Influence of inputs knowledge on Grey-box models for Demand Response in Buildings

Harald Taxt Walnum¹, Karen Byskov Lindberg¹,², Igor Sartori¹
¹ SINTEF Building and Infrastructure AS, Oslo, Norway
² NTNU, Dept. of Electric Power Engineering, Trondheim, Norway

Abstract
A set of grey box models for a passive single family house is evaluated to understand the influence of inputs (solar radiation, internal gains and ventilation) knowledge. The models are evaluated with both a log likelihood ratio test and evaluation of the long-term prediction RMSE. It is shown that solar radiation is a necessary input, while internal gains and ventilation gain show more ambiguous results. It is also shown that a strict log likelihood evaluation based on 1-step predictions is not suited for evaluating models for applications where longer predictions are necessary.

Introduction
The Clean Energy Package of the European union highlights the importance of utilising end-user flexibility to support the decarbonisation of the energy system (European Commission, 2018).

In Norway, about 80% of the buildings’ heat demand is met by electricity, either with heat pumps or direct electric heating (Boeng, 2005; Lindberg & Magnussen, 2010); and given the lack of a natural gas network there seems to be limited potential to substitute it with other energy carriers (Sandberg et al., 2017; Sartori et al., 2009). Hence, the heat demand of buildings comprises a large flexibility potential for the power system. For the building to control its load, i.e. provide demand response (DR) to the electricity grid, suitable methodologies for load forecast and load control are needed. Model Predictive Control (MPC) is one of the methodologies that provide optimal response to control signals – such as hourly energy price or CO₂ footprint – while keeping the building’s operation within the occupant’s satisfaction boundaries (Halvgaard et al., 2012).

In literature, there exist typically three main modelling approaches which are used for developing MPC models (Reynanders et al., 2014). White-box models are based purely on knowledge about the physical parameters of the building. However, it is often challenging to derive these physical properties from existing buildings to yield satisfactory results, in addition to the fact that complex models comprising hundreds of differential equations are not suitable for MPC uses. Black-box models are purely based on analysis of measurement data. A substantial amount of data is needed for appropriate results, and the models can diverge significantly when operating outside the range of conditions used in the training dataset. Grey-box models are a combination of these two, where statistical methods are used to derive the parameters of a reduced-order physical model.

Previous works on identifying grey-box models usually contain experiments with rich datasets of various measured inputs, i.e. detailed measurements on meteorological parameters (solar radiation, outdoor temperature, windspeed), and the usage of the building (ventilation heat and internal gains) (Bacher & Madsen, 2011; Bacher et al., 2013; Vogler-Finck et al., 2018). However, outside laboratory experiments it is difficult to obtain measurements for many of these parameters.

The aim of this work is to investigate which parameters and inputs are necessary in order to generate adequate grey-box models for MPC applications. The investigation is performed through model identification based on maximum likelihood estimations in CTSMR (a tool for continuous time stochastic modelling developed at DTU Compute in Denmark, (Kristensen et al., 2004). A selection of models, containing different knowledge about the building input and parameters are tested, and evaluated for both short-term (one timestep) and long-term (one week) predictions.

Test building and experimental data
The dataset used for model identification in this paper is based on an experiment performed on the ZEB Living Lab in Trondheim (Vogler-Finck et al., 2017) that makes use of a PBRS (Pseudo-Random Binary Signal). The ZEB living lab is an experimental facility designed to be a zero-emission single family house. The building envelope is highly insulated with 35-40 cm of rock-wool insulation and has a window ratio of about 20% of the heated floor area. The building is on a single floor and consists of 7 inhabitable rooms (whereof 4 are interconnected without doors), and with a total floor area of about 100 m². In addition, about 90 m² of PCM (phase change material) built into the inner walls (Goia et al., 2015).

The dataset contains data with a sampling rate down to 5 minutes, both meteorological data (ambient temperature, global solar radiation, wind speed and wind direction) and building performance data (heat input, ventilation heat, internal gains and indoor temperature). It should be noted that, for technical reasons, the ventilation heat is not measured directly, but calculated from a known air flow rate (assumed to be constant) and the air supply and extract temperatures. During the experiments, the building was heated with an electrical heater located in...
the middle of the building. The building was unoccupied during the experiments.

The full datasets from three experiments are available online (Vogler-Finck et al., 2017), and are named experiment 2, 3 and 4. In this work, the second part of experiment 3 (after the ventilation inlet temperature is set to 18 °C) is used as the training dataset, and experiment 4 is used as the validation dataset. The input and measurement data are shown in Figure 3. An important difference between experiment 3 and 4 is that during experiment 3, the doors to the two bedrooms where closed, while they were open in experiment 4. It should also be noted that the ventilation is turned off in the second part of experiment 4.

Method

Grey-box models can be described as a set of first order continuous stochastic differential equations (SDE) (in this case linear and time invariant). These equations can be written in a state space representation as shown in equations (1) and (2):

\[
\begin{align*}
    dX(t) &= A(\Theta)X(t) + B(\Theta)U(t) + \sigma(\Theta)d\omega \\
    Y(t) &= CX(t) + \epsilon
\end{align*}
\]

where \(X(t)\) is the state vector, which in building energy modelling usually represents internal temperatures. \(U(t)\) is the vector of measured inputs, both controlled ones (heating from radiator) and disturbances (solar radiation, internal heat gains, etc.). The stochastic part is represented by \(\omega\), which is a Wiener process; hence \(\sigma(\Theta)d\omega\) is the random walk error of the estimated states. \(A\) and \(B\) are matrices whose elements are functions of the parameters \(\Theta\), while \(C\) describes the relation between the model’s states (predicted temperatures) and the measured outputs \(Y(t)\) (measured temperatures), while \(\epsilon\) is the error of the measurements, assumed to be white noise.

The parameters \(\Theta\) in the grey-box model are estimated using, CTSMR, which employs an extended Kalman filter together with a maximum likelihood approach to estimate the parameters.

Linear and time invariant state space models are highly suitable for MPC applications, as they can be discretized and reformulated into a linear programming (LP) optimisation problem (Halygaard et al., 2012).

A backward selection strategy is applied. Starting with the full set of inputs as described in the following section, the model is reduced step by step, by successively removing disturbances while the order of the model is kept constant. The models are evaluated in two steps. A statistical evaluation of the identified models, and a long-term prediction evaluation.

To evaluate if there is a significant difference between two models, a likelihood ratio test as described by (Bacher & Madsen, 2011), is applied. Given two models where the model A is a simpler submodel of the more complex model B, the likelihood ratio test determines whether the complex model is significantly better than its simpler submodel. If the p-value of the \(\chi^2\) distribution is above a certain threshold, (typically 5%), the complex model is not significantly better than the simpler model.

The likelihood ratio from the parameter estimation is valid for a 1-step prediction, while in an MPC application, the prediction horizon of the optimiser must be long enough to ensure stability of the controller and meaningfulness of the results. To evaluate the model’s performance of long-term predictions (~1 week), the model is simulated by solving the deterministic part of the SDE using alternatively the training dataset and the validation dataset. In both cases, the initial conditions (temperatures) are set and the model predicts the evolution of the indoor temperatures for the whole period, assuming perfect knowledge of the disturbances and controlled input. The model most fitted for MPC applications is the model with the best trade-off between computational efficiency and prediction capability in both short (1-step ahead) and longer term (ca. 1 week, the possible prediction horizon in an MPC implementation). We therefore evaluate both the likelihood ratio for the 1-step ahead prediction and the RMSE of the long-term prediction.

The full grey-box model

The main purpose of the grey-box model is to give an adequate description of the thermal behaviour of the building (indoor temperature), while accounting for the controlled input (e.g. heating power) and the disturbances (e.g. solar gains).

A two-state model including internal gains, ventilation heat, solar gain, and windspeed is used as a starting point. An RC-diagram representation of the full model is shown in Figure 1. It contains two state variables:

- Ti (°C): Interior air temperature
- Tw (°C): Interior temperature of the building envelope

Each of these have the corresponding thermal capacities:

- Ci (Wh/K): Heat capacity of the interior
- Cw (Wh/K): Heat capacity of the building envelope

There are three internal resistances:

- Ri (K/W): between the building envelope and the interior
- Re (K/W): between the ambient and the building envelope
- Rin( K/W): Infiltration resistance described according to equation (3), where \(k\) is a constant parameter and \(Ws\) (m/s) is the windspeed.

\[
Rin = \frac{1}{k\cdot ws}
\]  

The model has one controllable input, and four disturbances:

- \(\Phi_h\) (W): Heat gain from the electric heater,
- \(\Phi_e\) (W): Internal loads
- \(\Phi_v\) (W): Heat gains from the ventilation
- \(\Delta w\Phi_h\) (W): Solar gains, which equals solar irradiation multiplied with the effective window area.
• Ta (°C): Heat losses due to the ambient temperature

![Diagram](image)

**Figure 1: RC-diagram representation of the full model.**

The RC-network yields the following SDEs:

\[ dT_i = \frac{1}{C_i R_i} (T_w - T_i) dt + \frac{k W s}{C_i} (T_a - T_i) dt \]
\[ + \frac{1}{C_i} (A_w \Phi_s + \Phi_w + \Phi_{ig} + \Phi_{h}) dt + \sigma_d d\omega_i \]  
\[ dT_w = \frac{1}{C_{w R_i}} (T_i - T_w) dt + \]
\[ \frac{1}{C_{w R_e}} (T_a - T_w) dt + \sigma_w d\omega_i \]  

Before deciding for this model configurations, some other typical configurations were tested (e.g. adding a capacitance and a resistance between the heater and the indoor temperature, and splitting the solar gain between the indoor temperature and the wall temperature). Without significantly improving the model.

With this as a starting point, we investigate the effect of removing each of the following parameters and inputs from the full model

- Wind – (removing R_w)
- Solar gains – (removing Aw*Φ_s)
- Ventilation gains – (removing Φ_v)
- Internal gains – (removing Φ_i)

The model with the highest log likelihood is used as a base model for further reduction. The log likelihood ratio test is used to evaluate if the reduction is significant. To investigate the impact of time resolution of the measurements and controller sample time, all model formulations are evaluated with both 15- and 60-minute time step (Ts), to evaluate the effect of sampling rate. Table 1 shows the models selected for detailed analysis. The #param column show the number of parameters that must be estimated by CTSM, and the #inputs column show the number of input and disturbance loads.

**Results**

Table 1 shows the log-likelihood of the full models and the likelihood ratio of the reduced models. The smaller the value of the likelihood ratio, the less difference there is between the base model and the reduced model. The model without wind performs almost equally well as the full model. For this model the value of k approaches zero and the p-value is equal to 1. This indicates that the wind speed has a negligible impact, which is justifiable in a highly insulated and airtight building such as the ZEB living lab. Based on this, windspeed is not considered in the remaining reduced models.

Figure 4 shows the estimated parameters for the different models with the uncertainties, corresponding to 2 standard deviations. In general, the estimated parameters are similar. However, in the model without solar radiation (B), the uncertainties of the estimated parameters are significantly higher, which is consistent with the higher likelihood ratios shown in Table 1. In comparison, the heat capacity of the indoor air and the total resistance has been estimated to 0.12 kWh/K 14 K/kW (Vogler-Finck et al., 2018).

The prediction capability is investigated for the most promising models, i.e. model A, C, D and E. In addition, model B without solar radiation is included to show the consequences of not taking solar gains into account.

Figure 5 shows predictions of the internal temperatures for the total dataset (training and validation data), for both 15- and 60-minutes time step models. Figure 6 shows the corresponding evolution of the root-mean-square-error (RMSE).

**Discussion**

**Influence of inputs**

Evaluating all the reduced models in Table 1, it is evident that solar radiation is the input that clearly reduces the likelihood of the model, when removed. This applies to all model combinations and gives a clear indication that the solar radiation is a significant parameter. Figure 2 shows the auto-correlation function (AFC) of the residuals for the full model (left) and model B without solar radiation (right) with 60 min time resolution of the data. Both plots show some diurnal lag, however it is much more significant for the model without solar radiation, and hence, we may conclude that relevant information is missing in the reduced model.

![AFC](image)

**Figure 2: AFC for full model (left) and model B without solar radiation (right)**

Also, from the log-likelihood ratio test in Table 1 it is clear that the solar radiation is a significant input to the model with a p-value << 0.05. The prediction results in Figure 5 and Figure 6 (orange line) also demonstrate the incapability of the models without solar radiation. It is interesting to see in Figure 5 how quickly model B (orange) deviate from the measured internal temperature, which indicates that it would be insufficient also for short prediction horizons.
The test also implies that removing information about the ventilation has a significant impact, with a p-value < 0.05. However, when looking at the prediction capability in Figure 5 and Figure 6 model C (green) seems to fit the internal temperature better than the full model, even with a lower likelihood in the parameter estimation.

Figure 6 shows that the RMSE for the whole prediction period is similar for the model with and without ventilation, i.e. respectively model A (blue) and model C (green). Therefore, as there are too many unknown and interlinked effects, it is not possible to conclude that the model without ventilation is superior to the models that take the ventilation into account, or if it will perform "good enough" in a real application. Hence, the results indicate that the models do not handle the impact of ventilation in a proper way. This could be due to the model structure, and/or the data quality of the ventilation heat gain measurements, which are not fully measured but calculated based on measured supply and return temperatures and the assumption of constant, nominal air flow rate. Based on the results, we may conclude that ventilation heat is only of value to the model if it is properly measured.

Internal gains show a more ambiguous behaviour, with a difference between the shorter time step (Ts=15min) and the longer time step (Ts=60min), see Table 1. This can be explained by a combination of model structure and the nature of the internal gains signal itself. The internal gains mainly fluctuate between 170W and 250W (not a strong variation compared to other inputs such as solar and active heating), with short spikes (lasting around 15 min) repeating with a relatively constant period (somewhat longer than one hour), as visible in Figure 3.

With 60 min time step, such a signal could be handled properly by the stochastic part of the model, i.e. the Wiener process, thus making it redundant to explicitly consider it as an input. Indeed, internal gains result in the grey area of significance; the p-value is below the threshold of 0.05 but is not so small as for other input, thus not allowing to be conclusive on their non-significance.

With 15 min time step, the results from the likelihood ratio test are unexpected in what the value is negative. Strictly interpreted, this would mean that the reduced model 'fits better' than the more complete one. However, a more meaningful interpretation is that the model does not handle properly signals with such a high frequency (the 15 min spikes). The model is of second order, and the value of the capacitance directly connected to the interior temperature, $C_i$, is determined by stronger, lower frequency inputs, such as solar gains and active heating. Thus, the model acts like a low-pass filter with respect to the internal gains, and the model appears to fit better when this input is eliminated altogether. Whether adding a further capacitance to capture the influence of higher frequency signals - thus moving to a third order model - would improve the overall trade-off between computational efficiency and prediction capability remains to be investigated. In the long-term prediction (Figure 6), we can see that the model without the internal gains (red) perform worse than the model that includes the internal gains (blue), in contradiction to what one would expect from the likelihood evaluation.

**Influence of time step**

In general, the models perform better on the training dataset than on the validation dataset. As mentioned in the dataset description, an important difference between the two datasets, is that the bedroom doors were closed in validation dataset. This might have significant influence on the dynamic behaviour of the building and contribute to the lower prediction performance on the validation dataset. The relative difference between the test and train dataset is larger for the models with shorter time step. One explanation for this can be that the longer time step smoothens out the short-term differences between closed and open doors, as time allows the temperatures to even out.

The results also show lower prediction errors for the models with longer time step. This contradicts the results found by Vogler-Finck et al. (2018). One explanation for this can be that in this work, the disturbances has been resampled from the 5 minute datasets to higher time steps by taking the mean values, while in Vogler-Finck et al. (2017) instantaneous values for solar radiation and ventilation heat was utilized for evaluating longer time steps. Another explanation can be that low order models are not able to handle the fast dynamics of the building, and by increasing the time step, the fast dynamics are of less importance, e.g. as with the closing and opening of doors.

**Validity of results**

The dataset that is used in this paper, is based on experiments of a highly insulated, air-tight single family house. This might influence the validity of the results in several ways.

Sub-section "Parameter estimation" concluded that the external wind speed was an insignificant input, which is expected for an airtight building with balanced ventilation. However, this conclusion might not be valid for a less air-tight and insulated building or a building with natural ventilation.

For highly insulated buildings, disturbances such as internal gains and solar radiations normally has a higher relative impact compared to the controlled heating, than in poorly insulated buildings. This would suggest that inclusion of these disturbances are of less importance with less insulated buildings. However, this must be investigated.

The investigated building has a balanced ventilation system with approximately constant air-flow and inlet temperature. For buildings with either demand-controlled ventilation or buildings that turn off the ventilation at night (typical non-residential buildings), we expect impact of ventilation to be more significant. In Figure 6 one can see that when the ventilation is turned off in the validation data set (after 95 hours), the errors increase for
the models not taking ventilation into account, i.e. model C (green) and G (purple).

As discussed above, the internal gains fluctuate with a more or less constant amplitude and wavelength throughout the experiment. It is questionable if this is realistic for normal operation of a single family house. According to the Norwegian technical specification on "Energy performance of buildings - Calculation of energy needs and energy supply" (Standard Norge, 2016), one could expect much larger diurnal variations. This would be more challenging for the model to handle.

Another issue that can influence the results, is the formulation of the full model. The cyclic patterns in Figure 2 indicate that there are some diurnal effects that are not properly handled. This is most likely effects of window location and shadowing objects around the building that affects the real solar radiation entering the windows. If this was handled better by the full model, the difference between the models with and without solar radiation would be even greater.

There are also indications that the models with longer time steps (Ts=60min) performs better than the models with shorter time step. This could be different, for a higher order model, that would be able to describe fast dynamic effects better.

Model application in MPC

It is difficult to define a strict limit for how well the models must perform to be suitable for MPC applications. However, when applying MPC for utilization of the thermal flexibility of a building, it is common to work with fluctuations of ±1°C on the indoor air temperature.

In MPC applications, perfect knowledge about the future is not available, and prediction models are therefore necessary. Forecasting models for weather related disturbances, such as ambient temperature and solar radiation are readily available, and relatively simple to implement, while user dependent disturbances such as internal gains, are much more difficult to predict. It would therefore be an advantage to have models independent for these disturbances.

Conclusions and further work

This paper evaluates the impact of removing parameters and inputs from the identification of a grey box model for a single-family house. The five reduced models are evaluated with respect to likelihood ratio test when compared to a higher order model, and how they perform when predicting the indoor temperature over a longer horizon.

The results show that for well insulated buildings with large window areas, knowledge about the solar radiation is essential to get adequate results. The necessity of information about ventilation and internal gains is more questionable. Especially for ventilation, it seems that it only has a small impact on the performance, as long as the flowrate and inlet temperature are constant.

It is also shown that a strict likelihood evaluation based on 1-step predictions is not suited for evaluating models for applications where longer predictions are necessary.

Further research is needed to fully understand several effects. New experiments with a more realistic representation of the internal gains should be carried out. In addition, studies on other types of buildings should be performed, both regarding the building’s energy standard and type of building (residential, offices, schools etc.). It is also necessary to evaluate the models in MPC applications, to understand how accurate the models need to be.

Acknowledgement

This work/article/report/paper/other has been written within the Research Centre on Zero Emission Neighbourhoods in Smart Cities (FME ZEN). The authors gratefully acknowledge the support from the ZEN partners and the Research Council of Norway.

References


Norwegian: “Tiltak og virkemidler for redusert utslipp av klimagasser fra norske bygninger”).

NVE.


Figure 3: Experimental dataset (Ts=15 min)
### Table 1: log-likelihood ratio test result from the model identification. (note that model B to H are without wind)

<table>
<thead>
<tr>
<th>Reduced model ID</th>
<th>Removed Inputs</th>
<th>Model</th>
<th>#param</th>
<th>#inputs</th>
<th>Reduced from</th>
<th>Ts=15min Ratio * (p-value)</th>
<th>Ts=60min Ratio * (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>0</td>
<td>Full model</td>
<td>11</td>
<td>6</td>
<td>-</td>
<td>1884.0 ** (1.00)</td>
<td>236.4 ** (1.00)</td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>NoWind</td>
<td>10</td>
<td>5</td>
<td>Full</td>
<td>-3.19E-06 (1.00)</td>
<td>6.45E-08 (1.00)</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>NoSol</td>
<td>9</td>
<td>4</td>
<td>A</td>
<td>700.58 &lt;1.0E-16</td>
<td>280.56 &lt;1.0E-16</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>NoVent</td>
<td>10</td>
<td>4</td>
<td>A</td>
<td>7.17 0.007</td>
<td>8.69 0.003</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>NoInt</td>
<td>10</td>
<td>4</td>
<td>A</td>
<td>-7.06 -</td>
<td>6.31 0.012</td>
</tr>
<tr>
<td>E</td>
<td>3</td>
<td>NoIntNoVent</td>
<td>10</td>
<td>3</td>
<td>D</td>
<td>6.31 0.012</td>
<td>8.46 0.004</td>
</tr>
</tbody>
</table>

* log likelihood ratio value  
** log likelihood value

Figure 4: Estimated parameters for the models, showing the estimated value and the 95% confidence interval.
**Figure 5:** Prediction of the indoor temperature, starting from initial conditions, for the training (left) and the validation dataset (right).

**Figure 6:** Evolution of the RMSE for the prediction of the indoor temperature, starting from initial conditions, for the training (left) and the validation dataset (right).