On the use of the transfer matrix method to evaluate sound insulation in complex building partitions

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Abstract

The transfer matrix method (TMM) represents a powerful tool to investigate wave propagation through different media, which could be particularly suitable to compute sound transmission through building partitions. Even though layered structures can be easily modelled by using the TMM approach, it is not always easy to determine the elastic properties of each layer the partitions is made of. Traditional partitions, generally made in masonry with clay or concrete bricks coupled together with mortar joints, are inhomogeneous and anisotropic structures whose elastic properties are difficult to measure. Again, cross-laminated timber (CLT) panels, due to their peculiar substructure, might exhibit a highly orthotropic behaviour. A homogenisation approach, based on a minimization algorithm of the transmission loss (TL) of the bare structure, is proposed in this paper. It allows to consider inhomogeneous or anisotropic materials as an equivalent elastic solid described by effective frequency-depended properties. The reliability of this approach is validated by comparing the TL of different building partitions computed using the TMM with the experimental sound insulation determined by means of laboratory measurements.

Introduction

The overall acoustic performance of a building depends on several factors, such as the acoustic properties of each building element, the effects of the flanking transmission paths, the mounting conditions of service equipment and also the workmanship (M. Caniato et al., 2018; Secchi et al., 2015; Zucherini Martello et al., 2015). In order to obtain good acoustic performances and meet the regulation requirements, a proper acoustic design of each single building partition is of fundamental importance. Building construction involves a great variety of technical solutions and materials: either traditional heavy weight structures, or lightweight elements. Besides, linings and additional layers are usually employed, both in heavyweight and in lightweight structures, in order to improve the thermal and acoustic performance of partition. As shown in a paper recently published by Santoni et al., 2018b, traditional prediction approaches used in building acoustics can only be applied to homogeneous monolitic structures. The transfer matrix methods (TMM) is a powerful tool to investigate wave propagation and sound transmission through different media (Allard and Atalla, 2009). Such a wave propagation-based method is already widely used in automotive and aerospace acoustic design, but has not gained the same popularity in building acoustics. The layered structure, which characterises several building partitions, can be easily modelled by using the TMM framework, although it is not easy to determine the elastic properties of each of the different layers a building partition is made of. Traditional partitions, generally made in masonry with clay or concrete bricks coupled together with mortar joints, are inhomogeneous and anisotropic structures whose elastic properties are difficult to measure. Lightweight partitions, such as cross-laminated timber (CLT) panels, have become quite popular in Europe in the last decade. In fact, CLT elements are nowadays widely used both as inner or façade walls and as floors. However, as shown by recent studies (M. Caniato et al., 2017a; Schoenwald et al., 2013), the sound insulation performance of CLT bare structures needs to be improved by using linings and properly designed acoustic treatments. CLT plates, due to their peculiar layered substructure, may exhibit a strong orthotropic behaviour. The TMM can be implemented to model orthotropic and transversely isotropic media (Kuo et al., 2008). However, it is not straightforward to determine the complete compliance matrix, expressing the strains-stresses relationship for such media. In this paper, we propose an alternative approach, in which inhomogeneous or anisotropic materials are considered as an equivalent isotropic elastic solid, described by effective frequency-depended properties. The homogenisation process is based on a minimisation algorithm of the sound insulation of the bare structure, implemented in the TMM framework. After a brief review of the TMM, the homogenisation approach, used to evaluate the equivalent elastic properties from the experimental sound insulation of the bare structure, is described. This approach was applied to predict the sound transmission loss provided by different partitions involving both masonry walls and CLT panels with linings and cladding systems. The reliability of
the proposed approach is finally evaluated by comparing numerical results with experimental data.

Transfer Matrix Method framework

To find the sound transmission coefficient of a laterally unbounded multilayer element, which separates two semi-infinite fluids, can be computed by means of the TMM. The investigated structure is excited by a plane acoustic wave, impinging with angle of incidence $\theta$, on the surface $S_1$ as shown in Figure 1. The acoustic field on the surface $S_1$ can be related to the variables describing the acoustic field on the opposite surface $S_2$ of the multilayer element according to the general formalism:

$$V(S_1) = [T] V(S_2) \quad (1)$$

The vector $V(S_1)$ represents all the variables needed to define the acoustic field on the surface on the source side: namely, sound pressure and particle velocity; while the vector $V(S_2)$ contains all the field variables associated to the interface surfaces between the different layers: such as pressure, particle velocity, normal and tangential stresses. The transfer matrix $[T]$ describes the wave propagation through the investigated layered element. The size of this matrix depends on the nature of each layer, for example elastic solid, fluid, or poroelastic media (Allard and Atalla, 2009).

For each given angular frequency $\omega$ and each incidence angle $\theta$ of the impinging plane wave, the system given in Eq. (1) can be solved in order to compute the sound transmission coefficient $\tau(\omega, \theta)$ of the modelled layered structure. Several extensions of the TMM framework have been published in the last years by different authors. It is possible to increase the accuracy of the method below the coincidence frequency, by including in the calculation a non-resonant radiation efficiency in order to take into account the finite dimension of the investigated structure (Bonfiglio et al., 2016; Rhazi and Atalla, 2010; Villot et al., 2001).

Besides, the TMM can be extended in order to take into account the contribution of the structure-borne sound transmission through the mechanical connections between different layers of double-leaf walls or cladding systems, which causes a reduction of sound insulation at the high frequencies. This can be done by means of a decoupled approach, as described by Vigran, 2010b and Santoni et al., 2017a). For any given angular frequency $\omega$, the transmission loss (TL) due to a perfectly diffuse acoustic field (DAF) can be computed by integrating the incidence-dependent transmission coefficient $\tau(\omega, \theta)$ over all the possible angles of incidence $0 \leq \theta \leq \pi/2$ as:

$$TL(\omega) = -10 \log \frac{\int_0^{\pi/2} \tau(\omega, \theta) \cos \theta \sin \theta \, d\theta}{\int_0^{\pi/2} \cos \theta \sin \theta \, d\theta} \quad (2)$$

Characterisation of the equivalent homogeneous medium

As already mentioned, many building partitions are inhomogeneous anisotropic structures. Masonry walls for example consist of clay or concrete bricks with layers of mortar to join the blocks, and they are usually finished with a plaster layer on both sides. The determination of the elastic properties, such as the elastic modulus $E$, the loss factor $\eta$ and Poisson’s ratio $\nu$, required as input data in the TMM model to characterise the brick-mortar system, is not trivial. Previous works tried to deal with this problem using different approaches. Maysenbölger and Haberkorn, 2003, calculated the sound transmission through a periodically inhomogeneous infinite plate under general conditions. This method is not easily applicable to realistic and practical purposes, requiring substantial computational resources. Jacq et al., 2011, presented a homogenised vibratory model to predict the acoustic properties of hollow brick walls starting from the elastic tensor material, measured using an ultrasonic technique, and a numerical model of the single brick. Dijckmans and Vermeir, 2013, computed the sound transmission loss of a brick wall, considered as an equivalent homogeneous isotropic elastic layer whose properties were determined from the surface mass of the wall, the measured coincidence frequency and the measured thickness resonance frequency.

The approach proposed the present paper does not consider the dynamic response of a single brick, but deals with the entire wall system, because the presence of mortar joints, or plaster layers, highly influences the dynamic behaviour of the structure.

CLT structures have become in the last decade a valuable alternative to traditional construction materials. However, bare CLT panels provide a poor sound insulation, as it was well documented by several studies in which CLT partitions were experimentally investigated (Barbaresi et al., 2016; M. Caniato et al., 2017b; Marco Caniato et al., 2017; Di Bella et al., 2017). CLT panels consist of an odd number of layers of timber beams glued together. The orientation of the fibres of each ply is rotated of 90° with respect to the adjoining plies. Due to such layered sub-structure and the properties of the wood material, CLT plates generally exhibit an orthotropic behaviour (Van Damme et al., 2017). This means that they have different elastic properties along mutually perpendicular directions. In structural engineering CLT panels are generally modelled either by using an equivalent orthotropic approach or according to a multi-layer shell model (Izzi et al., 2018). In vibro-acoustics CLT panels have successfully been modelled as equivalent orthotropic thin plates (Santoni et al., 2017b, 2019), described by frequency-dependent elastic properties in order to take into account the influence of rotatory inertia and shear deformation. How-
ever, in order to implement the orthotropic thin plate model the knowledge of the elastic properties associated to the principal directions is required (Santoni et al., 2018a, 2017c). In this study, a 3-ply CLT panel was modelled within the TMM framework as an equivalent homogeneous isotropic elastic medium, rather than as an orthotropic 3D elastic solid or an orthotropic thin plate.

In order to estimate the effective frequency-dependent elastic properties of an equivalent homogeneous isotropic layer, a numerical method has been developed within the TMM framework, based on the measured transmission loss. The tested inhomogeneous element, either constituted by bricks jointed with mortar, or layered wooden panels made of timber beams for example, is considered as a single equivalent homogeneous isotropic medium with the same density and thickness. A set of equivalent mechanical parameters is determined by using a minimisation procedure. In this process, as illustrated in the workflow chart given in Figure 2, the equivalent homogeneous layer is modelled within TMM. The geometric properties of the bare building partition, such as its density $\rho$, its thickness $h$ and its dimensions $L_x$ and $L_y$, are used together with the measured TL as input data, while the elastic properties $E$ and $\eta$ of the equivalent homogeneous layer represent the variables of the algorithm. By means of a non-linear optimization algorithm, based on the Matlab R2014b function fminsearchbnd (D’Errico, 2005 (Retrieved February 6, 2012)), the TL is computed iteratively by varying these variables within a given range. The algorithm tweaks the variables trying to minimise the implemented cost function, represented, in this case, by the sum of the absolute differences between the experimental TL $T_{i,exp}$ and the results of the TMM model $T_{i,TMM}$, computed for each one-third octave frequency band $i$:

$$\Delta_{TL} = \sum_{i=1}^{n} |T_{i,exp} - T_{i,TMM}| \quad (3)$$

where $n$ is the total number of frequency bands considered. The algorithm gradually converges towards a minimum providing, for each frequency band, the elastic properties $E$ and $\eta$ of the equivalent homogeneous layer as results. The result of this process might represents just one of the possible mathematical solutions, and have not a strong physical meaning in itself. In order to preserve as much as possible physical significance of the apparent elastic modulus and the loss factor, the choice of the limits in which the algorithm works to optimize the parameters is very important. The upper and lower limits should define a realistic range of values for the investigated element. Moreover, this also increase the robustness of the algorithm, since no significant differences in the resulting elastic properties are shown when the initial guess values are changed within such an interval. Since for all the investigated partitions the loss factor $\eta$ did not show a strong frequency-dependency, it was set as a constant value and the minimisation was performed only on the apparent elastic modulus $E$. The Poisson ratio $\nu$ as well was considered a constant, determined, for each of the investigated structures, from typical values found in the literature.

Investigated building partitions

A TMM model was implemented in order to investigate three different building walls, sketched in the diagram in Figure 3: W01_CHB_P – a clay hollow brick partitions lined with plasterboard; W02_CBB_E – a masonry clay brick wall clad with mineral wool-based ETICS (External Thermal Insulation Composite System); W03_CLT_P – a CLT wall lined with plasterboard. The experimental sound insulation spectrum of each bare structure, which was used to determine its effective elastic properties as described in the previous section, was obtained either from measurements into the sound transmission test facility of the Engineering Department of the University of Ferrara, or from laboratory reports. In any case, it was measured according to the standard ISO 10140-2, or the
ASTM E90, which is based on the same experimental procedure.

The basic wall of the first building element, hereafter referred to as W01_CBH_P, was constituted by horizontally perforated clay hollow bricks 80 mm thick, bound with mortar joints and plastered on both sides, for a total thickness of 100 mm and an effective density of approximately 880 kg/m³. This wall was lined with a single layer of plasterboard, 12.5 mm thick, and with a density equal to 740 kg/m³, fixed with fasteners to the metal studs, with a horizontal center to center distance of 600 mm and vertically spaced 250 mm. The cavity between the plastered surface of the basic wall and the plasterboard panels was filled with 45 mm of glass wool material with a density of 13 kg/m³. This fibrous material was modelled within the TMM model as an equivalent fluid according to the well known Johnson-Champoux-Allard model (Champoux and Allard, 1991; Johnson et al., 1987). The physical parameters required by this model were determined from the bulk density of the fibrous material and the average fibres diameter obtained from the literature, by using well established approaches (Bies and Hansen, 1980; Bonfiglio and Pompoli, 2013; Castagnede et al., 2000; Luu et al., 2017). The plasterboard layer and the basic wall were both modelled as isotropic elastic solids. The TL of the bare wall, required to compute the equivalent elastic properties, was obtained from a laboratory test report of the sound insulation performance. Sound transmission measurements were carried out, according to the EN 10140-1 standard, both on the bare brick wall and on the lined partition. Both the elements, with a surface area of approximately 10.8 m², were installed in a wall sound insulation test facility, satisfying the requirements given in the EN 10140-5. In order to include in the TMM calculation the structural transmission contribution through the studs, the decoupled approach proposed by Vigran, 2010b was applied.

The second investigated structure, named W02_CB_E, was a massive element used for external façades, consisting of a solid clay brick wall plastered on both sides, for a total thickness of 150 mm and an effective density of 1770 kg/m³. This bare structure was clad with 100 mm thick mineral wool slabs with a density of 78 kg/m³, finished with 5 mm of reinforced cement plaster. The plastered brick wall was modelled in the TMM as an equivalent homogeneous elastic layer, described by a frequency-dependent elastic modulus, given in Table 1. The finishing layer was also considered an isotropic elastic layer, characterised by constant elastic properties derived from the literature. The mineral wool layer was modelled as a poroelastic medium, considering the sound propagation through both the fluid and the solid phase. Its elastic and physical properties were fully characterised experimentally in the acoustic laboratories of the University of Ferrara. The structure-borne sound transmission due to the mechanical connections was taken into account by using the approach specifically developed for ETICS system by Santoni et al., 2017a. In the same reference, the elastic and physical properties of all the layers this partition consist of can also be found. The experimental sound insulation of the bare wall and the lined structure was measured in the sound transmission test facility of the University of Ferrara, on specimens with a surface area of approximately 10.8 m².

The last investigate partition, which is named W03_CLT_P, is constituted by a 3-ply CLT panel 78 mm thick, with a density of approximately 540 kg/m³. The wall, 3.6 m wide and 2.4 m high, was assembled from two smaller panel butt together but not systematically bonded. This element was tested,
together with other CLT walls and floors, as part of broad investigation of sound insulation in timber buildings, conducted by the National Research Council of Canada. The experimental results of several measured structures, both involving bare CLT panels and lined CLT walls, were published by Hoeller et al., 2017, presenting the TL of the bare structures and the improvement provided by the linings in terms of \( \Delta TL \). The considered 3-ply CLT panel was lined with a double layer of fire-rated plasterboard, screwed into the wood. Each layer of plasterboard was nominally 12.7 mm thick and with a density of 750 kg/m\(^3\). Unfortunately the research report published by the National Research Council of Canada did not provide all the materials’ properties and the information required as input data in the TMM model, therefore common practice values were used for these plasterboard panels.

Results and validation

In this section the reliability of the proposed homogenisation approach is investigated by comparing the TL computed by using the TMM with the experimental sound insulation of the three considered partitions. The frequency-dependent apparent elastic properties of the three bare structures, obtained from the minimisation algorithm, are given in Table 1. In Figure 4 the sound insulation provided by the partition defined W01_CHB_P is shown. As highlighted by the comparison between the numerical and experimental sound insulation of the bare brick wall, the frequency dependent elastic properties allowed for a remarkable agreement at the mid-high frequency range, while the accuracy slightly decreases at the lower frequencies. In fact, most of the fluctuations in the low frequency range are due to the modal behaviour of the testing rooms and the investigated structure, which are not taken into account in the TMM approach. Therefore, a smoother variation of the elastic properties is preferred in the low frequency range rather than an highly fluctuating curve that would provide a perfect match between numerical and experimental data. Moreover, a smooth variation of the frequency-dependent elastic properties ensures more reliable results when the equivalent homogeneous solid is coupled with other media; which is the final purpose of the proposed approach.

A good agreement is shown between the TL computed using the TMM model and the experimental sound insulation measured on the lined wall. The cavity between the basic wall and the plasterboard layer, filled with fibrous material, guarantees a mass-spring-mass resonance frequency below 100 Hz. In fact, the TL increases up to approximately 1500 Hz and drops at 2500 Hz due to the critical frequency of the plasterboard panel. Below the critical frequency, the numerical TL is slightly lower than the experimental sound insulation. This is a well-known effect caused by the assumption of massless and perfectly rigid structural connection between the basic wall and the plasterboard (Vigaran, 2010a), assumed by the implemented model, which overestimates the contribution of the structure-borne sound transmission.

In Figure 5 the TL of the solid brick wall is shown, both for the bare partition and the wall lined with the ETICS system. The TMM results are compared with the experimental sound insulation. The numerical results provided by the proposed model are in rather good agreement with the experimental data, within the entire frequency range. The mass-spring-mass resonant frequency of the building partition is correctly computed around 150 Hz, between the third octave bands centred on 125 Hz and 160 Hz. Above 2000 Hz, the reduction of sound insulation, partially caused by critical frequency region, but mainly due

Table 1: Elastic properties of the three bare walls used as input data for the equivalent homogeneous layer in the TMM model

<table>
<thead>
<tr>
<th>( f ) [Hz]</th>
<th>( E_{W01} ) [Pa]</th>
<th>( E_{W02} ) [Pa]</th>
<th>( E_{W03} ) [Pa]</th>
</tr>
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<tr>
<td>100</td>
<td>( 1.00 \cdot 10^{10} )</td>
<td>( 2.13 \cdot 10^{10} )</td>
<td>( 2.05 \cdot 10^{10} )</td>
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<tr>
<td>125</td>
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<td>( 1.77 \cdot 10^{10} )</td>
<td>( 1.20 \cdot 10^{10} )</td>
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<td>160</td>
<td>( 1.00 \cdot 10^{10} )</td>
<td>( 1.44 \cdot 10^{10} )</td>
<td>( 9.51 \cdot 10^{9} )</td>
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<tr>
<td>200</td>
<td>( 7.82 \cdot 10^{9} )</td>
<td>( 1.19 \cdot 10^{10} )</td>
<td>( 6.41 \cdot 10^{9} )</td>
</tr>
<tr>
<td>250</td>
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<td>( 9.89 \cdot 10^{9} )</td>
<td>( 4.53 \cdot 10^{9} )</td>
</tr>
<tr>
<td>315</td>
<td>( 3.01 \cdot 10^{9} )</td>
<td>( 8.15 \cdot 10^{9} )</td>
<td>( 3.44 \cdot 10^{9} )</td>
</tr>
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<td>( 6.68 \cdot 10^{9} )</td>
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<td>( 2.58 \cdot 10^{9} )</td>
<td>( 9.21 \cdot 10^{8} )</td>
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<td>( 5.28 \cdot 10^{8} )</td>
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</tbody>
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to the bridges’ action, is slightly overestimated due to the assumption of infinite stiff structural connections.

Finally, in Figure 6 the sound insulation of the 3-ply CLT panel is shown. The first coincidence and the critical condition, characterising the typical orthotropic behaviour of a CLT plate, fall within the 400 Hz and the 800 Hz third octave bands respectively. The equivalent isotropic model implemented with frequency-dependent elastic properties was proven to be suitable to predict this particular behaviour. The TMM also allowed to predict with good accuracy the TL provided by the CLT panel lined with a double layer of fire-rated plasterboard. As discussed in the Research Report published by Hoeller et al., 2017, due to the roughness of the CLT surface the plasterboard panels are decoupled from the CLT plate by means of a thin layer of air, which was necessary to include in the TMM computation, and connected through the wall only by screw fastening. At higher frequency, the critical condition of the plasterboard layer is clearly identified between the 2000 Hz and 2500 Hz frequency bands. In this case, the assumption of perfectly rigid structural bridges is suitable to accurately describe the fastening condition and provides a good approximation of the sound insulation of the building partition.

Conclusions

In this paper, the TMM approach has been applied to compute the TL of three building partitions. Each structure was constituted by a base element, such as a brick wall or a CLT panel, with linings or claddings. A homogenisation approach has been proposed in order to model by means of the TMM composite and anisotropic systems as equivalent isotropic elastic solids. The homogenisation approach, based on a minimisation algorithm of the sound insulation of the bare structure, was implemented within the TMM framework. It is based on a minimisation algorithm which computes, from the experimental TL of each investigated partition with the predicted results computed by using the TMM. In order to analyse the applicability of this method to different building technologies, three different base structures were investigated: a hollow clay brick wall (commonly used as inner partition); a solid brick wall (used for façades); a cross laminated timber element. It was possible to experimentally characterise only few of the materials the linings were made of, thus several properties were obtained from typical values found in the literature.
Despite these approximations, a very good agreement was found between the experimental and the numerical TL, proving that the TMM approach certainly represents a powerful tool to compute the sound insulation performance of building partitions. Moreover, the proposed homogenisation approach provides reliable equivalent elastic properties to model inhomogeneous and anisotropic partition as equivalent isotropic elastic solids, which can be easily determined by means of a minimisation algorithm based on the experimental sound insulation. Since the laboratory test report of the acoustic performance of common building elements is usually provided by the producers, this approach can be helpful in the acoustic optimisation design of several building partitions. Besides, the homogenisation approach proposed in this paper is suitable to be applied to a great variety of structures used in building construction or in other industrial fields.

References


