

VARIABLE CONVERGENCE IN EVOLUTIONARY OPTIMIZATION AND ITS RELATIONSHIP TO SENSITIVITY ANALYSIS

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ABSTRACT

Optimization methods provide building designers with optimized design solutions, whereas sensitivity analysis methods indicate the extent to which the design variables have an impact on building performance, this information being important when uncertainty exists in the design decisions. This paper investigates the extent to which the solutions found from a building design optimization can be used for a sensitivity analysis. It is concluded that the solutions found at the start of the optimization process can be used in a global sensitivity analysis. It is also concluded that the convergence behaviour of the search does not relate to the global sensitivity.

INTRODUCTION

Sensitivity analysis and model-based optimization are both used to inform design decisions. Sensitivity analysis (SA), identifies the design variables that have the greatest impact on the design objectives and constraints, whereas model-based optimization is used to find the combination of variable values that optimize the objectives while satisfying the design constraints. A designer using both methods would therefore be provided with an optimum design solution and an understanding of which variables have most impact on the solution (this being important when some uncertainty exists in the choice of design solution).

Global sensitivity (and uncertainty) analysis is typically applied using a Monte Carlo or Latin hypercube sampling of the solution space, with the sensitivity of the design objectives and constraints to changes in the variable values being derived from a linear regression fit or analysis of the variance of the samples. Among other research in the field of sensitivity and uncertainty analysis; Tian and De Wilde (2011) explored the uncertainty and sensitivity in the probabilistic prediction of future building performance under climate change; Moon and Augenbroe (2005) adopted a mixed simulation approach to analyse mould growth risk in buildings, with consideration of uncertainties in several building parameters; and Capozzoli et al. (2009), examined the sensitivity indexes of building energy

performance indicators for a set of different design variables and for different climatic zones; Hopfe and Hensen (2011), similarly studied the impact of design uncertainty on building performance.

Where the global sensitivity analysis is performed in relation to changes in the value of the design variables, it is logical to use a uniform distribution of samples, the assumption being that the "uncertainty" in the design decisions is uniform across the design space. Since the sensitivities can vary across the design space, they are likely to be influenced by the region over which the variables are sampled. Although they are not based on sensitivity analysis methods, it is probable that the higher ranked variables identified from a sensitivity analysis would be the fastest variables to converge in a numerical optimization (Marseguerra et al, 2003). Optimization processes also generate a number of solutions that might be used in a sensitivity analysis (Marseguerra et al, 2003), although rather than being uniformly distributed, they would be biased and centred around the optimum value of the design variables. Rather than being a disadvantage, this might be appropriate as the sensitivity analysis would be biased towards the region of most interest (the region of the optimum solutions). The majority of building optimization problems are constrained, so that the rate of convergence of any variable is influenced by multiple criteria (the design objective(s) and constraints), this potentially making it more difficult to extract the separate sensitivities from the sample solutions found from the optimization process.

The computational effort required to complete a combined design optimization and sensitivity analysis could be reduced if it is possible to either use a sample of the solutions from the numerical optimization in a global sensitivity analysis, or to use the rate of convergence of the variables during the optimization as a direct indication of the relative sensitivities. This paper therefore investigates the extent to which:

1. the solutions obtained from the building optimization process can be used for a global sensitivity analysis;

2. the rate of convergence of the variables during the optimization can be observed, and used as an indication of the relative sensitivity of the design objectives and constraints to changes in the variable values.

The particular results are dependent on the choice of optimization problem and the algorithm used to solve it. The analysis in this paper is for a single-objective constrained optimization problem that includes a range of problem variable types (discretized implementation of continuous variables, discrete variables, and categorical variables). The optimization problem has been solved using a conventional Genetic algorithms (GA) (Goldberg, 1989), since these probabilistic population-based optimizers are known to be effective in solving building optimization problems (Wetter and Wright, 2004). Their effectiveness has led to them being applied widely in building optimization research, including; HVAC system sizing (Hanby and Wright, 1989; Wright, 1996; Wright, et al., 2002); building envelope optimization (Caldas et al. 2003, Evins 2010, Jin et al. 2011); space layout planning (Jo 1998) building form and structure (Coley and Schukat, 2002; Geyer, 2009; Caldas, 2008); and building facades (Zemella et al., 2011).

EXPERIMENTAL APPROACH

The extent to which the solutions obtained from the optimization process can be used in a global sensitivity analysis is investigated by comparing the results of the global sensitivity analysis of a set of randomly generated solutions, with the same analysis applied to solutions generated by the optimization.

The convergence behaviour of the optimization algorithm, and whether the rate of variable convergence is an indication of the relative importance of the variable, is examined using a moving average and standard deviation of the variable values.

The sensitivity and algorithm convergence analysis is performed for both the objective function (building energy use), and solution infeasibility (based on thermal discomfort).

Sensitivity Analysis

The sensitivity analysis (SA), adopted here is based on a step-wise linear regression analysis of rank-transformed variable values (Saltelli et.al., 2008). The use of rank transformation data tends to mitigate against problems associated with fitting linear models to nonlinear data. A step-wise regression analysis provides an insight into the relative importance of the variables in several ways. The more important the variable, the earlier it will be added to the regression model. The magnitude of the change in the model coefficient of determination

(R^2), due to the inclusion of the variable in the model, is also an indication of the variables importance. However, here, we focus on the standardised rank regression coefficients (SRRC), for which the higher the absolute value, the greater the influence of the variable on the solution sensitivity. Care must however be taken in interpreting the SRRC's, as any correlation between the variables can result in a misleading indication of variable importance. In general, the results are also only considered valid for values of $R^2 > 0.7$

The robustness of the sensitivity analysis is dependent on the sample size and the manner in which the samples are generated. However, for a sample size of 100 and above, the difference in robustness of the results from different sampling methods reduces to the point that it is feasible to use simple random sampling of the solution space (Macdonald, 2009). In order to confirm that this is applicable to this study, the sensitivity study is performed using several different sample sizes:

- 1000 randomly generated samples.
- 100 randomly generated samples.
- 100 samples taken at the start and end of the optimization.
- All solutions generated during the optimization (a total of 3588 solutions including 3000 unique solutions).

Genetic Algorithm

The form of genetic algorithm and its operators used in this study are:

- Gray encoded binary chromosomes.
- Binary tournament selection.
- Uniform crossover (100% probability of chromosome crossover with 50% probability of gene crossover).
- Single bit mutation (a probability of 1 bit per chromosome).
- Stochastic Ranking fitness assignment (with a 45% probability of infeasible solution bias; (Runarsson and Yao, 2000)).
- Retention of a single elite solution.
- Population size of 20 individuals.
- Solution archive, used to eliminate the need to re-evaluate solutions found more than once during the search.
- Algorithm stopped after 3000 unique solutions have been evaluated.

A binary chromosome has been adopted since it allows the universal encoding of both discretized continuous variables and discrete variables (including the categorical variables). Tournament selection, uniform crossover, single bit mutation, and single individual elitism, are all very common algorithm operators. A population size of 20 with 3000 unique evaluations has previously been found to provide acceptable convergence for a problem of similar size

and complexity to the problem studied here (Wright and Alajmi, 2005; Alajmi, 2006).

The stochastic ranking fitness assignment is robust, easily implemented, and has become a benchmark algorithm against which other single objective constrained fitness assignment algorithms have been compared. One important characteristic of the algorithm is that it allows some infeasible solutions to remain their fitness and therefore the population is likely to include some feasible solutions. This is important in this study as it presents the possibility of the solution obtained during the optimization of being used to assess the sensitivity of the solution infeasibility to changes in the optimization variables.

Algorithm Convergence Analysis

The convergence behaviour of an algorithm can be examined through changes in the mean value and standard deviation of the variable values from one generation to the next. However, for consistency with the sensitivity analysis, we have adopted a sample size of 100 solutions in calculating the mean and standard deviation of the variable values, this sample covering 5 populations. The window of 100 solutions has been moved through the search results, one population of 20 solutions at a time (new information only being introduced with each new generation). The effect of this is to create a moving average of the results that tends to smooth short-term fluctuation in any trend.

Given that it is possible for the mean trend to exhibit some convergence, but the standard deviation in solutions to remain high (and vice versa), the extent to which both are converging is examined here using the coefficient of variation (CV), defined here to be equal to standard deviation divided by the mean.

The extent to which the certain variables have converged, is also considered by comparing the results of the sensitivity analysis of the first 100 solutions from the optimization, with the last 100 solutions (the implication being that the order of importance of the variables should change from the start of the search to the end).

Example building and performance model

The example optimization problem is based on a mid-floor of a commercial office building (Figure 1). Although the example has 5 zones, in this study, only the design variables associated with the perimeter zones are considered and optimized. The two end zones are 24m x 8m, and the three middle zones 30m x 8m. All zones have a floor to ceiling height of 2.7m.

The working hours are 9:00 to 17:00. Each zone has typical design conditions of, 1 occupant per 10m² floor area and equipment loads of 11.5 W/m² floor area. Maximum lighting loads are set at 11.5 W/m²

floor area, with the lighting output controlled to provide an illuminance of 500 lux at two reference points located in each of the perimeter zones. Infiltration is set at 0.1 air change per hour, and ventilation rates at 8 l/s per person. The heating and cooling is modelled using an idealized system that provides sufficient energy to offset the zone loads and meet the zone temperature setpoint during hours of operation; other than the free-cooling available from the fixed ventilation rate, no extra free-cooling potential is modelled. Both heating and cooling are available all year, although the operating hours are different for the colder (November to April) “winter” months than the warmer (May to October) “summer” months. The internal zone has been treated as a passive unconditioned space.

The building performance has been simulated using EnergyPlus (V7). The building is nominally located in Birmingham UK, with the CIBSE test reference year used in simulating the annual performance (CIBSE, 2002)

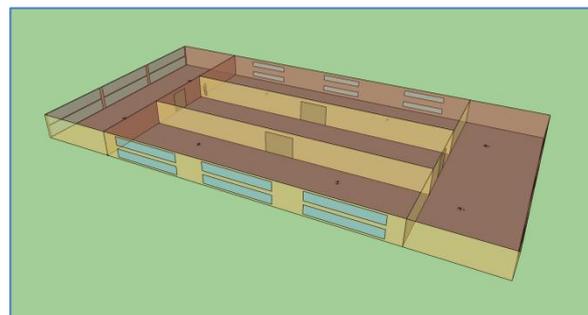


Figure 1 Example Building

Optimization Problem

Table 1 gives the optimization problem variables, and specifies their bounds and discrete increment in their value. The variables include orientation, heating and cooling setpoints (via the dead band), window-to-wall ratios, start and stop times, and construction type.

The building is orientated with the longest facades facing north (and south) when the orientation is set at 0°. The dead band has been optimized instead of the cooling setpoint to ensure that problem formulation does not result in an overlap of the heating and cooling setpoints. The window-to-wall ratios are applied by dividing the total window area across 6 equal size windows placed in three groups across each façade (Figure 1). The names given to the window-to-wall ratios in Table 1 reflect the general orientation of the façade for at the optimum solution (approximately that illustrated in Figure 1). The start and stop times are hours of the day.

The value of the categorical construction variables corresponds to a particular type of construction.

Three construction types are available for the external wall construction, a heavy weight, medium weight, and light weight construction (with each of these corresponding to a variable value of 0, 1, and 2). Similarly two floor and ceiling constructions (heavy and light weight), and three internal wall constructions (heavy, medium, and light weight) have been defined (with the heavy weight construction always corresponding to a variable value of 0, with the construction weight decreasing with increasing variable value). The alternative constructions have been taken from the ASHRAE handbook (ASHRAE, 2005). Two double glazed windows types are available, one having plain glass, and the second, low emissivity glass (the variable low emissivity glass corresponds to a variable value of 0).

*Table 1
Optimization Problem Variables*

Optimization Variable	Units	Lower Bound	Upper Bound	Increment	Optimum
Orientation	(°)	-90.0	90.0	5.0	25.0
Heating setpoint	(°C)	18.0	22.0	0.5	19.0
Heating set-back	(K)	0.0	8.0	0.5	8.0
Dead band	(°C)	1.0	5.0	0.5	5.0
North-East window-wall ratio	(-)	0.2	0.9	0.1	0.2
South-West window-wall ratio	(-)	0.2	0.9	0.1	0.5
South-East window-wall ratio	(-)	0.2	0.9	0.1	0.2
North-West window-wall ratio	(-)	0.2	0.9	0.1	0.9
Winter start time	(hrs)	1	8	1	8
Winter stop time	(hrs)	17	23	1	23
Summer start time	(hrs)	1	8	1	1
Summer stop time	(hrs)	17	23	1	23
External wall type	(-)	0	2	1	1
Internal wall type	(-)	0	1	1	0
Ceiling-floor type	(-)	0	2	1	0
Window type	(-)	0	1	1	1

The **objective function**, to be minimized by the optimization, is the building annual energy use. The energy use is given by the sum of the heating energy use, cooling energy use, supply and extract fan energy use, and the artificial lighting energy use.

The **design constraints** are that the thermal comfort in each of the perimeter zones should not exceed 20% predicted percentage of dissatisfied (PPD), for more than 150 working hours per annum. The constraint function are configured to return the number of hour above 150, or zero if the constraint is feasible. The **infeasibility** of a solution is equal to the sum of the square of the constraint values, so that an entirely feasible solution would have an infeasibility of zero, and an infeasible solution a positive value that is exaggerated by the worst of the violations.

Table 1, also gives the optimum solution found by the genetic algorithm. The optimum construction is generally of heavy or medium weight, with low emissivity windows. The longest façades of the building have been orientated 25° from true north (potentially reducing solar gain to through the south façade). The window areas are lowest in the east facing facades, with the largest windows being

placed on the north-west façade (this façade experiencing very little direct solar gain). The heating, cooling and ventilation systems are in operation for most of the day during summer, although the large dead band suggests that most of the unoccupied night operation corresponds to free-cooling (albeit at a fixed ventilation rate). The systems are also operated for a period after occupancy; this would be atypical, but might be driven by periods of free-cooling (confirmation of this requiring further investigation).

RESULTS AND ANALYSIS

Table 2 and 3 give the standardised rank correlation coefficients (SRCC), for the sensitivity of the energy use (Table 2), and solution infeasibility (Table 3), to the optimization variables. Four sets of SRCC's are given in each case; for a randomly generated sample of 1000 solutions, a randomly generated set of 100 solutions, the first 100 solutions from the optimization, the last 100 solutions from the optimization, and all solutions from the optimization. In both Table 2 and 3, the length of the bars indicates the relative magnitude of the SRCC and therefore the importance of the variable.

In relation to the choice of sample size from the randomly generated solutions, it can be seen from Table 2 and 3 that the smaller the sample size the fewer variables are included in the linear model (fewer variables have an SRCC value). However, it is also evident that the most important (having the highest SRCC values), are included in the model for the smaller sample size; this is certainly the case for the analysis of energy use (Table 2), but is marginally less true in the case of infeasibility (Table 3). It could therefore be concluded that a sample of 100 random solutions can provide an indication of the variables having the highest importance.

Table 2 and 3 also indicate that applying the sensitivity analysis to the first 100 solutions found by the optimization results in a similar indication of variable importance to the 100 randomly generated samples. The importance of this conclusion is that it suggests that the solutions found from the optimization can be used in a global sensitivity analysis and therefore it is not necessary to generate a separate random sample of solutions. This is perhaps surprising in that although solutions are taken early in the search, they span 4 iterations (5 generations), of the algorithm and therefore contain some biased solutions. A caveat on this conclusion is that the coefficient of determination R^2 is less than 0.7 for the infeasibility analysis (Table 3), and so the results for the infeasibility would normally be considered unreliable (even though they are similar to those for the 100 random samples).

Table 2
Standardised Rank Correlation Coefficients, for
Energy Use

Optimization Variable	+/-	Random Samples		Optimization Samples		
		1000	100	First	Last	All
Orientation	-	0.026				0.105
Heating setpoint	-	0.472	0.434	0.443	0.785	0.495
Heating set-back	-	0.129	0.114	0.174		0.046
Dead band	-	0.689	0.694	0.755		0.485
North-East window-wall ratio	+	0.053				
South-West window-wall ratio	+	0.117				
South-East window-wall ratio	+	0.213	0.234	0.176	0.126	0.146
North-West window-wall ratio	-	0.033			0.137	0.102
Winter start time	-	0.084	0.142			0.064
Winter stop time						
Summer start time	+	0.169	0.215		0.186	0.18
Summer stop time	-	0.095	0.122	0.13	0.207	0.099
External wall type	+					0.034
Internal wall type	+	0.037	0.106		0.149	0.029
Ceiling-floor type	+	0.085	0.134	0.147	0.117	0.063
Window type	-	0.197	0.214	0.111		0.043
R^2		0.839	0.844	0.867	0.819	0.721

Table 3
Standardised Rank Correlation Coefficients, for
Solution Infeasibility

Optimization Variable	+/-	Random Samples		Optimization Samples		
		1000	100	First	Last	All
Orientation	+					0.071
Heating setpoint	+	0.618	0.668	0.427	0.734	0.635
Heating set-back	+					0.04
Dead band	+	0.62	0.529	0.42	0.14	0.36
North-East window-wall ratio	+	0.1	0.137	0.302	0.343	0.227
South-West window-wall ratio	+	0.1			0.119	
South-East window-wall ratio	+	0.15	0.183		0.141	0.167
North-West window-wall ratio	-					
Winter start time	+					0.027
Winter stop time	-			0.134		0.041
Summer start time	+	0.088	0.111			0.118
Summer stop time	-	0.041			0.231	0.074
External wall type	-					
Internal wall type	-			0.126		0.043
Ceiling-floor type	+	0.027				0.1
Window type	-	0.108		0.275	0.138	0.107
R^2		0.849	0.832	0.678	0.721	0.624

Tables 2 and 3 also indicate that there is a change in the importance of the variables at the end of the optimization (based on the last 100 solutions of the search). In particular, due to the almost complete convergence of the deadband (Figure 3), its importance has been significantly reduced. In contrast, the importance of the heating setpoint has increased, which might be due to the increase in the spread of solutions (standard deviation) for this variable towards the end of the search (evident from the increase in the coefficient of variation shown in Figure 4). It is also interesting that a sufficient number of infeasible solutions exist at the end of the search to allow the sensitivity analysis to be completed with confidence ($R^2 > 0.7$).

Finally, the sensitivity analysis was applied to all solutions from the optimization. While there is some difference in the magnitude of the SRCC's for these samples and the 1000 randomly generated samples, and notwithstanding the fact that for the infeasibility, R^2 is less than 0.7, there is some overlap in the most important variables. The difference in the results is due to the bias in the optimization solutions around the optimum values (for example, Figure 2). As such,

it is possible that the analysis of the optimization solutions gives a better representation of the sensitivity local to the optimum that do the random samples; however, confirmation of this requires further research.

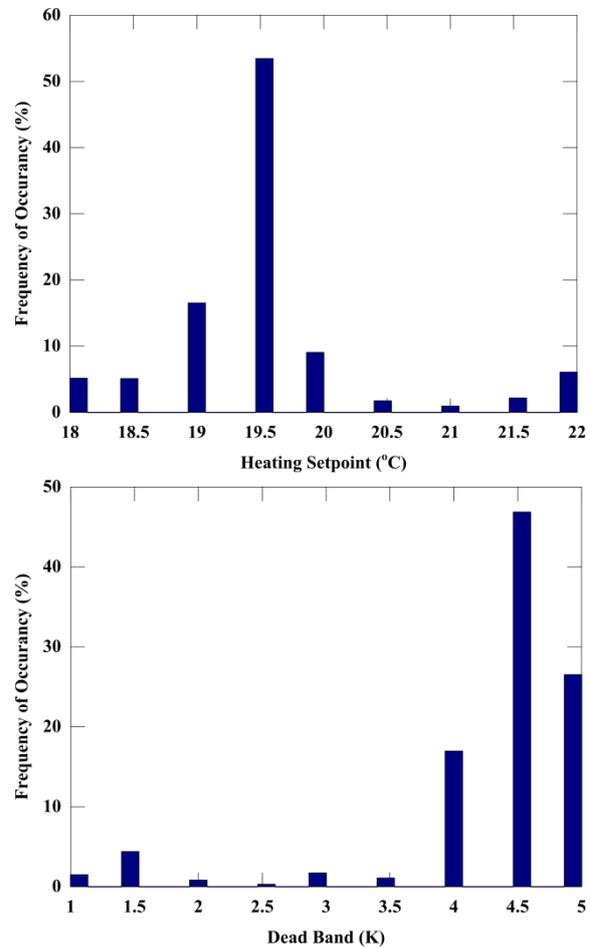


Figure 2 Variable Frequency Distribution (for 3000 unique solutions)

Figures 3 to 6 illustrate the convergence behaviour of a selection of the optimization variables. Each figure shows the change in the mean of a 100 samples and the coefficient of variation, the values having first been normalized to the variable bounds. The dotted line correspond to +/- 1.0 standard deviation on the mean value.

The extent to which the convergence behaviour can be used to indicate the importance of the variable has been investigated by examining the behaviour of: the two most important variables found from the global sensitivity analysis (deadband and heating setpoint, Figures 3 and 4); and two of the least important variables (North-West window to wall ratio, and the external wall type, Figures 5 and 6). Figure 3, illustrates that the most important variable found from the sensitivity analysis exhibits early convergence and that by the end of the search the

standard deviation is negligible (with the result that the coefficient of variation is close to zero). A question that arises in respect to this behaviour, is to what extent is the stable convergence driven by the importance of the variable, and to what extent is it a result of the solution lying on the upper bound of the variable range. A solution that is convergent on a bound is likely to have a lower standard deviation in the solutions as the search is unable to generate solutions that are beyond the bound; the clustering of the solutions around the upper bound is evident in Figure 2. Confirmation of the driver for stable convergence therefore requires further investigation.

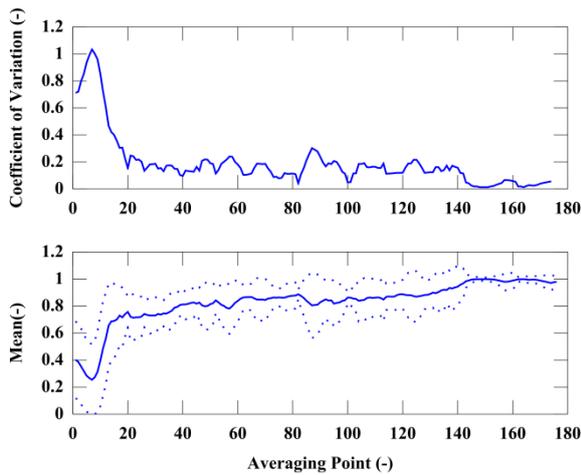


Figure 3 Deadband Convergence

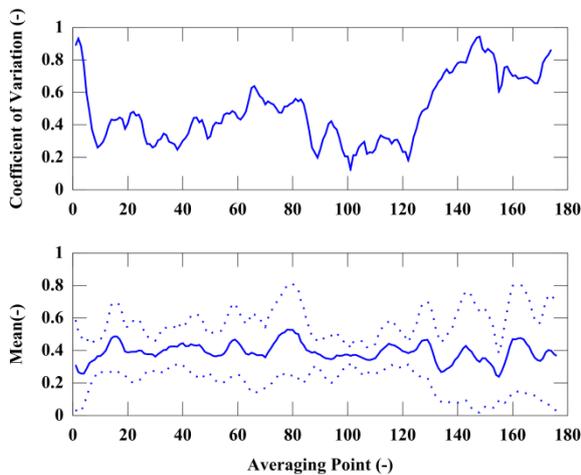


Figure 4 Heating Setpoint Convergence

It might be expected that the next most important variable, the heating setpoint, would exhibit similar behaviour, although perhaps with slightly delayed convergence. However, Figure 4 illustrates that this is not the case. The initial convergence is unexpectedly more rapid than for the deadband and becomes less stable with the standard deviation increasing significantly at the end of the search (this de-convergence being most noticeable through the increase in the coefficient of variation). It would therefore be difficult to conclude from this

convergence behaviour that the heating setpoint was of similar importance to the deadband.

The difficulty of using a sample mean and standard deviation to interpret variable importance is illustrated further in Figures 5 and 6. Although it has limited importance in terms of energy use (Table 2), and no impact on infeasibility (Table 3), the North-West window to wall ratio rapid initial convergence that is stable until the final stage of the search where the variable appears to change direction (indicated by a change in the mean, standard deviation, and increase in coefficient of variation; Figure 5). A comparison of the convergence of the heating setpoint (Figure 4), and the North-West window to wall ratio (Figure 5), might suggest that the North-West window to wall ratio is at least equally as important as the heating setpoint, if not more important. Tables 3 and 4, indicate that this is not the case, this further strengthening the conclusion that the mean and standard deviation convergence can not be used to judge the importance of a variable.

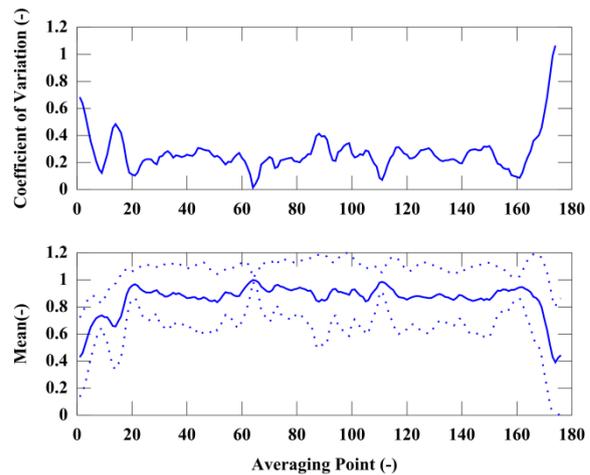


Figure 5 North-West Window-Wall Ratio Convergence

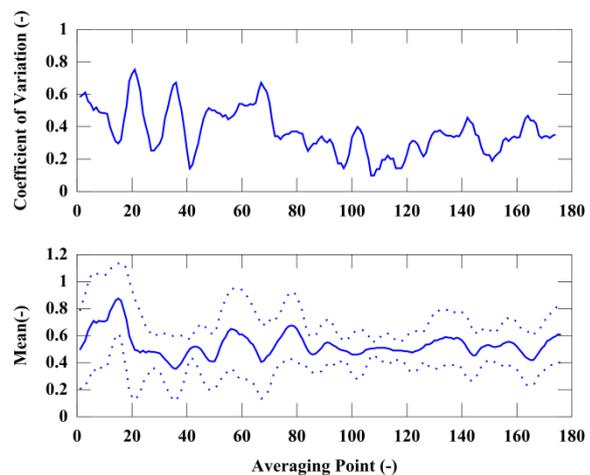


Figure 6 External Wall Type Convergence

Figure 6, illustrates the convergence behaviour of a variable (external wall construction type), that is of no importance in relation to either the energy use or infeasibility (Table 2 and 3). Although the convergence is weak, it is still difficult to rank its importance against the other variables considered here (especially, perhaps the heating setpoint; Figures 4 and 6).

Application to the Example Design Problem

The purpose of this study is to investigate the extent to which the solutions obtained from the optimization can be also be used to identify the most important design variables. Although the validity of the approach requires further research, in this example we assume that all solutions from the optimization have can be used for a global sensitivity analysis, and that the bias in these solutions provides an indication of sensitivity around the optimum. The proposed (optimum) design solution is that given in Table 1, and the variable importance that given in Tables 2 and 3.

Suppose that having been presented with the proposed design solution to the architect (or client), decides that 20% glazed area on the North-East and South-East facades, results in an unacceptable aesthetic form, and prefers the glazed areas on these façade to be increased to be the same as the South-West façade. In order to compensate for the increased cost of these changes, it is envisaged that the cheaper light-weight construction system would be used in place of the medium-weight system.

Although the true impact of these changes would require re-simulation of the building performance, the discussion could immediately be informed by the variables importance. In this case, a change to the North-East glazed area is likely to result in an increased chance of discomfort (Table 3), whereas changes to the South-East glazed area could impact on both thermal comfort (Table 3) and energy use (Table 4). In contrast, changes to the external wall construction are likely to have limited impact on building performance (Table 3 and 4; the reason for this possibly being due to the external wall performance being dominated by the glazed area).

DISCUSSION AND CONCLUSION

This paper examines the extent to which the solutions found from an evolutionary optimization of building design can be used in a global sensitivity analysis. It can be concluded that when the first 100 samples of the specified optimization process are used in a global sensitivity analysis (specifically based on a step-wise linear model of the ranked variables), that the most important variables are identified, and with similar relative magnitude, to those identified from using a 100 randomly generated samples. It was further concluded that the same sensitivity analysis applied to all solutions found during the optimization

resulted in the most important variables being identified, but with a different relative importance to those found using the randomly generated solutions, or solutions taken from the start of the optimization process. The cause of the difference in the results is thought to be the bias in the solutions round the optimum values of the variables, although confirmation of this requires further research. Further research is also required to confirm that the bias and sensitivity analysis results reflect the importance of the variables in a region local to the solution, rather than globally across the entire solution space.

The paper also investigates the extent to which the convergence characteristics of the variables, as represented by the mean and standard deviation in a moving set of 100 samples, can be used as an indication of the relative importance of the variables. It was concluded that, although the most important variable exhibited fast and stable convergence, it was not possible to identify the relative importance of other variables in the optimization. This particular result, as is the case for all results in this paper, is a function of the particular optimization algorithm used in the study. Although the algorithm is based on common evolutionary operators, it is possible that a different set of operators could result in different and more stable convergence characteristics that enable them to be used as an indication of variable importance.

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