Optimization Algorithms Supporting The Cost-Optimal Analysis: The Behavior of PSO

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Abstract
This work is within the wide context of simulation-based optimization methods applied to the cost-optimal analysis of nearly-zero energy buildings, with the objective of studying the behavior of the particle swarm optimization algorithm (PSO) in solving cost-optimal problems. After the presentation of the features of the involved design variables and of the resulting design space, the paper focuses on the application of PSO, implemented by coupling GenOpt\textsuperscript{®} to TRNSYS\textsuperscript{®}, to a typical cost-optimal problem for a single-family home. The algorithm performance related to different sets of algorithm parameters was analyzed and classified according to defined metrics. Best results are reached with a small number of particles and higher cognitive acceleration.

Introduction
The European Directive on the Energy Performance of Buildings (EPBD - 2010/31/EU) introduced the key-concepts of nearly Zero Energy Building (nZEB) and cost-optimality of the energy performance. In particular, the definition of a nZEB is based on the cost-optimal level of energy performance over a certain period of time and with appropriate boundary assumptions. This has given rise to a wide research activity across Europe (BPIE 2013) in order to define the energy performance level of such new nZEBs and, more, to study the design of high performing buildings through optimization methods that consider both the energy demand side and the energy supply side options (Bayraktar et al. 2012).

It is known that the introduction of the cost-optimal analysis, beyond its regulatory purpose that was addressed by each Member State of EU through the calculation of the cost-optimal levels for various reference buildings, has given a strong impulse to the scientific studies on the optimization of the performance of nZEBs. The application of the cost-optimal methodology can be seen as a complex optimization problem, where the objective function is the Global cost and the optimization variables are the various energy efficiency measures (EEMs) that are combined into a number of packages.

Some studies within the context of the cost-optimal analysis, aimed at studying large numbers of EEMs, introduced the use of simulation-based optimization methods (SBOM) for exploring a wider search space and identifying the point minimizing the global cost objective function (Attia et al. (2013), Hamdy et al. (2013), Ferrara et al. (2014a)). In fact, various packages of EEMs should be investigated in these studies, and the calculation method of the energy use for heating and cooling should be dynamic. The first aspect makes the optimization problem highly multi-dimensional, while the second aspect makes it highly computationally intensive, because of the simulation time of detailed energy models of large buildings into standard software tools like EnergyPlus or TRNSYS.

In their review, A.T. Nguyen et al. (2014) stated that the effectiveness of simulation-based optimization methods for cost reduction strictly depends on the objective function to be minimized and on the adopted algorithm, but there are still very limited results from the literature in this field. Machairas, M. et al. (2014) performed a detailed review of the algorithms that are used in optimization problems related to building design, however, the specific problem of the cost-optimal search was not included in the study.

Among the various algorithms adopted for the solution of building optimization problems, the Particle Swarm Optimization (PSO) algorithm is one of the most largely employed (Hamdy (2016)). PSO has been used for both envelope and systems optimization problems. Rapone et al. (2012) optimized the type and size of the glazing and the louvers of a double curtain wall façade. PSO was used to optimize heating terminal units (Hajabdollahi et al. (2012)) and heating and cooling systems (Wang et al. (2010)). Some studies addressed problems related to both demand side (envelope) and supply side (systems) options (Bichiu and Krarti (2012)).

Objectives of the work
This work aims at investigating the application of the PSO algorithm to the multi-dimensional optimization problem related to the search of the cost-optimized design of nearly zero energy buildings, by reporting the common features of the cost-optimal problem and the influence of the algorithm parameters in the optimization process effectiveness (ability of reaching the optimum) and efficiency (computation time).

In detail, this work aims at:

- Describing the features of the cost-optimal optimization problem of a nZEB;
• Analyzing the space of solutions (design space) related to the cost-optimal design problem, in order to appraise how the design variables influence the objective function and therefore the location of the solutions in the design space;
• Studying to what extent and according to which criteria it is possible to assess the performance and optimize the parameters of the PSO algorithm for application to this specific optimization problem in the context of building simulation research.

Methodology
The optimization objective
The objective of the cost-optimal analysis studies is the minimization of the global cost (EN 15459 (2007)), which takes into account both the investment and the operational costs related to the implementation of energy efficiency measures over the building economic lifecycle (EU Regulation (2012)). The global cost calculation leads to evaluate the net present value of the costs paid over a period of time (usually 30 years) taking into account the residual value of equipment having longer lifetime than the period of time of the analysis. Given the object of the EPBD, only the costs related to building components and systems that are able to impact the building energy performance must be included in the calculation.

The specifications of the so-called cost optimal methodology can be found in the Guidelines (EU Guidelines (2012)) accompanying the Regulation related to the Directive EPBD recast (EPBD (2010)). The equation of Global Cost can be written as

\[ C_g = CI + \left( \Sigma_j \left( C_{a,j}(j) \cdot R_d(i) \right) - V_f(j) \right) \]  

(1)

where:
• \( C_g \) - global cost;
• \( CI \) - initial investment cost
• \( C_{a,j}(j) \) - annual cost for component \( j \) at the year \( i \)
• \( R_d(i) \) - discount rate for year \( i \)
• \( V_f(j) \) - final value of component \( j \) at the end of the calculation period.

For each component \( j \), cost functions representing the related investment, maintenance, and replacement costs should be created. Then, a set of energy efficiency measures has to be selected for each component (e.g. different thicknesses of insulation are selected as energy efficiency measures and the related cost are calculated according to the insulation cost functions).

The final step of the methodology requires calculating and comparing the global cost and the related primary energy demand of different packages of energy efficiency measures, in order to define the energy performance level that is associated with the lowest global cost. The resulting building design leading to the lowest global cost value is defined as the cost-optimal solution of the cost-optimal design problem. If it is found that more than one package of energy efficiency measures lead to the lowest global cost value, the package with the lowest primary energy demand has to be chosen as the cost-optimal solution.

The design variables
The cost-optimal design problem involves several variables related to the building system and to the building envelope, such as insulation thickness, type of window glazing, window width and depth. Therefore, the design variables should be selected according to their availability on the market and should be optimized to the order of their variability on the market. Moreover, their variability should consider the scale of uncertainty of the variable itself due to the manufacturing and construction process and the marginal improvement of the energy performance of the building produced by the variation of the variables. In this perspective, the search of the cost-optimal solution is a problem dealing with discrete variables.

The insulation thickness in the opaque envelope components, depending on the material properties, can vary with a step of 1-2 cm, in a range imposed by construction feasibility and energy performance benefit. A sensitivity analysis for studying the impact of the variation of each parameter can be useful for determining the range, as it is known that, depending on the location of the building and the related energy need repartition in heating and cooling, the marginal improvement in energy performance tends to decrease when the insulation increases, as reported in Ferrara et al. (2014b).

In Fig. 1, an example of a typical cost function for the slab insulation is reported, which can be determined through interpolation of costs taken from price lists (details in Ferrara et al. (2014a)). As shown, the specific cost for one unit of incremental thermal resistance (expressed in €kJ/m²K) decreases for higher values of thermal resistance following an exponential function (black curve). Based on that, the cost function expressing the specific cost of 1 m² of slab insulation as a function of thermal resistance is shown in red.

![Figure 1: Example of cost function for slab insulation](image-url)
performances. If these properties were optimized as independent one of the other, the result would probably be a glazing type that is not available on the market, as these properties are interdependent and it is technically hard to create glazing with all the desired characteristics. Because the glazing types available on the market are packages of defined different physical properties, the window type variables are usually composed by a set of 4-6 window packages selected from the market.

For a given window type it is possible to create linear cost functions that have different gradients according to the type of glazing. However, constraints on the variability range should be imposed based on building layout, regulations (e.g. minimum amount of daylight entering the ambient), and the step of variation should account for the standard dimension of modular glass panels available on the market.

Concerning the energy systems, one or more different alternatives can be evaluated (e.g. heat pump, gas condensing boiler, wood boiler...) with their cost of investment, maintenance, replacement and the related energy costs according to the size of the system required to match the maximum building energy load, which is in turn related to the building envelope configuration. Therefore, the energy and cost model should account for this variability.

The algorithm

PSO is an algorithm inspired by social behavior of bird flocking or fish schooling. It was developed by Russell C. Eberhart and James Kennedy (1995).

For continuous design variables, the problem domain is seen as a multidimensional space populated by dimensionless particles. A position \( x_j \) and a velocity vector \( v_j \) are associated to each particle \( j = 1, \ldots, n \). The position, given as input to the cost function, allows for scoring and categorizing the particles from one to the other generation.

At the \( i \)-th step of the algorithm, the velocity \( v_j^{(i)} \) of each particle \( j \) is updated according to equation (2) and the position \( x_j^{(i)} \) is updated according to equation (3)

\[
v_j^{(i+1)} = v_j^{(i)} + c_1\rho_1^{(i+1)}(p_j^{(i)} - x_j^{(i)}) + c_2\rho_2^{(i+1)}(G^{(i)} - x_j^{(i)})
\]

\[
x_j^{(i+1)} = x_j^{(i)} + v_j^{(i+1)}
\]

where

- \( p_j^{(i)} \) is the best position ever attained by the particle itself (cognitive memory)
- \( c_1 \) is the cognitive acceleration constant
- \( G^{(i)} \) is the best position ever attained by the swarm as a whole (global, or social, knowledge)
- \( c_2 \) is the social acceleration constant

- \( \rho_1 \) and \( \rho_2 \) are uniformly distributed random numbers in the range \((0,1)\). A user-defined seed initializes the random number generator.

In the discrete implementation of the PSO algorithm, introduced by Kennedy and Eberhart (1997), the discrete design variables are encoded through Gray encoding, into a binary string, consisting of \( m \) bits.

Denoting by \( x_j^{(i)} \in \{0,1\}^m \) the binary representation of the discrete variable \( x_j^{(i)} \), and by \( p_j^{(i)} \), \( G^{(i)} \) the binary representation of \( p_j^{(i)} \) and \( G^{(i)} \), respectively, the discrete version of the PSO consists of a classical PSO applied separately to the single bits of \( x_j^{(i)} \). That is, the \( k \)-th bit of \( x_j^{(i)} \) is considered as a separate particle and updated according to equations similar to (2) and (3). In particular, the binary velocity is computed according to equation (5), and the velocity is saturated at a maximum velocity \( v_{\text{max}} \).

\[
v_j^{(i+1)} = g_{j,k}^{(i)} + c_1\rho_1^{(i+1)}(p_{j,k}^{(i)} - x_{j,k}^{(i)}) + c_2\rho_2^{(i+1)}(G^{(i)} - x_{j,k}^{(i)})
\]

\[
w_{j,k}^{(i+1)} = \text{sigmoid}(\text{sat}(g_{j,k}^{(i+1)}))
\]

\[
x_j^{(i+1)} = \begin{cases} 0, & \text{if } w_{j,k}^{(i+1)} \leq g_{j,k}^{(i+1)} \\ 1, & \text{otherwise} \end{cases}
\]

where \( g_{j,k}^{(i+1)} \) is a uniform random number between zero and one, sigmoid() is a sigmoid function and sat() is a saturation function that trims the velocity at \( v_{\text{max}} \).

In our study, we consider the binary PSO implementation of the generic optimization program GenOpt\textsuperscript{®} (Wetter (2011)), and for the purposes of our analysis, we fix the random generation seed to one (to allow repeatability) and adopt a von Neumann neighborhood topology of size 5. Refer to the GenOpt manual for further details.

As the cost-optimal problem is related to discrete design variables, the PSO, in its binary version, is chosen for this problem because of its ability to deal with discrete variables. In the present application of PSO, the problem space of solution is the above presented design space where the particles move through the design solutions (each representing a particle position) that are driven by the PSO algorithm towards the region of design space having low values of global cost. The particle having the best position \( p_j^{(i)} \) in one generation is associated to the design solution leading to the lowest value of global cost found in that generation, while the global best position \( G^{(i)} \) is related to the lowest global cost value ever found since the beginning of the optimization process.

Simulation

Case study description

In this work, the performance of the PSO algorithm in solving a cost-optimal problem for a Zero Energy Building is studied through an application to a reference
case study concerning the cost-optimization of a reference single-family house in France.

The building has two floors for a conditioned floor area of 155 m². The massive envelope is made of bricks and has insulation on the indoor side. A gas condensing boiler and an electric multi-split system were considered for heating and cooling energy supply. Refer to Ferrara et al. (2016) for additional details about the case study features and the cost-optimal calculation process.

The calculation of the energy demand and the related operational energy costs is made through a multi-zone building simulation model of the case-study building created into TRNSYS ®. For the purpose of this study, it is important to report that the running of the TRNSYS model of the case-study takes about 2 minutes (Intel® Core™ i7-4770 – 3.4 GHz, 8 MB cache, 4 core - HD 4600). The simulation-based optimization methodology used for the case-study cost-optimal problem is based on the coupling between TRNSYS, building simulation program, and GenOpt.

The cost-optimal problem addressed in this study involves 9 design variables related to the envelope, as shown in Table 1, each having its own related cost function. The energy system is sized according to the maximum heating/cooling loads related to the simulated building configuration. Refer to Ferrara et al. (2016) for details about the cost functions related to the case-study.

The design variables reported in Table 1 lead to a non-uniform 9-dimensional lattice search space, here named design space, composed of more than 10⁸ points, each representing a different design alternative.

The INI values in Table 1 constitute the reference configuration of the case-study building, which is taken as the initial point for the optimization (the initial position of the particles population).

**Reduction of the problem**

More than 130 years of simulations would be required for running one simulation for each design alternative of the design space of the aforementioned optimization problem, in order to explore the entire design space and find the minimum global cost (global optimum). Because this is not feasible, it becomes clear that the use of an optimization algorithm can help find solutions that minimize the objective function value with fewer simulation runs in a feasible computation time.

Therefore, in order to test and optimize the ability of the algorithm to reach the global optimum of a cost optimal problem, we need to test the algorithm performance on a problem where the global optimum is known. For this purpose, the original cost-optimal problem was reduced so that entire design space evaluation is computationally manageable.

The reduced problem resulted from reducing the number of points selected for each variable by choosing randomly some values from the range reported in Table 1. The dimension (number of points) of each variable is reduced by 30% (insulation parameters) or 50% (window parameters) with respect to the original problem. These values also represent, for each variable, the probability to include in the subset the value assumed by the variable in the optimal solution of the original problem. The number of variables was also reduced, as variables Bm, Hr and WTR were set to the values leading to the minimum value of objective function found in Ferrara et al. (2016). Since the area of the roof window was set to 0 (parameter Hr), the parameter WTR (window type roof) has no more meaning. Furthermore, the parameter Bm (dimension of one of the windows on the south façade) was set to its minimum value (0.2 m).

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### Table 1: Design variables for the original cost-optimal problem

<table>
<thead>
<tr>
<th>Variable name and description</th>
<th>Unit</th>
<th>Min</th>
<th>Max</th>
<th>Step</th>
<th>INI</th>
<th># of points</th>
</tr>
</thead>
<tbody>
<tr>
<td>ResO - Thermal resistance of wall internal insulation</td>
<td>[m²K/h]/[kJ]</td>
<td>0.25</td>
<td>5.00</td>
<td>0.25</td>
<td>1.75</td>
<td>20</td>
</tr>
<tr>
<td>ResR - Thermal resistance of roof insulation layer</td>
<td>[m²K/h]/[kJ]</td>
<td>0.25</td>
<td>5.00</td>
<td>0.25</td>
<td>3.50</td>
<td>20</td>
</tr>
<tr>
<td>ResS - Thermal resistance of slab insulation layer</td>
<td>[m²K/h]/[kJ]</td>
<td>0.25</td>
<td>3.00</td>
<td>0.25</td>
<td>2.50</td>
<td>12</td>
</tr>
<tr>
<td>Blr - Ground floor south window width (h= 2.15 m)</td>
<td>[m]</td>
<td>2.20</td>
<td>4.20</td>
<td>0.40</td>
<td>4.20</td>
<td>9</td>
</tr>
<tr>
<td>Bm - First floor south window width (h= 0.80 m)</td>
<td>[m]</td>
<td>0.20</td>
<td>5.40</td>
<td>0.40</td>
<td>2.20</td>
<td>11</td>
</tr>
<tr>
<td>Hr - Roof window height (w= 2.28 m)</td>
<td>[m]</td>
<td>0.00</td>
<td>4.80</td>
<td>0.60</td>
<td>4.72</td>
<td>9</td>
</tr>
<tr>
<td>WT – Window Type North, East, West</td>
<td>-</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>WTS – Window type South</td>
<td>-</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>WTR – Window type roof</td>
<td>-</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

### Table 2: Design variables for the reduced cost-optimal problem

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Unit</th>
<th>Values</th>
<th>Space dimension</th>
<th>Reduction</th>
<th>OPTr</th>
</tr>
</thead>
<tbody>
<tr>
<td>ResO</td>
<td>[m²K/h]/[kJ]</td>
<td>1.25 – 2 – 2.75 – 3.5 – 4 – 4.75</td>
<td>6</td>
<td>0.3</td>
<td>1.25</td>
</tr>
<tr>
<td>ResR</td>
<td>[m²K/h]/[kJ]</td>
<td>1 – 1.25 – 2.25 – 3 – 3.5 – 3.75</td>
<td>6</td>
<td>0.3</td>
<td>1.25</td>
</tr>
<tr>
<td>ResS</td>
<td>[m²K/h]/[kJ]</td>
<td>0.75 – 1 – 1.25 – 2</td>
<td>4</td>
<td>0.33</td>
<td>2</td>
</tr>
<tr>
<td>Blr</td>
<td>[m]</td>
<td>2.2 – 3 – 3.4</td>
<td>3</td>
<td>0.375</td>
<td>2.2</td>
</tr>
<tr>
<td>Bm</td>
<td>[m]</td>
<td>fixed at 0.2</td>
<td>1</td>
<td>-</td>
<td>0.2</td>
</tr>
<tr>
<td>Hr</td>
<td>[m]</td>
<td>fixed at 0</td>
<td>1</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>WT</td>
<td>-</td>
<td>2.4</td>
<td>4</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>WTS</td>
<td>-</td>
<td>1.2</td>
<td>4</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>WTR</td>
<td>-</td>
<td>as Hr=0, it has no meaning</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
The resulted design variables of the reduced problem are reported in Table 2. The reduced design space is composed of 1728 points, which can be all evaluated, leading to know the true optimum of this reduced problem, (OPTr).

It has to be noted that this method of problem reduction leads to a probability of including the global optimum of the original problem within the design space of the reduced problem of 0.3%. This is not high in absolute, but it is $10^8$ higher if compared to the probability of finding the global optimum by randomly extracting 1728 points from the design space of the original problem, that is equal to $3 \times 10^{-9}$.

Analysis of the design space of the reduced problem

In order to explore the n-dimensional design space of the reduced cost-optimal problem and investigate the nature of the global cost function, the GenOpt “parametric algorithm on a mesh” was run, leading to run 1728 simulations for the evaluation of all the points in the design space.

The minimum value of the objective function of the reduced problem is equal to 521.37 €/m² and the related primary energy demand is equal to 67.55 kWhρυ/m². The values of the design variables in the optimal point are reported in Table 2.

In Figures 2, 3, 4 and 5 the design space of the reduced problem is represented in a cost-optimal diagram, which has the energy performance on the x-axis expressed in terms of annual primary energy need, and the global cost objective function on the y-axis. The different colors and shapes of points highlight how points related to selected design variables are located in the design space and how costs associated to those variables are able to increase or decrease the objective function and the related energy performance.

In Figure 2 it is shown that the increasing Blr variable (representing the south window area) has not a positive effect either in lowering global cost or in moving towards better energy performance with similar cost values. Moreover, the window type 2 (WT2) is present in all the points located in low positions. From Figure 3 it is clear that the lowest global cost, and solutions in its neighbourhood, are reached for ResO=1.25, and it is clear that higher values of outwall insulation lead to an increase of the global cost while reducing the primary energy demand. From Figure 5, it emerges that the increase of the slab insulation (ResS) leads to an opposite trend, as it increases both the global cost and the primary energy demand.

Most of the variables are spread in the cloud, meaning that there is not a leading variable in determining the global cost value, but it results from the many mutual relationships between the design variables.

The way the parameter values are spread within the design space indicates that the sole parametric analysis
for optimizing one design variable at a time would not be enough for reaching the cost-optimal point, as the optimal value of one variable is strictly dependent on the others.

Based on Figures 2-5, it is clear that an optimization process is required for the cost-optimal search so that the mutual relationships between the involved design variables can be taken into account.

**Evaluation of the algorithm performance**

When performing simulation-based optimization for solving a cost-optimal design problem, time is the main constraint researchers and/or professionals have. They may want to minimize the objective function as much as possible within the minimum possible computational time. This is why the performance of the algorithm should not only be evaluated in terms of ability in reaching the global optimum, but also in terms of time needed for reaching the optimum. Moreover, within the time constraint, they may want that the neighborhood of the optimum is evaluated as much in details as possible, so that at the end of the simulation process they have a set of design options that are very close to the optimum in terms of global cost and can be selected according to other criteria, first of all the primary energy demand.

Within a simulation-based optimization process, one iteration corresponds to one update of the velocity and position in the design space of one particle, while one simulation corresponds to the run of the building simulation tool for calculating the objective function value associated to that point of the design space (representing one building configuration). Within an optimization run, the number of iterations may not correspond to the number of simulations, because the algorithm may lead the particles to return to the same position in the design space more than one time. When it happens, the objective function value is reported without running the simulation program another time for the same building configuration.

In this context, let $n_i$ denote one objective function evaluation within the optimization process, where $i \in \mathbb{N}$ is the iteration number in the range $1 \leq i \leq i_{\text{max}}$ ($i_{\text{max}}$ is the total number of iterations within an optimization process, depending on the number of particles $n_P$ and the number of generation $n_g$), and $s \in \mathbb{N}$ is the simulation number in the range $1 \leq s \leq s_{\text{max}}$ ($s_{\text{max}}$ is the total number of simulation within an optimization process). Let $C_g(n_s)$ be the objective function value related to iteration $i$ and simulation $s$ and let $U = C_g(n_s) + u$ be an upper bound of the neighborhood of the optimum where $u$ is a user-defined acceptable increase of the global cost value. The proposed metrics are defined as follows:

- $i_{\text{OPT}}$ is the iteration number related to the $1^{\text{st}}$ occurrence of the global optimum (if it is known and is reached within the defined number of iterations) within an optimization run;
- $s_{\text{OPT}}$ is the simulation number occurring within an optimization run when $i=k$;
- $S_u$ is the number of simulations, for which $C_g(n_s) < U$, occurred within an optimization run until $i=k$.

Different values can be assigned to $u$ and $k$. In this work:

- $u$ is taken equal to $2 \text{ €/m}^2$;
- $s_{\text{OPT}}$ denotes the simulation number occurring when $i=i_{\text{OPT}}$ ($k = i_{\text{OPT}}$, if the optimum is known and reached);
- $s_{\text{MIN}}$ denotes the simulation number occurring when $i=i_{\text{MIN}}$ ($k = i_{\text{MIN}}$, when the optimum is unknown);
- $S_{u_{\text{OPT}}}$ denotes the number of simulations, for which $C_g(n_s) < U$, occurring within an optimization run before the first occurrence of the optimum value ($k = i_{\text{OPT}}$, if the optimum is known and reached);
- $S_{u_{\text{MIN}}}$ denotes the number of simulations, for which $C_g(n_s) < U$, occurring within an optimization run before the first occurrence of the minimum value ($k = i_{\text{MIN}}$, when the optimum is unknown);
- $s_{500}$ denotes the simulation number occurring when $i=500$ ($k = 500$);
- $s_{1500}$ denotes the simulation number occurring when $i=1500$ ($k = 1500$);

When comparing the different optimization runs, according to the objective of minimizing computational time and maximizing the exploration of design space near the optimum, the algorithm performs better when lower values of $i_{\text{OPT}}$ are reached with higher values of $S_u$ and $s_u$.

**Variation of the PSO parameters**

For testing and optimizing the performance of PSO in solving the cost-optimal problem, different optimization processes were run with different combinations of algorithm parameters. The stopping criteria for all the optimization runs was set to $i=1500$.

As a first step, four runs were made for different number of particles ($5$, $10$, $20$, $40$) with $v_{\text{max}}$ fixed to $4$ and four combinations of values assigned to $c_1$ and $c_2$ so that $c_1+c_2=4$ ($c_1=1.5$ and $c_2=2.5$; $c_1=2$ and $c_2=2$; $c_1=2.5$ and $c_2=1.5$; $c_1=3$ and $c_2=1$).

Then, for each number of particles, the $c_1$ and $c_2$ values leading to the lowest value of $i_{\text{OPT}}$ are used for running the optimization process with $v_{\text{max}}=2$. As the last step, with $v_{\text{max}}$ fixed to 2, another optimization process was run proportionally reducing the $c_1$ and $c_2$ values so that $c_1+c_2=2$. 
**Results and Discussion**

**Reduced problem**

The different combinations of PSO settings used for carrying out different optimization process of the reduced problem are reported in Table 3 together with the resulting performance metrics.

**Table 3: Reduced problem - Optimization runs with different settings.**

<table>
<thead>
<tr>
<th>BP</th>
<th>c₁</th>
<th>c₂</th>
<th>vₘₐₓ</th>
<th>iₒₜₚ</th>
<th>sₒₜₚ</th>
<th>sₒₙ</th>
<th>Suₒₜₚ</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.5</td>
<td>2.5</td>
<td>4</td>
<td>161</td>
<td>89</td>
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Only the combination P5_2-2-4 (5 particles, cognitive and social accelerations equal to 2 and vₘₐₓ equal to 4) was not able to reach the optimum of the reduced problem. All the other runs ended up finding the optimal solution, but the performance metrics show many differences depending on the algorithm parameters. In fact, iₒₜₚ range between 96 and 595, with Suₒₜₚ ranging between 4, for iₒₜₚ=96, and 14, for iₒₜₚ=338 (note that only 29 points in the all design space have Cg values lower than U=523.37 €/m²).

Figure 6 and 7 report examples of optimization runs conducted by PSO algorithm in solving the cost-optimal problem, reporting the iteration number i on the x-axis and the global cost values on the y-axis.

**Figure 6: Reduced problem - Optimization processes for nP=5 and different values of c₁, c₂ and vₘₐₓ**

The lowest iₒₜₚ is reached with 5 particles and a cognitive acceleration c₁ higher than social acceleration c₂. Figure 6 reports the sequence of objective function evaluations occurring in 3 runs with 5 particles. If the combination P5_2.5,1.5,4 leads to faster occurrence of the optimum (lowest iₒₜₚ), but with limited exploration of the optimum neighborhood (low Suₒₜₚ), the combinations with vₘₐₓ=2 lead to larger exploration of the design space but require more objective function evaluations (and thus more time) for reaching the optimum, which is however reached in a reasonable computation time in all cases.

Figure 7 compares the sequences of objective function evaluations occurring in the best run for each number of particles. It is interesting to note that better performance is reached with c₂>c₁ for small number of particles, and for c₁>c₂ with high number of particles.

**Figure 7: Reduced problem - optimization processes for different values of nP, c₁, c₂ and vₘₐₓ.**
Table 4: Original problem - optimization runs with different settings -

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<th>C^\text{MIN}</th>
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Figure 8: Original problem - Comparison of optimization process with different algorithm parameters, with indication of the first occurrence of the minimum objective function value

Figure 9: Original problem - Part of design space as explored within the optimization processes

Figure 10: Original problem - Neighborhood of optimum (Su) as found in optimization processes
Original problem

The same combinations of settings that were analyzed for the reduced problem were applied to the original problem. As already mentioned, the global optimum of this problem is unknown and the different settings could lead the algorithm to converge to different minimum objective function values. Therefore, the different combination of algorithm settings were compared, first of all, based on the reached minimum objective function value \( C_{\text{MIN}} \) and then on the iteration number of its first occurrence \( b_{\text{MIN}} \). Then, given the same minimum \( C_{\text{MIN}} \) value, the other performance metrics \( (S_{ik} \text{ and } S_{i}) \) were considered in the analysis.

In Table 4, results related to the optimization runs of the original problem are reported. The same minimum global cost \( C_{\text{MIN}}=518.47 \text{ €/m}^2, \text{EP}=71.9 \text{ kWh/m}^2 \) was reached for 5, 10 and 20 particles, with different combinations of accelerations and velocity. With 40 particles, the algorithm converged on a solution that has a higher objective function value.

As reported in Table 4, the values of design variables associated to the minimum global cost denote a building that has 10 cm of wall insulation, 12 cm of roof insulation and 20 cm of slab insulation. The selected window types are WT3 (triple glazing, \( g=0.5 \)) for south orientation, and WT1 (double glazing, \( g=0.7 \)) for other orientations.

Figure 8 reports, similarly to what Figure 7 reports in for the reduced problem, the best run for each number of particles (in this case, the best run is selected according to lowest \( C_{\text{MIN}} \) values) for the original problem. The colors in Table 4 highlight the runs that are represented in Figure 8. As for the reduced problem, it is shown that a better performance is reached with \( c_2 \geq c_3 \) for small number of particles, while with \( c_1 \geq c_2 \) for high number of particles.

Figure 9 reports the same 4 runs of Figure 8 on a cost-optimal diagram. From Table 4, it is clear that the higher the number of design variables, the larger the exploration of design space \( (S_{1500} \text{ ranging from 709 to 1357}) \). However, in Figure 9 it is shown that the different runs lead to explore different points within the design space, moving towards concentrating in the cost-optimal region \( (C_{2} \leq U) \).

A zoom view of this region is shown in Figure 10, where the only points of Figure 9 that are below the U line are reported (U is calculated on the minimum found objective function value, that is \( U=518.47+2=520.47 \text{ €/m}^2 \)). It is interesting to note that the energy performance of these points, representing solutions that have very similar values of global cost, may vary from 62 kWh/m² to 78 kWh/m² (variation range of 21%), where \( \text{EP}=71.9 \text{ kWh/m}^2 \) is associated to \( C_{\text{MIN}}=518.47 \text{ kWh/m}^2 \).

Table 4 shows that the values assumed by some design variables (all the insulation variables and the south window type) may be very different. If focusing only on the search of \( C_{\text{MIN}} \) without extending the analysis to the neighborhood of the minimum, this aspect may be neglected, while a solution with a very similar global cost may have a significant reduction in the energy use (e.g. the point with a global cost 0.7 €/m² higher than and optimum but with a primary energy demand decreased by almost 10 kWh/m²) and may be preferred, leading to more performing building construction.

Conclusions

This paper demonstrated that, even in large cost-optimal problems, the PSO algorithm seems to perform better, in terms of time needed for reaching the minimum global cost, with a small number of particles (5-10), close to the number of involved design variables. Moreover, low numbers of particles perform better with cognitive acceleration higher than the social acceleration, while high numbers of particles perform better if the social acceleration is higher than the cognitive acceleration.

It was also demonstrated that the design space of the cost-optimal design problem is highly interrelated by the mutual relationships between the design variables and many solutions that are highly different in terms of energy performance may have very similar global cost values. This is the reason of the importance, when applying a simulation-based optimization method to the cost-optimal analysis, of focusing not only on the search of one minimum global cost value, but also on the neighborhood region, as mentioned in EU guidelines.

Since, for most complex and large building design optimization problems the determination of the global optimum by exploring the entire design space is far from being feasible from the point of view of the simulation time, research on the behavior of appropriate algorithms for such problems is of the foremost importance.

Moreover, if research has to have impact on the building industry, optimization methods have to be derived so that they that can be applied by professionals and are sufficiently reliable.

This is why this paper aimed at studying the reliability of the solutions obtained with the PSO algorithm, that is one of the most used in building optimization problems and why further research should be directed towards the creation of new algorithm versions that are tailored on the features of the cost-optimal problem.

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References


for the simultaneous optimization of energy demand and energy supply in buildings. *HVAC&R Research* 18(1-2) 67-87.


BPIE – Building Performance Institute Europe (2013). Implementing the cost-optimal methodology in EU countries


