Weather Scenario Generation for Stochastic Model Predictive Control Using Vector Autoregressive Prediction

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Abstract

Conventional building energy simulation utilizes characteristic locational weather data to illustrate the typical operation of the modeled facility, generally to provide design or capital investment insight. Because of the uncertainty in the weather, the assumptions behind typical meteorological year (TMY) data tend to perform poorly in building energy modeling applications for real-time control such as model predictive control (MPC) of passive building thermal mass. To account for weather uncertainty in such operational context, we present a strategy for creating an arbitrary number of plausible near-future weather scenarios via a vector autoregressive (VAR) time-series prediction framework. This approach allows us to preserve the relationships between several spatiotemporally interrelated weather variables, for example dry-bulb temperature and absolute humidity, by capturing the variance in the joint time-series. Results from several climates are presented for 24-hour predictions of psychrometric and solar weather variables for a range of samples sizes and the application to stochastic MPC is highlighted.

1. Introduction: MPC for Space Temperature Setpoints

In a perfect world, in which we had perfect forecasts of future weather events, a commercial building could be controlled by adjusting the space temperature setpoints and other operational parameters in order to both keep occupants comfortable and minimize either energy consumption or utility cost according to the conditions that the building will experience in the near future. These optimized strategies would take advantage of the thermal capacitance of construction materials (structural steel and concrete) and interior furnishings (e.g. system furniture) to store heat and release it at a later time. This type of control would require:

1. A model to represent the behavior of the building in response to weather variables
2. A mechanism for forecasting future weather and other uncertain driving variables

Building energy modeling is common today and is typically employed during the design phase of a new construction project or during retro-commissioning of an existing facility. Unfortunately, perfect weather forecasts do not exist. Statistical models that represent plausible weather forecasts have been in use for decades; the difference between these plausible scenarios and the weather events that will eventually unfold represents our uncertainty in using this data for MPC. We can attempt to account for this uncertainty by using a range of plausible weather scenarios to account for what may occur, as opposed to a single forecast. Here, an MPC strategy would implement a control strategy that performs best over the entire range of possibilities in an attempt to protect ourselves from the mismatch between forecast and actual weather. The established methods of accounting for this range of possibilities are robust MPC, which examines and prepares for the extreme events that may occur, and stochastic MPC, which uses random but statistically likely events instead in an attempt to reduce the conservatism inherent in robust MPC.

2. Uncertainty Characterization

A typical energy model can be considered as a "grey box" representation of a building. That is, it uses physics based models of heat transfer phenomena enhanced by empirical data and statistical models to represent equipment performance and other facets of building operation. These models are ambitious, attempting to capture the effects of weather, occupants, internal gains such as computers and lights, construction assemblies, stratification of air inside spaces, thermal mass of furnishings and more. Attempting to capture the behavior of so many features of modern buildings means that simplifications necessarily take place; occupants are modeled through immutable schedules as are lights and internal heat gains such as computers, weather is assumed to be “typical” weather rather than historic or forecast, air is assumed to be fully mixed or arbitrarily “layered”, and so on. What these energy models do not account for is the uncertainty in all of these other characteristics; the framework proposed here attempts to account for the uncertainty in weather so that these models may be used in a stochastic MPC context.

2.1 TMY Data

Energy models used for design or commissioning purposes typically make use of TMY weather data for a location reasonably close to the modeled site. The goal of TMY data is to represent the typical characteristics and patterns of local weather–it is a composite of many years of empirical data for the given location. In this case, “typical” means that many years (typically 30) of data are collated to capture a wide range of weather phenomena experienced by a location while still presenting an annual average that corresponds to the average long-term weather trends for that location. Because of this, one cannot assume that the hourly TMY weather data will match the actual weather at any given time but one can reasonably assume that the long-term data trends will tend to be similar to the actual weather.

For engineers and architects designing buildings and building systems, performing a simulation using TMY data is sufficient to determine the performance impact of design decisions and to create arguments to justify one design over another. For the operation of the building, however, one cannot assume that TMY will match the current or future weather; if we would like to optimize the operation of our HVAC systems, we would like to know how weather will behave in the short-term. Predicting short-term weather trends is, of course, a famously difficult problem.

2.2 Uncertainty with Scenarios

The difficulty in predicting short-term weather trends is an uncertainty we wish to account for in MPC. Classically, this uncertainty might be represented by the extremes encountered for a particular location–the so called robust MPC framework (Kouvaritakis and Cannon, 2015). This however introduces a considerable amount of conservatism in our model; since extreme weather patterns are unlikely to materialize, in most situations we would expect that optimizing building operations to account for these extremes would mean that some savings are “left on the table” – meaning that there is still potential for further optimization.

Instead of relying on extreme events to characterize a location’s short-term weather patterns, we can use data to develop a hypothetical weather pattern that is likely to occur by examining the recorded weather pattern leading up to the current moment. In this approach, we assume a future time horizon, F, for which we would like to optimize over as well as a window of time directly preceding the current time for which the optimization is taking place, H. The past time window H represents the historical data that we will use to generate a likely weather scenario for the future time horizon F.

The scenario approach is naturally deterministic, unlike other stochastic MPC strategies such as the use of chance constraints, for which the feasible set of solutions have the potential to be non-convex and difficult to express (Schildbach et al., 2013). Scenario based approaches have been shown to be an effective method in estimating the solution of nonlinear optimal control problems (Mesbah, 2015), which optimization problems involving buildings tend to be.

3. Vector Autoregressive Framework

Because weather variables are not independent (i.e. dry bulb temperature has a relationship with absolute humidity), any effective modeling approach used to produce a weather scenario must account for the joint variance between the modeled variables. Here, we use a vector autoregressive (VAR) model to represent this dependence. The VAR framework has been shown to generate useful weather scenarios that account for the stochastic nature of local weather pattern (Verdin et al., 2014). The VAR model can be summarized as an extension of the classical autoregressive time series model which allows explaining each variable's evolution in terms of its own lags (the value of the variable at previous time intervals) as well as the lags of each additional exogenous variable assumed to be interrelated. To illustrate this approach, let us first consider an example in which two variables, say dry bulb temperature (T) and absolute humidity (W) are modeled as lag 1, or rather that the current value of these variables is only a function of their values at the previous observation:

$$\begin{bmatrix} T_t \\ W_t \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix} \begin{bmatrix} T_{t-1} \\ W_{t-1} \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \tag{1}$$

Here, c_1 and c_2 are some constants and e_1 and e_2 are error terms. The A matrix is solved via a least squares minimization. The current values of temperature and absolute humidity are assumed to depend only on the values at the previous time step; the number of lags represent the time window used to create the forecast, in this example the window is 1 hour.

3.1 Lag Selection

Of course, predicting future temperatures based solely on the value of temperature and humidity an hour ago are not likely to be very accurate. We would like to choose a number of lags that maximizes our confidence in the model. Following (Hastie et al., 2008), let us define our total prediction error E as in Equation 2:

$$E = \frac{1}{N}\sum_{n=1}^{N} L\left(X(n), \hat{f}(n)\right) \tag{2}$$

where N is the number of samples, $\hat{f}(n)$ is the fitted model, and L is some loss function relating the observed test value $X(n)$ with the prediction. Since the testing error will nearly always be smaller than the error for new predictions, it is useful to quantify our confidence in the model and thus how optimistic our testing error is. We define this optimism O as:

$$O = E_{in} - E \tag{3}$$

where E_{in} is our in-sample error, an estimate that combines the prediction error with a term that penalizes complicated models; the justification being that complex models may be over-fitted to the training data. Two common estimates for in-sample error E_{in} are Akaike's Information Criterion (AIC) and the Bayesian Information Criterion (BIC). We can define both criteria in terms of the final prediction error FPE for autoregressive models (Akaike, 1969), which is based on the mean squared error of residuals $\overline{R^2}(A^{(M)})$ in Equation 4, the VAR model order or number of lags M:

$$\overline{R^2}(A^{(M)}) = \frac{1}{N}\sum_{n=1}^{N}\left[\tilde{X}(n) - \sum_{m=1}^{M} A_m^{(M)}\tilde{X}(n-m)\right]^2 \tag{4}$$

Where $A^{(M)}$ is the matrix of covariates from the fitted VAR model and $\tilde{X}(n)$ is the deviation from the data mean:

$$\tilde{X}(n) = X(n) - \frac{1}{N}\sum_{n=1}^{N} X(n) \tag{5}$$

with $X(n)$ being the observed data. Using Equation 4, we can estimate our final prediction error as:

$$FPE = \left(1 + \frac{M+1}{N}\right) S_M \tag{6}$$

where S_M is:

$$S_M = \frac{N}{N-M-1}\overline{R^2}(A^{(M)}) \tag{7}$$

Using our estimate of the final prediction error we can now formulate an information criterion to guide the selection of lags:

$$AIC = \ln|FPE| + \frac{2}{N}MP^2 \tag{8}$$

$$BIC = \ln|FPE| + \frac{\ln N}{N}MP^2 \tag{9}$$

where P is the number of variables jointly modeled. Equations 8 and 9 use similar penalty terms, BIC penalizing complex models slightly more than AIC due to the $\ln N$ term versus factor 2 as illustrated in Fig. 1. Ideally we would like to minimize how optimistic our model is, so we should select a number of lags that minimizes either the AIC or BIC, whichever we elect to use. Because of the slightly larger penalty term in BIC, models using this selection criteria will generally have fewer lags than those using AIC.

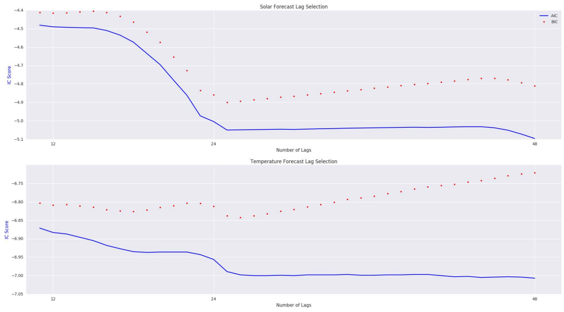

Fig. 1 – Information criterion scores for solar forecasts (top) and temperature forecasts (bottom)

After we select the number of lags, we write the general form of the VAR framework for P variables and M lags as in Equation 10.

$$X_t = c + \sum_{m=1}^{M} A_m^{(M)} X_{t-m} + e \tag{10}$$

or in long form for each variable $x_p, p \in [1, \dots, P]$

$$\begin{bmatrix} x_{1,t} \\ \vdots \\ x_{P,t} \end{bmatrix} = \begin{bmatrix} c_1 \\ \vdots \\ c_P \end{bmatrix} + \begin{bmatrix} A_{1,1}^{(1)} & \cdots & A_{1,P}^{(1)} \\ \vdots & \ddots & \vdots \\ A_{P,1}^{(1)} & \cdots & A_{P,P}^{(1)} \end{bmatrix} \begin{bmatrix} x_{1,t-1} \\ \vdots \\ x_{P,t-1} \end{bmatrix} + \cdots$$

$$+ \begin{bmatrix} A_{1,1}^{(M)} & \cdots & A_{1,P}^{(M)} \\ \vdots & \ddots & \vdots \\ A_{P,1}^{(M)} & \cdots & A_{P,P}^{(M)} \end{bmatrix} \begin{bmatrix} x_{1,t-M} \\ \vdots \\ x_{P,t-M} \end{bmatrix} + \begin{bmatrix} e_1 \\ \vdots \\ e_P \end{bmatrix}$$

where c is some constant and e is some error.

4. Scenario Generation

Using the VAR framework, we can take advantage of either locally measured historical or typical weather data to forecast a range of scenarios to represent what the local weather of a given location may look like over the next 24-hours.

The general process for producing weather scenarios is as follows:

1. Fit a VAR model to the data by selecting a number of lags via AIC and solving for the A matrix like that shown in Equation 1
2. Find the standard deviation σ for each weather variable over the number of lags preceding the current time t
3. For each desired scenario, perturb the observed weather data during the time window (based on the selected number of lags) with a random variable between ±σ and use the fitted VAR model to predict a weather scenario using the perturbed data.

The development of the stochastic weather generator gives us a tool for creating any number of plausible weather scenarios that largely relies on two parameters:

1. The length of the historical window W (the number of lags used in VAR process);
2. The number of scenarios to be generated.

These parameters can be thought of high-level tuning parameters specific to the application, building and data at hand but some general guidance on these questions is warranted. Fig. 2 illustrates how the range of predictions changes depending on the number of scenarios, in particular it indicates a widening interquartile range as the number of scenarios increases. Indeed, as the number of scenarios approaches infinity, a scenario based stochastic MPC solution will begin to approximate the robust MPC solution (Zhang et al., 2013). In addition to the temperature and humidity data we have seen thus far, we can also predict solar data like that shown in Fig. 3, which shows predictions for global horizontal and direct normal irradiance using data from Golden, Colorado.

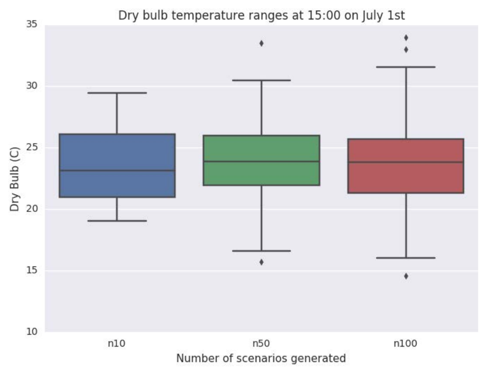

Fig. 2 – Example prediction ranges generated for an increasing number of scenarios for dry bulb temperature during 15:00 on July 1st for Golden, CO

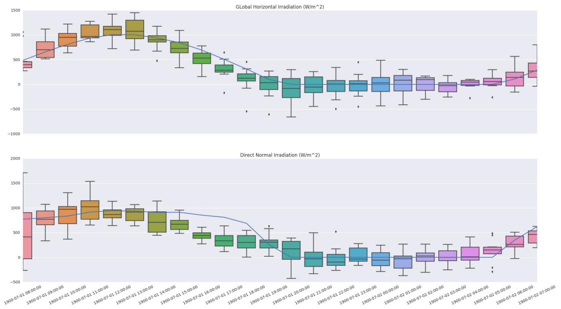

Fig. 3 – Forecast of 10 direct normal and global horizontal irrradiance scenarios for 24 hours starting on July 1st in Golden, CO

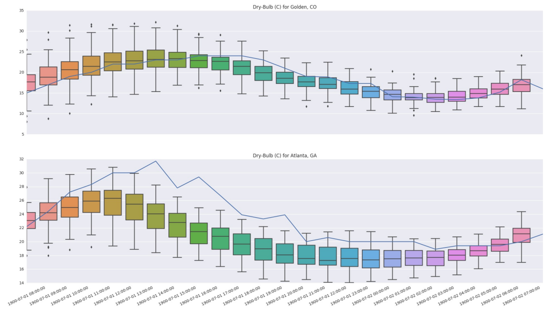

Fig. 4 – Forecast of 50 dry-bulb temperature predictions for Golden, CO (temperate, dry) and Atlanta, GA (hot, humid)

5. Summary and Conclusions

The VAR framework provides an accessible and generally applicable method of representing uncertainty with scenarios. Indeed, the VAR solution is tractable (as evidenced by rapid scenario generation times) and easy to use, the only requirement being relatively stationary time-series test data, which the seasonal nature of weather phenomena is well suited for. While the proposed VAR framework provides a simple tool for generating forecasts of short-term weather trends for MPC applications, care must be exercised in choosing which variables are assumed to be related. Assuming two tangentially related variables are dependent on each other can produce poor fits and misleading results. Additionally, the selection of an appropriate time-horizon is important for capturing the daily swings in weather behavior; locations that see large diurnal temperature differences may require a longer historical window to produce acceptable fits. Fig. 4 illustrates this using predictions for a hot, humid climate (Atlanta, Georgia) during the summer season where the diurnal temperature swing is relatively large as compared to a temperate climate (Golden, Colorado) where the diurnal swing is approximately 5 °C on the presented day.

The solution presented in this paper was developed as part of the stochastic MPC python package smpc (Currie, 2016), the goal of which is to provide a set of open-source tools for performing stochastic time-series analysis and investigating stochastic MPC problems.

Nomenclature

AIC	Akaike's information criterion
BIC	Bayesian information criterion
TMY	Typical Meteorological Year
VAR	Vector autoregressive
$A^{(M)}$	Covariate matrix of the fitted VAR model
c	Constant
e	Error
E	Total prediction error
$\hat{f}(n)$	Fitted VAR model
FPE	Final prediction error
F	Future time horizon
H	Past time window
M	VAR model order, lag order
N	Number of observations
O	Model optimism
P	Number of features to model
$\overline{R^2}(A^{(M)})$	Residual mean squared error
T	Dry-bulb temperature
W	Absolute humidity
$X(n)$	Observed data
$\tilde{X}(n)$	Deviation of the observed data from mean

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