# Recurrent Neural Network based Deep Learning for Solar Radiation Prediction

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## Abstract

In the past decade, the data based solar radiation prediction models such as artificial neural network (ANN) model, support vector machine (SVM) model, state space model (SS), Bayesian network model (BN), and autoregressive with exogenous terms (ARX) model appeared in abundance along with the advanced computing technologies and large amount of data storage devices. These inverse models performed relatively well in the solar radiation prediction. Currently, how to further improve the accuracy of the solar prediction algorithms for building application such as model predictive controls remains as a challenge. In the era of big data, deep learning is being explored widely. This is a new area of machine learning research with an objective of moving machine learning closer to one of its original goals: Artificial Intelligence. To further investigate the solar radiation prediction using neural network, recurrent neural network (RNN) based deep learning algorithm is proposed and compared with other data-driven methods: ARX, SS, ANN, and BN methods. It was concluded that RNN method has the best performance in terms of the accuracy of solar radiation prediction for the selected case study.

## Introduction

In the United States, 40% of the nation's total energy consumption and 75% of the total electricity consumption are due to commercial and residential buildings (Min et al, 2015). Among numerous approaches to improve building energy efficiency in operation stage, model predictive control (MPC) applications in the buildings were approved to be an effective method to reduce building energy consumption (Chen et al, 2015; Oldewurtel et al, 2012; Li et al, 2015). MPC has been studied by the HVAC community with a number of experimental results including optimizing building operation with a large water storage tank (Ma et al. 2012a), optimizing low-lift chiller for thermo-active building systems (Gayeski, 2010), optimizing building heating systems (Siroký et al., 2011), optimization of conventional HVAC systems in a full-scale building (Narayanan, 2011; Bengea et al., 2014), multi-objective optimization scheme for commercial offices (West et al., 2014), and radiant cooled building (May-Ostendorp et al. 2013). MPC utilizes a dynamic model to predict the future response of a plant (e.g., building, its HVAC equipment and system) and makes control decisions to minimize a predefined cost function, which was largely influenced by the weather forecasting. Weather conditions are critical thermal boundary conditions for the dynamic model and the modelling accuracy is a key enabler for effective and robust controller performance. Thus, weather forecasting including outside air dry bulb temperature, solar irradiation, etc. plays a vital role for the MPC controller performance.

Dong et al. (2011) designed a nonlinear model prediction control method based on the weather forecasting model and occupant behaviour pattern prediction model. The building cooling energy consumption can be saved by 17.8% compared with the commonly used daily set point and night setback temperature control strategy. Zong et al. (2012) implemented a MPC methodology to support the introduction of wind renewable energy penetration and optimize the utilization of the current distributed energy resource to make a full use of the available capacity. Using dynamic electricity prices and integrated weather forecasting data, it indicated that the MPC control strategy is able to shift the large electricity load to periods with low prices. Zavala et al. (2009) establish an on line optimization framework to exploit weather forecasting information in the operation of energy systems. Adding weather forecasting information provides a mechanism to compute proactive operation that can lead to significant cost reduction and building energy consumption saving. As evidenced by the literature, weather forecasting including the solar radiation prediction is a key procedure for an effective MPC application in buildings. The accurate solar radiation prediction could significantly affect the MPC controller performance. Inaccurate forecasting could result in the MPC controller making sub-optimal and even wrong decisions, leading to higher costs and energy consumption. This is one key motivation for the study presented in this paper.

In addition, the renewable solar energy application in buildings is also an effective way for building source energy reduction. Solar energy is free, clean and abundant in most places throughout the year and is important especially at the time of high fossil fuel costs and polluted atmosphere by the use of these fossil fuels. The accuracy of solar radiation prediction is an important component to evaluate the performance of the solar photovoltaic (PV) systems. This is especially true for applications such as integrated the building PV and the power grid. Su et al. (2012) developed a new real time prediction model for the output power and energy efficiency of solar PV systems. The prediction models selection and accuracy were shown


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to have a great influence on the building energy consumption.

The commonly used solar radiation prediciton models developed in the past are based on linear and nonlinear models. These models give a correlation between solar radidaiton flux on a horizontal surface and some meteorological variables such as shining hours, ambient air dry bulb temperature and relative humidity. The linear models use a simple linear function while the nonlinear models use a polynomial function of the third or higher order. A commonly used linear model for this purpose which defines the global solar energy in terms of the extraterrestrial solar energy, day length and number of shining hours (Chineke, 2008). The global solar energy models based on the Angström model determine the model coefficients by using the MATLAB curve fitting tool. A nonlinear term is added to the Angström model to obtain a quadratic model. For the global solar energy model, many linear models used the clearness index as the inputs. For the nonlinear model, quadratic and cubic terms are added to the linear model to estimate the diffuse solar energy (Khatib et al. 2011).

In the past decade, the data-driven based solar radiation prediction models such as ANN model, SVM model, and ARX model have received a lot of attendtion in both academia and practice. In general, these inverse models performed well in the solar radiation prediction.

Mellit and Pavan (2010) presented a practical method for solar irradiance forecasting using artificial neural network. The proposed Multilayer Perceptron MLP-model makes it possible to forecast the solar irradiance on a base of 24-hr using the present values of the mean daily solar irradiance and drybulb air temperature. An experimental database of solar irradiance and air temperature data was used to test and validated the proposed MLP model. The results indicated that the proposed model performs well, while the correlation coefficients are in the range 98–99% for sunny days and 94–96% for cloudy days. As an application, the comparison between the forecasted solar irradiance and the energy produced by the PV plant installed on the rooftop of the municipality of Trieste shows the goodness of the proposed model.

Voyant et al. (2012) proposed an original technique to predict the global radiation using a hybrid autoregressive–moving-average (ARMA)/ANN model and data from a numerical weather prediction model (NWP). Multi-layer perceptron (MLP) was particularly considered. After optimizing the proposed architecture with NWP and endogenous data previously made stationary and using an innovative pre-input layer selection method, it was combined into an ARMA model from a rule based on the analysis of hourly data series. This model has been used to forecast the hourly global radiation for five places in Mediterranean area. This technique outperforms classical models for all the places. The normalized root mean square error (nRMSE) for the proposed hybrid model MLP/ARMA is 14.9% compared to 26.2% for the naïve persistence predictor. Note that in the standalone ANN case, the nRMSE is 18.4%.

Ji and Chee (2011) proposed a new approach that contains two phases to predict the hourly solar radiation series. In the detrending phase, several models are applied to remove the non-stationary trend lying in the solar radiation series. To judge the goodness of different detrending models, the Augmented Dickey–Fuller method is applied to test the stationarity of the residual. The optimal model is used to detrend the solar radiation series. In the prediction phase, the AARMA model is used to predict the stationary residual series. Furthermore, the controversial Time Delay Neural Network (TDNN) is applied to do the prediction. Because ARMA and TDNN have their own strength respectively, a novel hybrid model that combines both the ARMA and TDNN, is applied to produce a better prediction. The simulation result shows that this hybrid model can take the advantages of both ARMA and TDNN and give better results.

Some researchers used SVM and support vector regression (SVR) for developing global solar irradiation (GSR) predictor models. Zeng and Qiao (2013) proposed a least-square (LS) SVM-based model for short-term solar power prediction (SPP). The inputs of the model were historical data including atmospheric transmissivity sky cover, relative humidity, and wind speed. The output of the model was the predicted atmospheric transmissivity, which then was converted to solar power according to the latitude of the site and the time of the day. Their results demonstrated that the proposed model not only significantly outperformed a reference autoregressive (AR) model but also achieved better results than a radial basis function neural network (RBFNN)-based model in terms of the prediction accuracy.

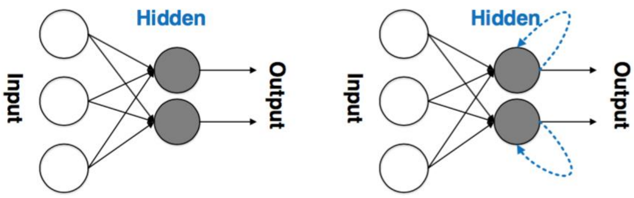
Yacef et al. (2012) presented a comparative study between Bayesian Neural Network (BNN), classical Neural Network and empirical models for estimating the daily global solar irradiation. An experimental meteorological database from 1998 to 2002 at Al-Madinah (Saudi Arabia) was used. Four input parameters were required: dry bulb air temperature, relative humidity, sunshine duration and extraterrestrial irradiation. Automatic relevance determination (ARD) method has investigated in order to select the optimum input parameters of the NN. Results show that the BNN performs better that other NN structures and empirical models.

To further investigate the solar radiation prediction using neural network, a RNN based deep learning algorithm is investigated in this study. The results using solar data from a local weather station indicate that the RNN model is the most accurate for solar radiation prediction. However, the RNN model requires a longer computation time compared with other models.

## Methodology

### Recurrent Neural Network Algorithm

RNN is a class of artificial neural network where connections between units form a directed cycle. This creates an internal state of the network that allows it to exhibit dynamic temporal behavior. Unlike feedforward neural networks, RNNs can use their internal memory to process arbitrary sequences of inputs, as shown in Figure 1.




*(a) Feed forward NN* *(b) RNN*

*Figure 1: Neural network*

The idea behind the RNN is to make use of sequential information. In a traditional neural network, we assume that all inputs (and outputs) are independent of each other. However, for many tasks that is a bad idea. RNNs are called *recurrent* because they perform the same task for every element of a sequence, with the output being depended on the previous computations. Another way to think about the RNNs is that they have a memory which captures information about what has been calculated so far. Figure 2 illustrates a typical RNN:

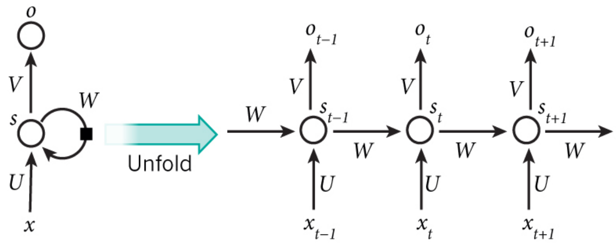



*Figure 2: A recurrent neural network and the unfolding in time of the computation involved in its forward computation (Wildml, 2016)*

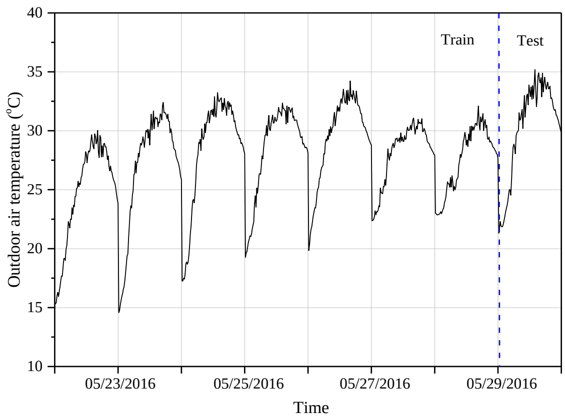
Figure 2 shows the RNN being unfolded into a full network. By unrolling the RNN, the network is written out in a complete sequential format. The formulas that govern the computation happening in a RNN are as followings:

- $x_t$ is the input at time setp *t*.
- $s_t$ is the hidden state at time step *t*. It is the memory of the network. It is calculated based o the previous hidden state and the input at the current step: $s_t = f(Ux_t + Ws_{t-1})$. The function *f* usually is nonlinear. $s_{-1}$, which is required to calculate the first hidden state, is typically initialized to all zeros.
- $o_t$ is the output at step *t*.

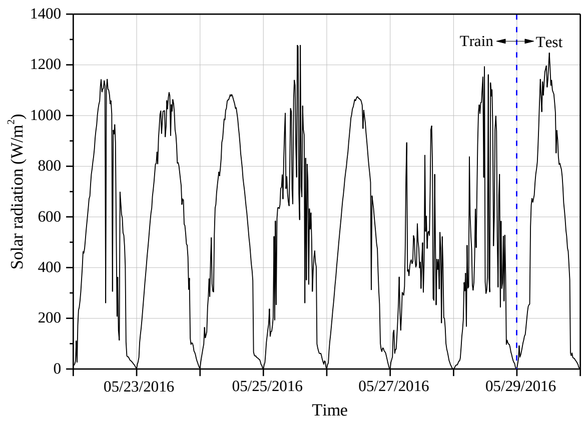
### Data sets

In this case study, weather data is from a local on-site weather station as shown in Figure 3. This includes global solar radiation, outdoor air-dry bulb temperature, relative humidity, dew point, wind speed, etc. This data set is used for training and testing the different data-driven models. In this application, the hour of day and outdoor air temperature were selected as input variables. Data from May 22nd to May 29th, 2016 are used for solar prediction training and testing, as shown in Figure 4. The data in the first seven days are selected as the training data set to predict the solar radiation in the following one day. The sampling frequency of all data from the weather station is 2-minute. In this study, processed data with different frequency is used to see the impacts of sampling frequency on the prediction performance.

*Figure 3: The on-site weather station*




(a)




(b)

*Figure 4: (a) Training and testing data set of outdoor air temperuare; (b) Training and testing data set of solar radiation*

### Evaluation Metrics

In this paper, the proposed RNN model is compared with other data-driven models of state space (SS), autoregressive with exogenous terms (ARX), artificial neural network (ANN) and continuous Bayesian network (BN) models (Niu et al. 2015). In order to quantitatively evaluate the solar prediction performance of each model, $R^2$, RMSE, CV-RMSE and NMBE are used to indicate the statistic feature of the prediction (ASHRAE 2012). $R^2$, which measures the proportion of the total variation explained by the fitting regression model, is computed from:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y}_i)^2} \quad (1)$$

The Root Mean Squared Error (RMSE) is computed from:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n-p}} \quad (2)$$

The coefficient of variation of the root mean square error (CV-RMSE) is computed from:

$$CV - RMSE = 100 \frac{RMSE}{\bar{y}} \quad (3)$$

The normalized mean bias error (NMBE) is computed from:

$$NMBE = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)}{(n-p) \times \bar{y}} \times 100 \quad (4)$$

## Results

Figure 5, 6, 7, 8 and 9 present the solar radiation prediction by ARX model, SS model, ANN model, BN model and RNN model, respectively. For all these results, the data sampling interval is 10-minute.
These results reveal that ARX, SS and BN models have the relatively bad performance for the solar radiation even they captured the pattern of the solar radiation. The SS model which did not predict well in the whole day has the worst prediction performance. ARX and BN models did not predict well at the low and large solar radiation areas. ANN and RNN models have the better performance in the solar prediction during the whole prediction period. The RNN model is the best model for the solar radiation among the five models.

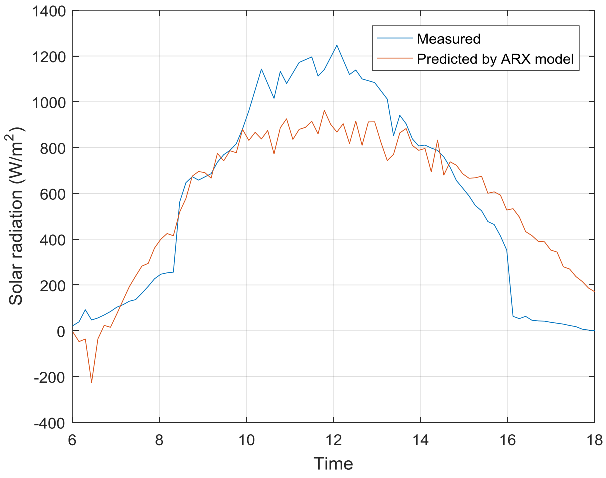



*Figure 5: Solar radiation prediction by ARX model*

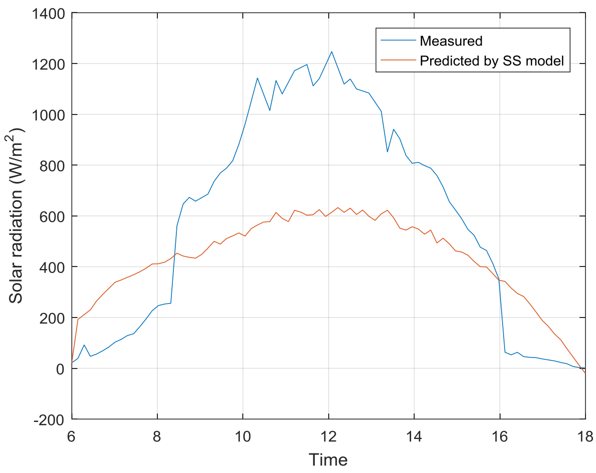



*Figure 6: Solar radiation prediction by SS model*

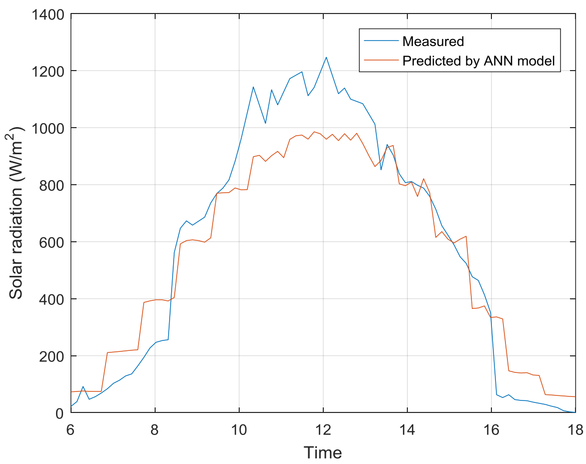



*Figure 7: Solar radiation prediction by ANN model*

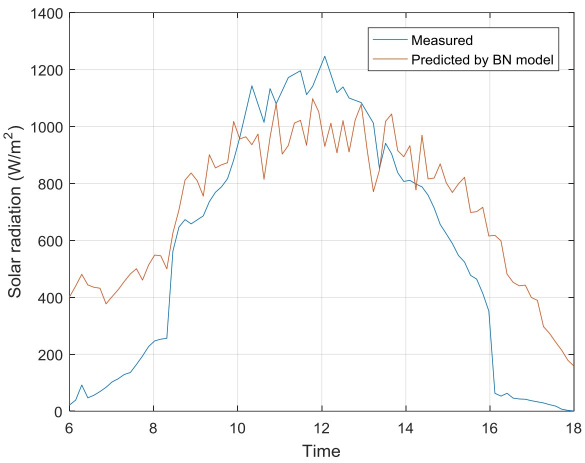



Figure 8: Solar radiation prediction by BN model

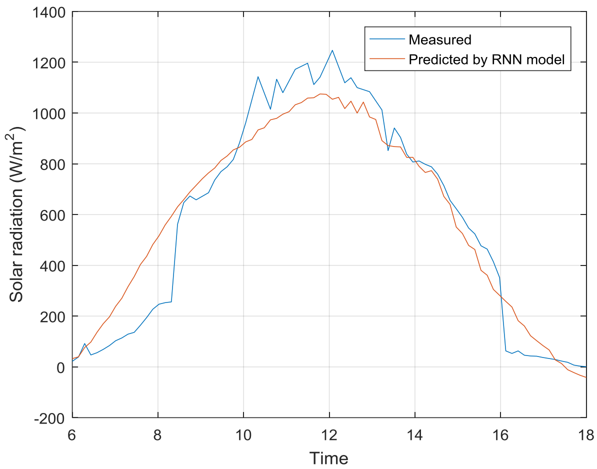



Figure 9: Solar radiation prediction by RNN model

In order to quantify the solar prediction of each model, the indices R2, RMSE, CV-RMSE and NMBE are calculated and shown in the Figures 10, 11,12 and 13 respectively. The R square value of the SS model is only 0.42. The ARX and BN models' R square values are in the range of 0.6 and 0.8. Moreover, the R square values of ANN and RNN are all above 0.9. The RNN model has the highest R square value of 0.925. Figures 11 and 12 show that the RMSE and CV-RMSE of the SS model are the highest. While,the RNN's has the lowest RMSE and CV-RMSE. It can be seen from the Figure 13, the NMBE of the SS and BN models are relatively high which are above 20%. The ANN and RNN's NMBEs are all less than 1%. The NMBE of the RNN model is the lowest among the five models. Base on this index analysis, the SS model for the solar radiaiton predciton is the worst. The RNN model is most accurate for the solar radiation predction.

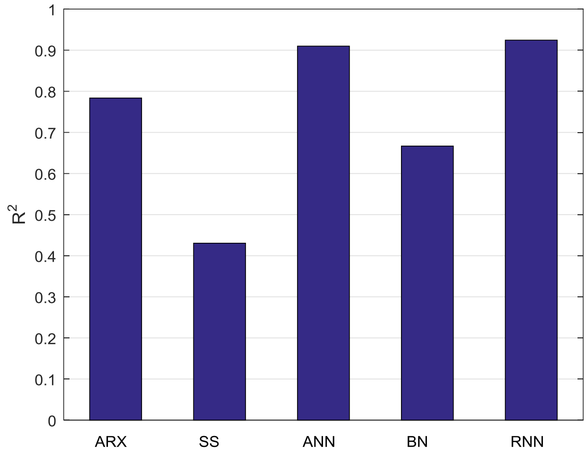



Figure 10: R square of all the prediction models

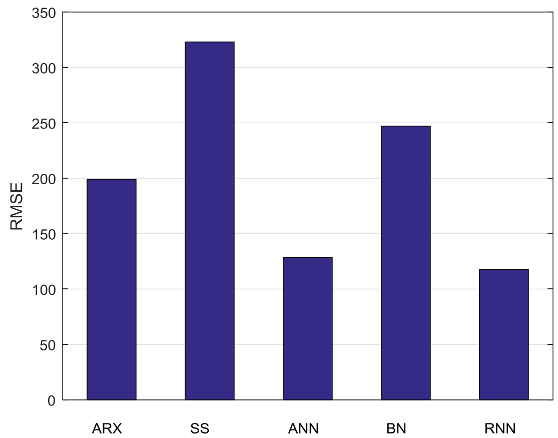



Figure 11: RMSE of all the prediction models

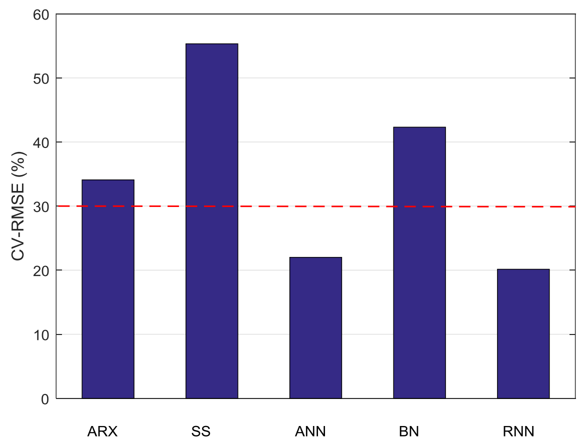



Figure 12: CV-RMSE of all the prediction models

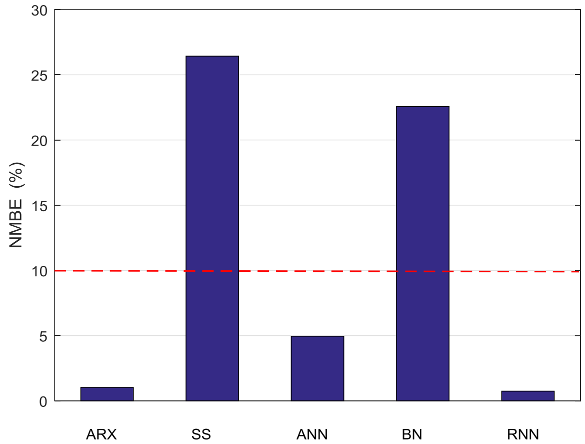



*Figure 13: NMBE of all the prediction models*

Table 1 lists the required computational time of five models. For this study, a computer with Intel Xeon CPU 3.2GHz, 8 GB memory and 64-bit operation system is used. The ARX is the least time-consuming. The computational cost for the RNN models is relatively large which is up to 530 seconds. Although the RNN model has the highest accuracy for the solar radiation prediction, the high computational cost is its limitation for the real application. Through the balance of the accuracy and computational cost of all models, it seems that the ANN model is an appropriate method for the solar radiation prediction based on this case study, which is the current industry practice.

*Table 1:Computational time for different prediction models*

| | SS | ARX | BN | ANN | RNN |
|---|---|---|---|---|---|
| Computation time (second) | 7.1 | 0.5 | 1.2 | 17.5 | 530 |

### Data Sampling Frequency Effect

In the aforementioned analysis, the data sampling frequency is ten minutes. In general, different data acquisition systems have different settings for data sampling frequency based on various functions. The data sampling frequency could have impacts on the prediction accuracy from different data-driven models. Therefore, two more data sampling frequencies with half an hour and an hour intervals are selected for a further investigation. Figure 14 and Figure 15 show the solar radiation prediction of all models with the data sampling frequency of half an hour and an hour intervals. Comparing with results of a ten-minute sampling method, the solar radiation prediction of some models (i.e., SS and ARX models) are improved. Some models' performance are getting worse (i.e., RNN model). It is also noted that some model's prediction performance (i.e., BN and ANN models) is not significantly influenced by the sampling frequency.

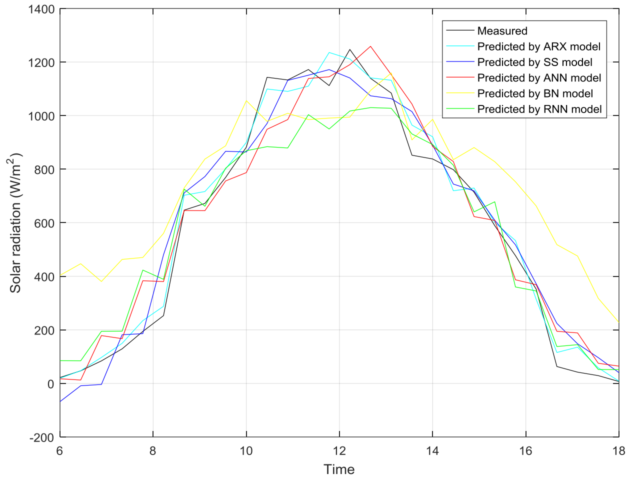



*Figure 14: Solar prediction of all the prediction models with data sampling frequency of half an hour*

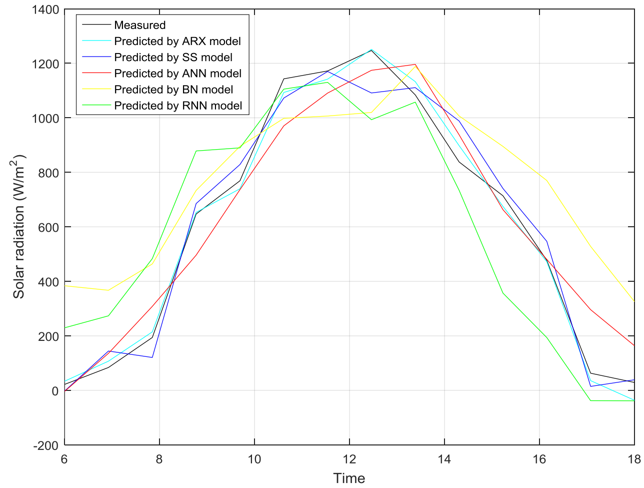



*Figure 15: Solar prediction of all the prediction models with data sampling frequency of an hour*

In order to quantify the performance, the assessment based on the indices $R^2$, RMSE, CV-RMSE and NMBE are calculated and compared, as shown in the Tables 2, 3 and 4. The solar prediction performance of SS and ARX models are getting better along with the decreasing of the data sampling frequency. The $R^2$ of SS and ARX models can reach up to 97.3% and 99.4%, respectively. For the BN and ANN models, the prediction performances under different data sampling frequencies are similar. It indicates that as long as the range of training data can cover range of prediction parameters, the data sampling frequency has less influence on the results for these types of data-driven models. For the RNN model, the accuracy of solar radiation prediction decreased significantly when the data sampling frequency decreased. It means that the RNN needs sufficient historical data set to train the prediction model to get a high accuracy. The results also suggest that the simple SS or ARX models can be used when the data set is not sufficient with a low data sampling frequency. When data sampling frequency is high, the advanced prediction models such as ANN and RNN can be applied. Moreover, another interesting finding is that even the training data is sufficient, the simple models such as SS and ARX models can be still used by processing the original data set into the low sampling frequency data set. Then, linear or nonlinear interpolation algorithms can be applied to obtain the high frequency data. In this way, predictions with a better accuracy can be achieved using the simple models requesting less computational cost.

*Table 2:Assessment of different prediction models based on data frequency with ten minutes*

| | SS | ARX | BN | ANN | RNN |
|---|---|---|---|---|---|
| $R^2$ (%) | 43.1 | 78.4 | 66.7 | 91 | 92.5 |
| *RMSE* | 324 | 199 | 247 | 128 | 118 |
| *CV-RMSE (%)* | 55.3 | 34.1 | 42.3 | 22 | 20.1 |
| *NMBE (%)* | 26.4 | 1.0 | 22.6 | 5.0 | 0.74 |

*Table 3:Assessment of different prediction models based on data frequency with half an hour*

| | SS | ARX | BN | ANN | RNN |
|---|---|---|---|---|---|
| $R^2$ (%) | 95.7 | 98.5 | 66.5 | 95.0 | 92.1 |
| *RMSE* | 89.6 | 53.2 | 248.5 | 96.1 | 121 |
| *CV-RMSE(%)* | 15.1 | 9.0 | 42.2 | 16.3 | 20.5 |
| *NMBE(%)* | 3.4 | 3.5 | 26.4 | 3.4 | 1.4 |

*Table 4:Assessment of different prediction models based on data frequency with an hour*

| | SS | ARX | BN | ANN | RNN |
|---|---|---|---|---|---|
| $R^2$ (%) | 97.3 | 99.4 | 68.6 | 93.5 | 80.8 |
| *RMSE* | 73.3 | 35.8 | 249.3 | 113.2 | 195.0 |
| *CV-RMSE(%)* | 12.1 | 5.9 | 41.1 | 18.7 | 32.2 |
| *NMBE(%)* | 0.8 | 0.8 | 24.8 | 1.9 | 2.8 |

## Conclusions

To further investigate the solar radiation prediction, a RNN based deep learning algorithm is investigated. In addition, the effect of data sampling frequency on the prediction performance with the different models are conducted. The following conclusions were summarized:

(1) A new solar radiation prediction algorithm based on RNN deep learning method was proposed.

(2) Neural network based prediction models have the high prediction accuracy compared with the SS, ARX and BN models using a high data sampling frequency of 10 minutes.

(3) The accuracy of RNN solar prediction decreased when the data sampling frequency decreased.

(4) The solar prediction accuracy of SS and ARX models significantly increased along with the data sampling frequency decreasing.

(5) The limitation of RNN model is related to the significant computation cost.

(6) The simple SS or ARX models can be used when the training data set is not sufficient with low data sampling frequency. When data sampling frequency is high, the advanced prediction models such as ANN and RNN can be applied.

## Future work

In this study, the RNN based deep learning solar prediction was proposed, and had a very high prediction accuracy. However, the input variables of the models only considered the outdoor air temperature and the hourly number of one day. More input variables will be considered in future such as cloud coverage, sunshine duration, etc. In addition, the moving window algorithm based prediction-training model will be investigated to better predict future solar radiation in a real-time fashion.

## Nomenclature

x = input

y = meaured data

$\bar{y}$ = mean of y

$\hat{y}$ = estiamted value

o = output

s = state

CV-RMSE = coefficient of variation of the root mean

NMBE = normalized mean bias error

NRMSE = normalized root mean square error

RMSE = root mean squared error

R2 = coefficient of determination

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