MODELLING OF HEAT EMITTERS EMBEDDED WITHIN THIRD ORDER LUMPED PARAMETER BUILDING ENVELOPE MODEL

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ABSTRACT
A dynamic modelling approach for heat emitters embedded within an existing third order lumped parameter building envelope model is reported in this work. The dynamic model has been found to provide more accurate results with negligible expense of computational time compared to a conventional quasi-dynamic model. The dynamic model also is preferred over the quasi-dynamic model as it allows for modelling emitters with high thermal capacity such as under-floor heating. Recommendation for this approach is justified through a series of analyses and comparative tests for various circuit options, timesteps and control volumes.

INTRODUCTION
With the increasing processing power in personal computers, sophisticated building energy simulation tools are widely adopted at early design stages. These tools are used to predict and inform building designers of the impact that system selection and building design has on building energy consumption throughout a building’s life. There has also been an increase in recent years for mandatory energy benchmarking using building energy simulation tools (such as energy performance certificates in the UK).

Many such simulation tools are however time-intensive both in model construction and, subsequently, in model implementation and run time. For instance, Hensen & Lamberts (2011) reported that an approximately 200 and 100 hours are spent on loads calculation and energy analysis respectively for a large design project. Therefore the amount of time required for building energy simulation often deters building designers from exploring different design/systems/materials options which could potentially be used to achieve optimum building design in terms of energy.

Hence, a significant reduction in time spent on setting up and simulating the building with HVAC systems model is needed. In addition, efficient modelling methods are needed in the search for optimum HVAC system configurations for every new building (Wright et al., 2008).

There is therefore a need for new simulation tools that permit both easy model construction and low run time whilst not compromising on model detail and rigour.

AIMS AND OBJECTIVES
The aim of this work is to develop and evaluate a new heat emitter model capable of being embedded within a third-order lumped-parameter building model and suitable for simulating a range of alternative capacity control methods whilst maintaining its computational efficiency.

The objectives are as follows:
- Develop an alternative quasi-dynamic heating system model using conventional algebraic equations and a fully-dynamic heating system model using differential-algebraic equations
- Impose alternative control strategies of external weather-compensation (feed-forward), local flow-control (feed-back) and a combined feed-forward and feed-back scheme
- Embed the alternative models into an existing third–order lumped-parameter building model
- Evaluate the alternative heating modelling methods and control strategies taking account of alternative time steps, emitter control volumes, and emitter types

REVIEW OF LITERATURE
From an engineering perspective, it is necessary to undertake a holistic approach to accurately predict the thermal interaction between the building and its services in order to implement HVAC systems efficiently.

Building energy simulation tools have undergone 40 years of developments and the earliest is referred to as the “handbook” approach by Clarke (2001). Later tools adopted the Load, System, Plant and Equipment (LSPE) approach using weighting factors examples of which include DOE-2 and BLAST (Sowell & Hittle, 1995; Crawley et al., 2001). Despite low computation cost, the limitation for such approaches lay in the use of a single set of constant pre-defined factors which neglect the vitally important interactions between load, systems and plant (Al-Homoud, 2001). Therefore, a third generation of
tools made significant improvement by simulating the building envelope and plant equipment simultaneously (Hong et al., 2008). In addition, such tools were the first to implement a graphical user interface (GUI).

Building energy simulation tools (whether giving a rigorous treatment of plant and controls or not) have historically been classified according to the method by which they deal with conduction through construction elements. Numerical solutions of the governing equations using, for example, finite differences were developed by Clarke et al., (1986). Alternative analytical methods such as the response factor method (first proposed by Stephenson and Mitalas (1967)) and, later, the use of conduction transfer functions continue to be widely used in North America. Similarly, frequency response techniques interpreted, for example, as the admittance method have been adopted by the CIBSE for UK practice (Clarke 2001; Underwood & Yik, 2004).

A further method, again analytical, is the use of lumped parameter modelling in which model order can be considerably reduced whilst most of the dynamics of interest are captured (e.g. Lorenz and Masy (cited in Gouda et al., 2002); Tindale, 1993; Crabb et al., (1987)). Lumped parameter modelling has the potential to give high computational efficiency however it is a relatively under-researched approach and, in particular, little is understood about the accuracy of this approach over extended time horizons. This method has been adopted in the present work due to its computational efficiency with the intention of quantifying long-term accuracy as a key objective (though this aspect is not reported here).

The method of building energy modelling was first proposed in 1980 by Laret and the first attempt made for modelling was by Lorenz and Masy (cited in Gouda et al., 2002) and, subsequently, applied by Crabb et al., (1987). Later, as highlighted by Levermore (1988) the need to have a third time constant, which was developed by Tindale (1993). Tindale (1993) identified the inadequacies of the previous versions (2nd order) of the lumped-parameter model and proposed straightforward solutions to improve its accuracy without compromising its simplicity or speed. Modifications made include separate treatment of radiative and convective heat through developing the rad-air model, modelling of partitions for inter-zone conduction and ground floors and addition of a third time constant to improve dynamic response, hence the name 3 Time Constant, 3TC.

Due to its computational efficiency, the lumped parameter method offers significant promise for dealing with plant and controls with a reasonable high level of detail and rigour. Previous work in this area is due to Crabb et al., (1987), Gouda et al. (2000) and Hanby (2008). However, with the exception of Crabb et al.’s contribution, the previous work is mainly bespoke with no generic tools emerging from the research done and in most cases, led to either single zone models or restrictions in zone numbers.

In the present work, the intention is to create a fully-integrated building modelling package including a lumped parameter building envelope model with no restrictions on the number of zones and a detailed heating plant and controls modelling procedure. It is intended that the integrated model will be equally applicable to practitioners as well as researchers.

**INTEGRATED MODEL**

The newly developed heat emitter model, written as a standalone class using object oriented programming language C++, is embedded within the existing third order lumped parameter simplified building envelope energy model developed in C++ previously.

At each timestep, the lumped parameter building envelope energy model calculates the internal room condition (i.e. zone’s floating temperature) for the simulated zone based on current external boundary conditions.

Checks are performed to see if the zone’s heating setpoint is achieved based on the zone’s internal temperature. If not, additional heating demand \( Q_{\text{heating}} \) is subsequently calculated by summing the convective \( Q_{\text{conv}} \) and radiative \( Q_{\text{rad}} \) plant loads using Equations (1) and (2) respectively (Tindale, 1993).

\[
Q_{\text{pa}(z)} = \frac{r_{\text{pr}}(z) - \theta_{\text{a}(z)}}{a_1 + a_2 R_F}\]

(1)

\[
Q_{\text{pr}(z)} = R F Q_{\text{pa}}\]

(2)

This heating demand generated by the building envelope energy model is used to determine the mass flow rate for variable mass flow rate circuits. Variable temperature circuits on the other hand use the external dry bulb temperature as an input parameter to control the flow water temperature for the heat emitter model.

The heat emitter model embedded within the zone loop of the lumped parameter model then used these updated parameters to calculate for emitter’s return water temperature. With all the necessary parameters calculated, the actual heating delivered by the emitter for the zone can be determined.

The actual heat delivered by the heat emitter model is firstly split into radiant and convective heat before feeding it back to the building envelope model. The typical proportions of radiant and convective heat from emitters are dependent on the emitter type can be found in Table 5.4 of CIBSE Guide A (CIBSE, 2006).

The radiative and convective heat delivered are then fed back to the lumped parameter building envelope energy model and is used to recalculate the zone’s internal room temperature within the same timestep.
Unlike commonly adopted heating setpoint tracking method that assumes the heating demand by the building is constantly met by the plant/emitter, this approach imposes the actual heat output capable of being delivered by the emitter at each time step. Hence, this approach provides a better, more realistic representation of the interaction between the building’s fabric and plant equipment for an annual building energy simulation.

**HEAT EMITTER MODEL**

Two heat emitter models were developed; a simplified ‘quasi-dynamic’ and a more rigorous fully dynamic model. The first of these models was developed based on the well-established calculation method presented in BS 3528 (British Standards Institution, 1977) to calculate the thermal output from any space heating appliances operating with steam or hot water. ASHRAE (ASHRAE, 2004) and CIBSE (CIBSE, 2005) guides have provided further guidance and example calculations steps.

The energy of the air side of the heat exchanger is given by:

\( Q = K \left( \frac{T_{r} - T_{u}}{2} \right)^{n} \) \hspace{1cm} (3)

Where the typical recommended index values (n) by ASHRAE (2004) are as follows – Radiators: 1.2, Baseboard radiation: 1.31, Convectors: 1.42, and Underfloor heating: 1.0.

K is the heat emitter’s coefficient and is to be calculated using design conditions using Equation (4) which is rearranged from Equation (3). The emitter coefficient is assumed to be constant.

\[ K(z) = \frac{Q_{Des(z)}}{\left( T_{rDes(z)} - T_{SP(z)} \right)^{n(z)}} \] \hspace{1cm} (4)

Similarly, the design mass flow rate for each zone is calculated from Equation (5) using design conditions. It should be used to either set the flow rate for zones with constant flow rate circuit or adjusted accordingly for varying flow rate circuits.

\[ \dot{m}_{Des(z)} = \frac{Q_{Des(z)}}{C_p(T_{rDes(z)} - T_{rDes(z)})} \] \hspace{1cm} (5)

**Heating Control Options**

Three alternative control configurations have been included in the heating model:

1) Constant Temperature, Variable Flow Rate (CTVF)
2) Variable Temperature, Constant Flow Rate (VTCF)
3) Variable Temperature, Variable Flow Rate (VTVF)

For circuit options with constant flow temperature (CT) or constant flow rate (CF), the design flow temperature and design flow rate for the particular zone are applied respectively.

**Variable Flow Water Temperature (VT)**

The emitter’s variable flow temperature is determined by using the external dry bulb temperature at the current timestep as a control input parameter. It is linearly proportioned between the heating design external dry bulb temperature and a balance point temperature (i.e. a classical ‘weather-compensating schedule’) with reference to a minimum practical emitter flow temperature and design flow temperature (Equation (6)). This method of control is frequently adopted in the UK because it leads to reduced installation cost. However, the controller takes no account of useful heat gains in the zone which often result in over-heating.

\[ T_{f(z)} = \min \left( T_{fDes(z)}, \max \left( T_{fMin(z)}, T_{fDes(z)} - \left( T_{rDes(z)} - T_{rMin(z)} \right) \times \left( \frac{T_{as(z)} - T_{asMin(z)}}{T_{BP(z)} - T_{asMin(z)}} \right) \right) \] \hspace{1cm} (6)

**Variable Mass Flow Rate (VF)**

The heating demand by the zone is used to determine the varying flow rate into the emitter with reference to the heat capacity of the emitter \( Q_{Dem(z)} \) selected for zone. The zone’s heating capacity is referred to the maximum heating output capacity by the emitter. By using the zone’s heating capacity as a reference point for design mass flow rate, the intermediate heating demand of each zone can be linearly proportioned accordingly.

\[ \dot{m}_{(z)} = \dot{m}_{Des(z)} \times \frac{Q_{Dem(z)}}{Q_{Des(z)}} \] \hspace{1cm} (7)

**Quasi-Dynamic Model**

Once the emitter model’s parameters have been defined, the return water temperature of the emitter for the zone can be determined applying Newton-Raphson method. This is interpreted in Equation (8) where \( x \) is the variable whose value is to be determined (i.e. return water temperatures at the current timestep), \( f(x) \) is a function of the variable formed by an energy balance involving Equations (3) and (5) and \( f'(x) \) is the first derivative of the function with respect to \( x \).

\[ x_{(i+1)} = x_{(i)} - \frac{f(x_{(i)})}{f'(x_{(i)})} \] \hspace{1cm} (8)

**Dynamic Model**

The dynamic heat emitter model adopts the following energy balance:

\[ C \frac{dT_{r}}{dt} = mC_p(T_{r_{(n-1)}} - T_{r_{(n)}}) - AU(T_{r_{(n)}} - T_{al}) \] \hspace{1cm} (9)
\[ T_{r(n)} = C_1 \left[ T_{r(n)}^{-} + mC_2 T_{r(n-1)} + C_3 T_{a(z)}^{-} \right] \]  
(10)

Where \( C_1, C_2 \) and \( C_3 \) are constant coefficients and are known as:

\[ C_1 = \frac{c}{\Delta t m C_p + \Delta t U/V}, \quad C_2 = \frac{\Delta t C_p}{c}, \quad C_3 = \frac{\Delta t U/V}{c} \]

From Equation (10), at each timestep, \( T_{r(n)} \) will be known but \( T_{r(n-1)} \) and \( T_{a(z)} \) will be unknown. Hence an iterative loop is used to solve for \( T_{r(n-1)} \) ... \( T_{r(N)} \) until convergence.

**Model Input and Output Parameters**

The table below summarises the input and output parameters of the heat emitter model. The number of inputs will vary depending on the choice of calculation method. A convergence tolerance of 0.01 has been applied for all test cases in this paper. Results are analysed based on validity, accuracy and computational speed.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Heat Emitter Model’s Inputs and Outputs</th>
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<tbody>
<tr>
<td>Inputs</td>
<td>Outputs</td>
</tr>
<tr>
<td>( T_{f(Des)} )</td>
<td>( C_p )</td>
</tr>
<tr>
<td>( T_{r(Des)} )</td>
<td>( n )</td>
</tr>
<tr>
<td>( T_{Min} )</td>
<td>No. of Segments</td>
</tr>
<tr>
<td>( T_{SP(z)} )</td>
<td>Material Mass</td>
</tr>
<tr>
<td>( Q_{Des(z)} )</td>
<td>Material Cp</td>
</tr>
<tr>
<td>( Q_{Dem(z)} )</td>
<td>Volume (l)</td>
</tr>
<tr>
<td>( T_a0 )</td>
<td>Convergence Tolerance</td>
</tr>
<tr>
<td>( T_{a0} )</td>
<td>( \Sigma Q_{Del(z)} )</td>
</tr>
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</table>

**TEST CASE BUILDING**

For the purpose of this research, a test case building model located in London has been used based on hourly 2005 London Gatwick weather data. The building model is a single zone, single storey (3.5m) office building with a flat roof and a total floor area of 154m² facing in a North-South direction.

The floor consists of 4 layers with a total U value of 0.249W/m²K made up of formaldehyde foam (132mm), cast concrete (100mm), floor screed (70mm) and timber wood flooring (30mm) with an assumed constant ground temperature of 14°C.

The roof is made up of (from external to internal), asphalt (10mm), medium weight glass wool (rolls) (140mm), an air space and plasterboard (13mm) with an overall total U-value of 0.25W/m²K.

The walls are made up of 100mm of brickwork outer leaf, 70mm of XPS extruded polystyrene, 100mm of concrete block and 13mm of gypsum plastering with a U-value of 0.35W/m²K.

All four walls facing each direction are double glazed with 10 percent wooden frame and have an overall U-value of 1.97 W/m²K. The panel on the outer surface is made up of 3mm clear glass with an emissivity of 0.84 followed by 13mm of air and another 3mm of clear glass.

Casual gains for the test case building include occupant at 9m²/person from 8 to 7pm, 5 days per week, lighting at 19W/m² from 8am to 7pm, 5 days per week and office equipment operating at full load (1.8kW) from 7am to 7pm with standby operation (5%) throughout the year.

Ventilation to the zone includes 10litres/s/person (1.1 air changes per hour (ACH)) of fresh air mechanically supplied during occupied hours with a constant 0.7ACH of external air infiltration.

The heat emitter model is scheduled to operate from 7am to 7pm, 5 days per week taking into consideration two hours of pre-heat during the heating season.

The underfloor heating example is applied assuming an additional layer of 40mm screed topping with a density of 1200kg/m³ above 30mm diameter pipe spaced at 300mm across the length of the zone over the whole floor area. For the radiator example, a 5kg of material mass and 5litres of emitter water is assumed.

**Evaluation**

The heat emitter model has undergone an extensive series of checks for robustness and accuracy. A code-checking procedure has been carried out at random timesteps whilst stepping through the code. Computer generated results are compared against manual calculations over a range of possible scenarios ensuring the robustness and rigour of the model is not compromised whilst maintaining its flexibility and functionality. It is intended to carry out more detailed verifications tests in later work.

The heat emitter model is capable of providing the flexibility for a combination of emitter types for a multi-zoned building.

Results generated from an annual hourly simulation generated are extracted starting from the second week (\( 8^{th} \) day) ensuring all initial condition assumptions will have no impact on the accuracy of results.

**RESULTS AND DISCUSSION**

Results have been generated for a series of possible heat emitter configurations. Figure 1 presents the internal room temperature for the various circuit options simulated using the quasi-dynamic and dynamic models for both underfloor heating and radiator over a period of 48 hours. To allow for comparisons to be made, both models are simulated at one timestep per hour (i.e. a 1-hour integration interval) achieving heating setpoint. It is clear that the quasi-dynamic model can be seen generating unstable internal room temperature around the heating setpoint resulting in a “saw-tooth” effect. The dynamic model however, can be seen to achieve the
heating setpoint better than the quasi-dynamic model in most cases.

A further significant disadvantage of the quasi-dynamic heating model is that during conditions of light load, the algorithm predicts a return water temperature less than the internal room temperature which is clearly invalid for heat transfer to take place. This arises because the model assumes that the emission rate varies as a function of the mean water temperature. As the mean water temperature reduces at light loads, the return temperature falls below the room temperature.

This can be further explained as the quasi-dynamic model calculates the return water temperature by assuming a uniformly temperature across the emitter (‘stirred tank’ energy balance) which only occurs during ideal conditions and hence, return water temperature falling below the room temperature on occasions with light loads.

The dynamic model on the other hand has its advantage as it more realistically predicts a gradual loss of heat (reduction in emitter water temperature) across the emitter’s segments/length to its surrounding from inlet to outlet over time. Therefore, the gradual reduction in water temperatures generates a higher mean water temperature which prevents invalid scenarios where the return water temperature falls below the room temperature.

Achieving Heating Setpoint
It is evident that perfect set point tracking has not been achieved by internal room temperature for test scenarios presented in Figure 1. This can be explained by the well-understood ‘proportional offset’ from the proportional control method used to vary flow temperature and flow rate. This offset can be eliminated by adding integral control action.

However, there are two key reasons that preclude the implementation of integral control into the heat emitter model. Firstly, the model would have to be solved at smaller time intervals of a few seconds for the integral action which is considered unfeasible as it would significantly add to the computational time. In addition, emitters using common controls such as TRV (Thermostatic Radiator Valve) adopt proportional control so integral action is not required.

Overheating for Weather Compensated Control
It can be noted that variable flow temperature circuits with weather compensated control tend to overheat with up to 3°C for underfloor heating and 5°C for radiators. Such occurrence can be predicted of these circuits as the temperature sensors are located externally instead of within the occupied space. As a result, the emitter will often continue to supply heat into the space even when the zone’s temperature is above heating setpoint as the controller does not have a feedback control signal on the current zone’s internal temperature and thus, overheating.
Figure 2 Results for Various Timesteps from Dynamic Model

is experienced. This is a common flaw with weather-compensating control in that it conventionally fails to account for adventitious heat gains to the space, resulting in over-heating.

Computational Cost

From the above analysis, the dynamic model seems to be generating more accurate results over the quasi-dynamic model. However, the computational cost of the dynamic model might be expected to be higher. Therefore, comparison tests on computational time taken to simulate an annual hourly 8760 timesteps using both heating models have been carried out.

It is noted that there is no disadvantage in terms of computational time for the dynamic model as the time taken for both fully dynamic and
quasi-dynamic models are approximately 15(±11) seconds for an annual simulation for a single zone, single storey building.

The surprising similarity in computational cost can be explained by the fact that both the quasi-dynamic and dynamic model solve iteratively for different reasons. In addition, it appears that both methods require a similar number of sweeps at each time step through their respective iterative loops to reach convergence.

Advantages using the Dynamic Model

With such negligible differences in computational time and advantages of achieving setpoint better by taking into account the building’s thermal capacity, the dynamic model is preferred.

Moreover, the dynamic model eliminates the problem with the low return water temperature and it more realistically captures an emitter’s response over time which is vitally important for high thermal capacity emitter types such as underfloor heating. Therefore, for a better, more realistic representation of an actual heat emitter’s performance in a building, the dynamic model is recommended.

Varying Timesteps and Segments

Results generated using the dynamic model for each of the circuit types across a range of timesteps is presented in Figure 2. This test was carried out to investigate whether increasing the number of emitter time steps had an impact on results and computational time. The algorithm used maintained to the time step for building envelope calculations at 1h but permitted up to 60 smaller integration timesteps per hour (i.e. an integration interval of 1min) to be applied to the emitter – in effect a nested 2-speed solution method.

The increment of timesteps indicates insignificant improvements for the internal room temperature over a range of circuit and emitter types generated by the dynamic model.

A further investigation into the impacts number of segments (1, 3 and 5 segments) has on accuracy based on total heating delivered has been carried out for both emitters and results are given in Figure 3.

Results indicate negligible differences with the timestep increment for actual heating delivered which is also found in achieving setpoint for internal room temperature.

A trend can be observed in the actual heating delivered across the number of segments for both emitter types. The results reveal a significant jump in the actual heating delivered between 1 and 3 segments with little improvements in accuracy gained using 5 segments for all circuit types and both emitters.

Therefore the findings for this paper suggest that \( n = 3 \) segments will in most cases generate acceptable results which are similar to the findings and recommendation by Underwood and Yik (2004).

In summary, it was found that increasing the number of timesteps dedicated to emitter calculations and increasing the number of model segments (control volumes) from 3 to 5 had a negligible effect both on overall computational cost and on the accuracy of the results obtained.

CONCLUSION

Extensive comparative tests and analysis have been conducted on the heat emitter model embedded within the third order lumped parameter building envelope energy model.

Two methods for modelling heat emitters have been investigated; a quasi-dynamic method and a fully dynamic method.

This research found that the fully-dynamic heating system model provided more stable and results when coupled with this type of building envelope model particularly when dealing with high thermal capacity heating such as underfloor heating. A further conclusion is that the additional computational cost due to the dynamic modelling approach was negligible.

FUTURE WORK

Further investigation on the performance of the integrated annual building energy package for multi-zone building types with a variety of thermal capacity and occupancy patterns shall be carried out.
Primary system plant shall be incorporated (including boilers, heat pumps and other sources) and the validity of the results verified using inter-program comparison tests. The BESTEST method shall be adopted to compare results generated from the integrated third order lumped parameter with HVAC against other software that allows for intermediate complexity modelling of HVAC systems and plant equipment shall be applied.

NOMENCLATURE

- \( T_{BP} \) Balance point outside air temperature (°C);
- \( K \) Emitter coefficient;
- \( T_{Fr} \) Emitter flow water temperature (°C);
- \( T_{Rr} \) Emitter return water temperature (°C);
- \( T_{fMin} \) Emitter minimum flow temperature (°C);
- \( n \) Emitter index,
- \( T_{Set} \) Heating setpoint temperature (°C);
- \( T_{ai} \) Internal dry bulb air temperature (°C);
- \( \dot{m} \) Mass flow rate (kg/s);
- \( T_{ao} \) Outside dry bulb air temperature (°C);
- \( A_U \) Overall heat transfer coefficient (W/m²K) * surface area (m²);
- \( Q \) Rate of heating energy (kW);
- \( C_p \) Specific heat capacity (kJ/kgK).

Superscript

- \( ^{-} \) Value of variable from previous timestep.

Subscript

- \( \text{Curr} \) Current timestep;
- \( (z) \) Current \( z \)th zone;
- \( (n) \) Current \( n \)th segment of dynamic model;
- \( (l) \) Current iterative loop;
- \( \text{Des} \) Design condition;
- \( \text{Dem} \) Demand of heating energy;
- \( \text{Del} \) Delivered heating energy;
- \( \text{N} \) Total no. of segment in dynamic model.

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REFERENCES


