j & -dxdy\rho c\omega \\
0 & 0 & 0 & 0 & .\;.\;.\;. & 0 & 0 & dxdy\rho c\omega & \sum\limits_{j\neq N}^{n} s_j
\end{bmatrix}
\begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ . \\ . \\ . \\ . \\ u_{N-1} \\ v_{N-1} \\ u_N \\ v_N \end{bmatrix}
=
\begin{bmatrix}
\sum\limits_{j\neq 1}^{n} s_j u_j \\ \sum\limits_{j\neq 1}^{n} s_j v_j \\ \sum\limits_{j\neq 2}^{n} s_j u_j \\ \sum\limits_{j\neq 2}^{n} s_j v_j \\ . \\ . \\ . \\ . \\ \sum\limits_{j\neq N-1}^{n} s_j u_j \\ \sum\limits_{j\neq N-1}^{n} s_j v_j \\ \sum\limits_{j\neq N}^{n} s_j u_j \\ \sum\limits_{j\neq N}^{n} s_j v_j
\end{bmatrix} \tag{4}$$

Where: $\theta$ is the temperature in frequency domain (℃), $\bar{\theta}$ is is the temperature in time domain (℃), $t$ is the time (s), $i$ is the symbol of $\sqrt{-1}$, $\omega$ is the frequency (rad $s^{-1}$),

$\Phi$ is the phase angle (°), $u$ is the real part of the complex quantity (℃), $v$ is the image part of the complex quantity (℃), $\rho$ is the density of concrete (kg/m$^3$),$c$ is the specific heat capacity of concrete (J/kg·℃), $s$ is the conductance between two nodes (W/m·k), $dy$ is the incremental in the $y$-direction (m), $dx$ is the incremental in the $x$-direction (m), $\lambda$ is the thermal conductive of concrete (W/m·k), $N$ is the total number of cells, subscripts $j$ is the number of the cells or boundary nodes.

For the variables of $u$ and $v$, they can be denoted as Equation (5). The variable of $s$ can be denoted as Equation (6).

$$u = \bar{\theta}\sin\Phi, v = \bar{\theta}\cos\Phi \tag{5}$$

$$\mathrm{s_j} = \frac{dy_0}{\frac{dx_j}{2\lambda_j} + \frac{dx_0}{2\lambda_0}}(\mathrm{j} = 1{,}3), \mathrm{s_j} = \frac{dx_0}{\frac{dy_j}{2\lambda_j} + \frac{dy_0}{2\lambda_0}}(\mathrm{j} = 2{,}4) \tag{6}$$

In this equation, the real part of the complex quantity ($u$) and the image part of the complex quantity ($v$) are unknowns, and need to be solved based on the boundary conditions (disturbances). The structure is usually imposed by three disturbances, i.e., the temperature of the external wall surface, the temperature of the internal wall surface and the circular temperature of the fluid in the pipe. To obtain the frequency characteristic of this structure, it is assumed that one of these three disturbances is given, and the remained two disturbances are assumed to be zero. The response by the disturbance indicated by the values of $u_{j,}$ and $v_j$ of each cells or boundary nodes can be solved by Equation (4). Then, the amplitude and phase angle of the frequency characteristics can be obtained as Equation (7) and Equation (8) respectively based on the solved $u_{j,}$ and $v_j$. Therefore, the temperature and heat flux numerical solutions on the external wall surface, internal wall surface and internal surface of embedded-pipe in frequency domain can be obtained easily.

$$A = \sqrt{u_j^2 + v_j^2} \tag{7}$$

$$\Phi_j = \arctan\frac{v_j}{u_j} \tag{8}$$

For solving $u_{j,}$ and $v_j$ in Equation (4), this equation can be further transformed to a large nonsymmetrical sparse matrix. There are many mathematical methods for calculating a large nonsymmetrical sparse matrix, such as LU-Decomposition method (Mittal and Al-Kurdi. 2001), Bi-CGSTAB method (van der Vorst, 1992) and generalized minimal residual method (usually abbreviated GMRES) (Saad and Schultz,et al.1986) etc. GMRES is an iterative method for the numerical solution of a nonsymmetrical system of linear equations, and approximates the solution by the vector in a Krylov subspace with minimal residual. The big advantage of GMRES is the guarantee to compute the approximate solution with minimum residual norm (Saad et al.2000al. 2000). For solving Equation (4), a FORTRAN code by using GMRES algorithm developed and used.

## 4. Modeling of this structure using CFD for reference

For evaluate the performance prediction of the active pipe-embedded building envelope by using FDFD model, another numerical model is developed for performance prediction for comparison.

For the complicated and irregular structure or the structure having internal heat sources, numerical analysis method is usually used for heat transfer analysis. The literature (Oppelt et al. 2010, Wu et al. 2010) used CFD technology to establish numerical simulation model for analyzing the unsteady heat transfer of the ground-coupled heat exchanger of a heat pump system. In this paper, CFD modeling is used to simulate the dynamic thermal performance of this structure for reference. In FDFD model, frequency ($\omega$) is used. For performance prediction comparison, a periodical signal may be assumed to impose on CFD model. The period may relate to frequency as Equation (9). Conventional CFD software can’t deal with periodical boundary conditions automatically. In this study, a CFD interface program was developed to import the periodical temperature boundary conditions since periodical heat transfer of the building envelope model is needed to be calculated.

$$T = \frac{2 \times \pi}{\omega} \tag{9}$$

Where: $T$ is the period (s), $\omega$ is the frequency (rad $s^{-1}$)

In CFD model of this active building envelope, the first boundary conditions are used (i.e., the temperatures external wall surface, internal wall surface and the internal surface of embedded-pipe are specified). In this simulation, the disturbances are periodic change, and the period is determined by the frequency which is used in FDFD model. With the output of each cell (or node) in one period, the amplitude and phase angle can be calculated easily.

## 5. Performance prediction comparison between CFD model and FDFD model

In this study, the predicted heat flux on the external wall surface, internal wall surface and internal surface of the embedded-pipe is used for performance prediction comparison between CFD model and FDFD model. To obtain the frequency characteristic of this structure, the disturbances should be specified in advance. The disturbances of concern are the temperature of the external wall surface, the temperature of the internal wall surface, and the circular temperature of the fluid in the pipe. And usually three combinations of these three disturbances are used. They are as follows. The first combination is that the external wall surface temperature is regarded with the amplitude of 1 and the frequency of $\omega$, and the remained two disturbances are assumed to be zero. The first combination is that the external wall surface temperature is regarded with the amplitude of 1, the phase angle of 0, and the frequency of $\omega$, and the remained two disturbances are assumed to be zero. The second combination is that the internal wall surface temperature is regarded with the amplitude of 1, the phase angle of 0, and the frequency of $\omega$, and the remained two disturbances are assumed to be zero. The third combination is that the internal surface temperature of the embedded pipe is regarded with the amplitude of 1, the phase angle of 0, and the frequency of $\omega$, and the remained two disturbances are assumed to be zero.

In this paper, first disturbance combination is used to predict the performances of this structure by using CFD model and FDFD model. These boundary conditions are imposed on CFD model and FDFD model respectively, and the dynamic thermal performance of this structure can be obtained easily in the frequency range of normal concern ($10^{-8}$ to $10^{-3}$ rad $s^{-1}$) (Xu and Wang 2007). In order to obtain comprehensive performance prediction comparison by using CFD model and FDFD model, one frequency point with $1.04 \times 10^{-5}$ rad $s^{-1}$ (time period of 168 hour) is selected in this paper..

Figures 3 show the heat flux of the external wall surface, internal wall surface and internal surface of the embedded-pipe predicted by CFD model and FDFD model respectively with the frequency of $1.04\times10^{-5}$ rad $s^{-1}$(T=168h). Although the amplitude of the heat flux of the internal wall surface predicted CFD model and FDFD model can be noticed, the difference is small and the relative error is less than 7%. The heat flux on the external wall surface and the internal surface of the embedded-pipe agree well. It can be concluded that the thermal performance prediction by using FDFD model agree with that by using CFD model. It means that FDFD model can be used for thermal performance of this active pipe-embedded building envelope in frequency domain for frequency characteristic analysis.

It is noted that the computation time by using FDFD model is much less than by using CFD model. For a common configured desktop computer with Intel Pentium G630 of CPU frequency of 2.7GHz, CFD model needs about 3 hour to calculate the heat flux of each surface imposed by the sine temperature on the external surface with frequency point of $1.04\times10^{-5}$ rad $s^{-1}$ (time period of 168h). When the frequency becomes more low (i.e., time period becomes more longer), the computation time will be more. However, FDFD model just needs about 5 minutes to calculate the heat flux in frequency domain regardless of that the frequency is high or low. Therefore, FDFD model is good and quick for frequency characteristic analysis.

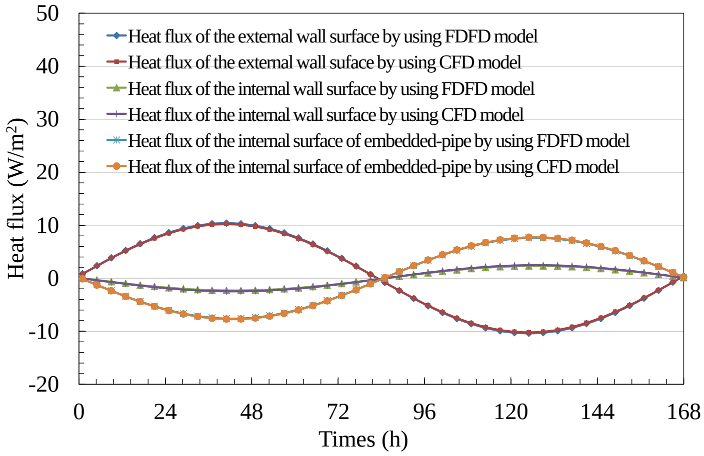


***Figure 3*** *Predicted heat flux comparison with frequency of $1.04\times10^{-5}$ rad $s^{-1}$ (T=168h)*

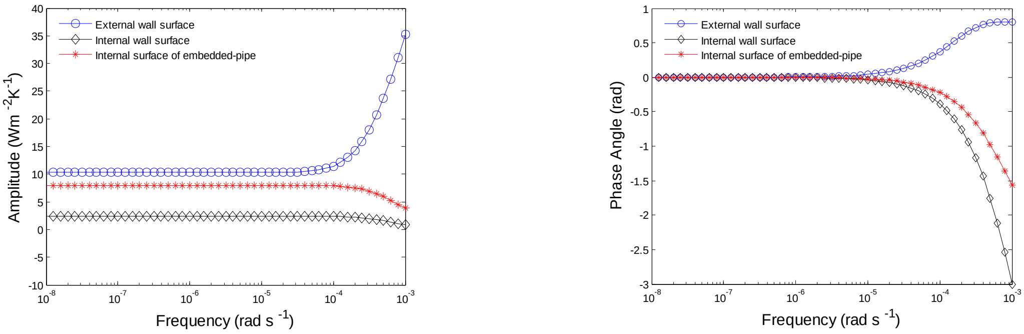
## 6. Frequency thermal performance of this structure

For the frequency characteristics analysis of conventional and regular walls without heat/sink source, it is usually assumed that one side is imposed with temperature of frequency of ω, and the other side temperature is zero (Wang and Chen 2001). For the frequency thermal performance analysis of this structure, similar means are used.

FDFD model is used for the frequency thermal performance prediction of the active pipe-embedded building envelope imposed by various disturbances. In this study, only the frequency characteristics of heat flux on the external wall surface, the internal wall surface and the internal surface of the embedded pipe imposed by the first disturbance combination (details given in the section 5)are presented. The frequency range of concern is from $10^{-8}$ to $10^{-3}$ rad $s^{-1}$.

Figure 4 shows the heat flux performance in frequency domain (i.e., frequency response) of these three surfaces of the building envelope imposed by the first disturbance combination in the frequency range of concern. These heat flux are indicated by amplitude and phase angle. The frequency responses of these surfaces are almost kept constant in the low and medium frequency regions. In the medium and low frequency regions, the amplitude of the heat flux on the external wall surface, internal wall surface and the internal surface of the embedded pipe are 10.2W/m$^2$,2.4 W/m$^2$ and 7.9 W/m$^2$ as shown in Figure 4(a) respectively while the phase angle of the heat flux on these surfaces are almost zero as shown in Figure 4(b).

However, in the high frequency regions, the frequency response is changed significantly with the increase of frequency. As shown in Figure 4(a), the amplitude of the external wall surface heat flux is increased as the increasing of the frequency, and the phase angle of the external wall surface heat flux leads the given external disturbance in the high frequency region as shown in Figure 4(b). The frequency responses of the internal wall surface and the internal surface of embedded-pipe are different from that of the external wall surface. In the high frequency region, the amplitude of heat flux of the internal wall surface and the internal surface of embedded-pipe is decreased as the increasing of the frequency as shown in Figure 4(a) while the phase angle lags the given external disturbance as shown in Figure 4(b).



(a) (b)

***Figure 4** Frequency responses of the active pipe-embedded building envelope*

## 7. Conclusion

Active embedded-pipe building envelope has the potentials to use low-grade energy source for building energy consumption, frequency thermal characteristics is of importance for performance evaluation. In this study, FDFD model is proposed for the frequency response analysis of the active embedded-pipe building envelope under various disturbances.

For evaluation of the accuracy and effectiveness of the FDFD model for frequency characteristics analysis, CFD model is developed for calculating the thermal response of this structure under sine temperature wave on the external surface with a frequency point for reference. The results show that the thermal responses of these three surfaces predicted by FDFD model agree well with that predicted by CFD model. On the other hand, FDFD model requires much less computation time than CFD model although CFD model may predict these frequency responses indirectly.

With FDFD model, the frequency thermal performance of the active pipe-embedded building envelope can be predicted quickly with acceptable accuracy and good reliability. The results shows the frequency responses of these surfaces are almost kept constant in the low and medium frequency regions. However, in the high frequency regions, the frequency response is changed significantly with the increase of frequency. With these frequency characteristics, simplified models may be developed, and the parameter may be identified by frequency characteristics matching. The further work is still in progress.

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