EVALUATING CONTROL PERFORMANCE ON BUILDING HVAC CONTROLLERS

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ABSTRACT

Most controllers in Heating, Ventilation and Air-conditioning (HVAC) applications remain the Proportional-Integral (PI) type. Poorly performing control loops is a common issue, resulting in wasted energy, reduced occupant comfort, and excessive and unnecessary wear of actuators. In this paper, different methods are evaluated for assessing the controller performance in HVAC applications. Each assement method is evaluated statistically using the simulation data from Modelica based heating and cooling coil models, which denote the good and two bad controls. Two performance indices are selected based on evaluations using the simulation data. These indices are further evaluated using real data from building HVAC control loops, which inlcude the hot water supply temperature control, hot water loop differential pressure control, room (served by VAV box) air temperature control, Air Handling Unit (AHU) supply air pressure control.

INTRODUCTION

HVAC systems are used to control environmental variables such as temperature and humidity in the built environment. Although some intelligent controllers (e.g., fuzzy logic controllers, pattern recognition adaptive controllers, etc.) have been developed over the past two decades, the most commonly used controller in HVAC applications remains the Proportional-Integral (PI) type (Seem 1998, Zhao et al. 2013). Indeed, 95% of industrial controllers are of the Proportional-Integral-Derivative (PID) type even though most of loops are actually PI control (Åström and Häggland 1995). The PI/PID controller has proven to be simple to implement and sufficient for most HVAC applications. However, numerous studies show that, while effective in regulating the built environment, HVAC systems implementing these controllers with poor performance often result in inefficient energy use (Barwig et al. 2002). Poorly performing control loops are a common issue across various industries, resulting in wasted energy, reduced occupant comfort, and excessive and unnecessary wear of actuators. In a 2000 Honeywell report (Edgar 2007), the author lists the following performance assessment number for installed controllers based on surveys: Of the 64% of controllers that utilized closed-loop feedback, 25% were rated as having excellent performance, 23% as acceptable, 34% as fair, and 16% as poor.

Cost and performance are always the biggest concerns in the HVAC industry, and for engineers installing and commissioning a system, time spent tuning control loops can significantly add to the overall expense. Controllers are typically shipped with default tuning parameters that are determined through manufacturers' lab tests. Without retuning, those parameters could result in poor control performance since the actual HVAC systems will almost certainly have nonlinear and varying dynamics that are different from those existing at the manufacturers' test facilities (Federspiel and Seems 1996). Loads for a given HVAC system will often vary with time due to different seasonal or job-schedule loads (summer vs. winter and week-day vs. weekend, for example). To better optimize the performance, tuning parameters in the controller should be changed to accommodate such major process parameter variations. The tunning depends much on the performance metrics to decide if the current control loop is acceptabe.

In the process control industry, the first performance metric for a feedback control system was developed using Minimum Variance Control (MVC) which represents a method for comparing the actual performance of a controlled system against the best achievable performance (Harris 1989). The PI/PID controllers are compared with this controller (Underwood 1999). Due to large variations in the manipulated variable in MVC, field-implementation is not always practical. Other common performance metrics implemented in the process control industry are: IAE (Integral of the Absolute Error), ISE (Integral of the Square Error), ITAE (Integral of the Absolute Error Multiplied by Time) and ITSE(Integral of Time Multiplied by Squared Error). However, these have not been widely applied in the HVAC industry although some studies are available from both industry (Boysen 2013) and academia (Underwood 2009; Zhen 2006). Moreover, there is a need to define thresholds for these metrics, which can be used to set tolerances for an acceptable tuning.

This paper presents a systematic assessment of performance metrics for HVAC closed control loops. The focus is the realization of normal loop operation after a reasonable time to recover from a disturbance. Below is the outline of this paper.

- Methodology: Seven performance indices are reviewed with corresponding principles.
- Control Data Generation: Simulation data from Modelica-based models and real building HVAC control data are introduced.
- Results and Discussion: The assessments of control loops from simulation data are conducted and two performance indices are further selected and evaluated using data from real data of building HVAC control loops.
- Conclusions and further work

METHODOLOGY

Seven methods of assessment of HVAC control loop performance are listed in Table 1, followed by detailed explanations of these indices.

Table 1
Performance Indices

Indices	Description
IAE	Integral of the Absolute Error
ISE	Integral of the Square Error
ITAE	Integral of the Absolute Error Multiplied by Time
ITSE	Integral of Time Multiplied by Squared Error
EWMA	Exponentially Weighted Moving Average
Harris	Harris
VarBand	Variance Band

The performance indices are categorized into three groups. The IAE, ISE, ITAE, and ITSE are integral based performance indices. The EWMA of Error makes the control error smooth by using exponential weights. The Harris and VarBand indices is variance based performance indices.

1) IAE:

$$IAE = \int_0^t \left|\mathrm{y(t)} - r(\mathrm{t})\right| \mathrm{dt} \qquad (1)$$

where y is the control output and r is the set point. The IAE treats the magnitude of the control error. It is used for online controller assessment. Hägglund (1995) proposed an automatic approach based on IAE to determine if a control loop will oscillate with load disturbance occurring frequently.

2) ISE:

$$ISE = \int_0^t (\mathrm{y(t)} - r(\mathrm{t}))^2 \, \mathrm{dt} \qquad (2)$$

With ISE, errors with large values are weighted more heavily than the errors with smaller values. Large errors usually occur immediately following a disturbance and are seen in the form of overshoot or too sluggish of a response. The ISE is one of the most commonly used criteria to reduce overshoot.

3) ITAE:

$$IATE = \int_0^t \left[\left|\mathrm{y(t)} - \mathrm{r(t)}\right| \mathrm{t}\right] \mathrm{dt} \qquad (3)$$

ITAE heavily weights larger errors that occur late in time while less emphasis is placed on the initial error (usual large).

4) ITSE:

$$ITSE = \int_0^t t\left[y(\mathrm{t}) - \mathrm{r(t)}\right]^2 dt \qquad (4)$$

ITSE heavily weights larger squared errors that occur late in time while less emphasis is placed on the initial error.

5) EWMA:

$$\overline{e}_t = \overline{e}_{t-\delta} + \lambda(\mathrm{e}_t - \overline{e}_{t-\delta}) \qquad (5)$$

Where λ is the exponentially weighting factor. The value of λ is usually set between 0.2 and 0.3 (Hunter, 1986), although this choice is somewhat arbitrary. Lucas and Saccucci (1990) have shown that the smoothing factor λ used in an EWMA chart is usually recommended to be in the interval between 0.05 to 0.25. Thus, in this study, we select the value of λ to be 0.2. e_t is the process error which is defined as (Seem and House, 2000):

$$e_t = y(\mathrm{t}) - r(\mathrm{t}) \qquad (6)$$

Smaller weighting factor will puts more weights on the old data and less weight on new data. Seems and House (2009) used EWMA to assess Air Handler Unit (AHU) controller performance with the purpose of controller fault diagnosis.

6) Harris index:

$$H = 1 - \frac{\sigma_{mv}^2}{\sigma_y^2} \qquad (7)$$

where σ_{mv}^2 is the minimum variance of the control output obtained by maximum likelihood estimation method (Box and Jenkins, 1976). σ_y^2 is the variance of the control outputs with respect to the set point. It is given by:

$$\sigma_y^2 = \frac{\sum_{t=1}^{n} (y(\mathrm{t}) - r(\mathrm{t}))^2}{n} \qquad (8)$$

where r is the set point value.

Harris index has a range of [0, 1], with “0” indicating the worst performance and “1” indicating the best performance. It was proposed by Harris (1989) with an assumption that the minimum variance control is the best contorl that a given control loop can achieve.

7) VarBand:

$$V_b = 1 - e^{-\frac{\sigma_y^2}{\sigma_b^2}} \quad (9)$$

where σ_b^2 is the variance for the predefined error band. σ_y^2 is the variance of the control outputs with respect to the set point.

This index is based on the concept of comparisons of the allowed control error and actual output error. The exponential function is applied to normalize the index value to the range of [0, 1].

The predefined error band used in this case study, is listed in Table 2 based on domain knowledge and literature review.

Table 2

Error Band for VarBand

Control Variables	Error Band	Unit
Hot water supply temperature	-1 ~ 1	F
Hot water loop differential pressure	-0.1 ~ 0.1	PSI
VAV room temperature control	-1 ~ 1	F
Supply air static pressure control	-0.1 ~ 0.1	inH2O
Heating/Cooling coil outlet air temperature (Modelica-based simulation study)	-1 ~ 1	F

CONTROL DATA GENERATION

1) Simulation Data

In this study, there are three simulation data sets from control loops of heating coil and cooling coil respectively: good control, bad control I, and bad control II. Modelica-based models in Dymola are used to generate the data. The free open source LBNL Modelcia building library (Wetter and Zuo, 2014) is utilized for this case study due to its flexibility and capability for dynamic modelling of control components. The heating coil model is composed of a hot water loop, an air loop, a heat exchanger, a water valve, a motor, a temperature sensor, and a PI controller. In this study, the supply hot water temperature is 185 ºF and the return hot water temperature is 149 ºF. The source air temperature is 41ºF and the sink air temperature is 70ºF. The outlet air temperature is controlled at a setpoint of 64.4ºF with a PI controller. Table 3 lists the PI controller parameters for three cases of heating coil.

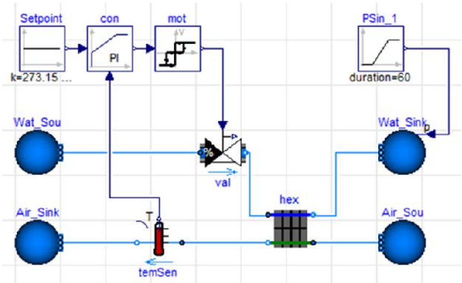

Figure 1 Diagram of Modelica Models for Heating Coil Model in Dymola

Table 3

PI Parameter of Heating Coil

Case	Good	Bad I	Bad II
P	1	1	0.02
I	100	1	250

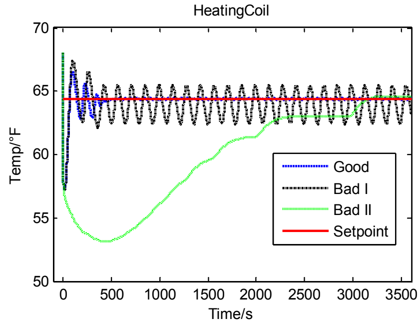

Figure 2 Heating Coil Simulation

Figure 2 shows three data sets of outlet air temperature generated from Modelica-based heating coil model: the blue dash line for the good control case, the black dot line for the bad control I, and the green dot line for the bad control II. The outlet air temperature control setpoint of 68ºF is indicated by the red line. By observations, both the good control case and bad control case I have a settling time of 200 seconds. The bad control I case is also oscillated drastically. The bad II control case is in good control states from t=2250s to t=3600s.

Similarly, the Modelica-based cooling coil model is composed of a cooling water loop, an air loop, a heat exchanger, a water valve, a motor, a temperature sensor, and a PI controller. In this study, for cooling coil, the supply chilled water temperature is 41ºF and return chilled water temperature is 68 ºF. The source air temperature is 86ºF and sink air temperature is 50 ºF. The outlet air temperature is controlled at a setpoint of 59 ºF with a PI controller. Table 4 lists the PI controller parameters for three cases of cooling coil.

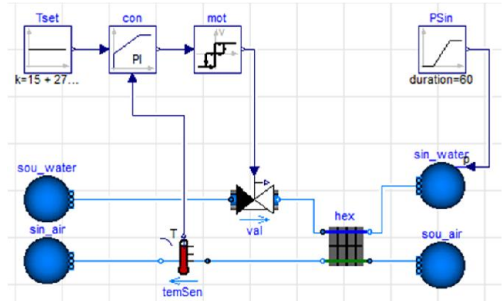

Figure 3 Diagram of Modelica Models for Cooling Coil Model in Dymola

Table 4

PI Parameter of Cooling Coil

Case	Good	Bad I	Bad II
P	1	0.05	0.2
I	100	1	5000

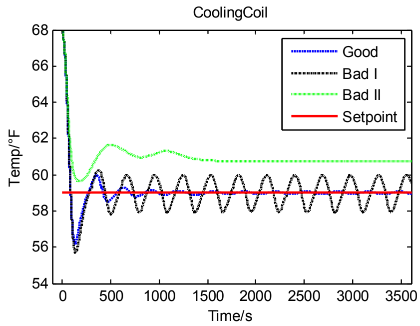

Figure 4 Cooling Coil Simulation

Figure 4 shows three data sets of outlet air temperature generated from Modelica-based cooling coil model: the blue dash line for the good control case, the black dot line for the bad control case 1, and the green dash line for the bad control case 2. The outlet air temperature control setpoint of 59°F is indicated by the red line. By observations, the good control case has a settling time of 450 seconds. And the bad control I case has a settling time of 500 seconds. The outputs from bad control II case are deviating away from the setpoint by about one Fahrenheit.

2) Real Data

Data from real HVAC control loop in one building were obtained for this case study. The sampling frequency for all the data is 5-minute. Only data from the period when HVAC equipment is on is being used. Figure 5 shows the real data for two days of four different control loops:

- Heat exchanger hot water supply temperature control with a three-way water valve
- Hot water loop differential pressure control with a variable speed pump
- Room temperature control with a hot water reheat 2-way valve (VAV box)
- AHU supply air static pressure control with a variable speed fan.

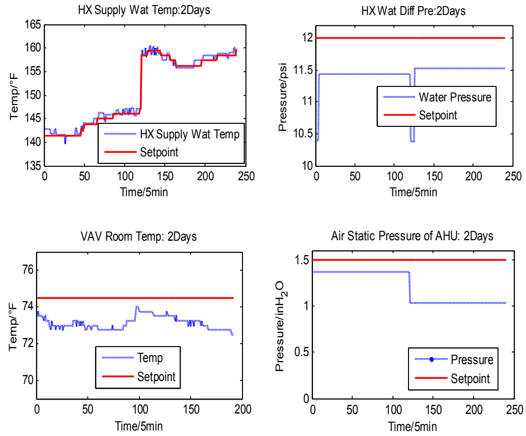

Figure 5 Control Loops from Real Building

RESULTS AND DISCUSSION

1) Control Loop Assessments for Simulation Data

The simulation data is evaluated by using seven performance indices described previously.

a. Heating Coil Control

The evaluation of heating coil control are divided into 3 groups: integral based indices (Figure 6), EWMA of Error (Figure 7), and variance based indices (Figure 8).

Subplots (a) to (d) in Figure 6 demonstrate the integral based indices. The integral based indices are denoted by red dot (good control), blue line (bad control I), and green dashed line (bad control II) respectively. The performance indices for the good control loops are almost constant at about zero. The indices for the bad control I loops are slightly oscillated. The indices for bad control II loops are varying greatly.

Subplot (a): The IAE index calculates the absolute error of the control outputs relative to the setpoint. The IAE index remains about zero for good control loops. This is because the errors between the control outputs and the setpoint included in the integration function, are close to zero, leading the integration values close to zero. The IAE index for the bad control I loop, has slightly oscillations but with a constant amplitude. This is because the control errors are slightly oscillating with a constant amplitude. The IAE index is synchronous with the control outputs in regarding to the oscillation. The IAE index for bad control II case remains almost a constant of 200 between the time t=2250s and t=3000s. After t=3000s, the IAE index decreases to zero with the control outputs error further decreasing.

Subplot (b): The ISE index calculates the square of the control errors, comparing to the IAE index for the errors. The square of the error is almost constant for the good control case, which leads a constant ISE index value. The ISE index for the bad control I case, has slightly oscillations. The ISE index for bad control

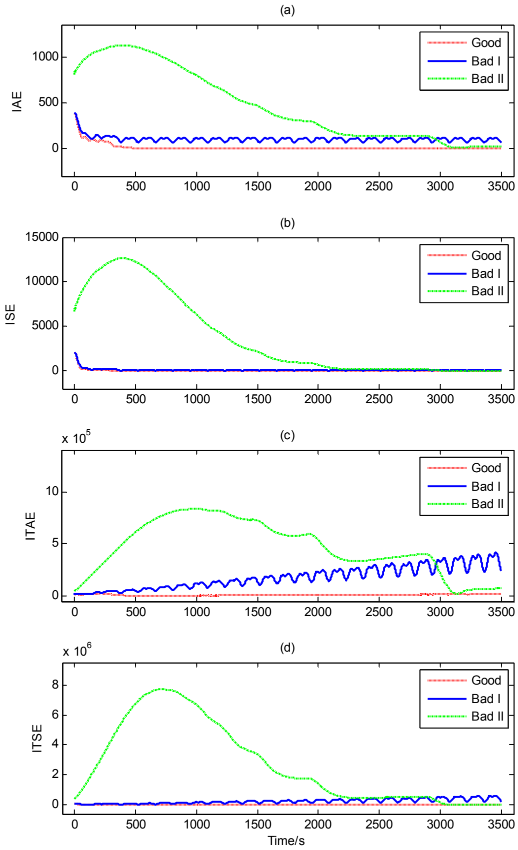

Figure 6 Integral Based Index for Heating Coil Control

II case is varying greatly between the time t=0s and t=2250s. After t=2250s, the ISE index decreases with the decreasing control outputs error.

Subplot (c): The ITAE index is based on the IAE index but with the time weighted in the formula. The ITAE index remains constant close to zero for the good control case. This is because the absolute error is almost constant close to zero. The ITAE index for the bad control I case, is oscillated with increased amplitudes. The oscillation is also synchronous with the control oscillations. Due to the time weight included in the ITAE index, the index values are increasing with time increasing which leads to the increased amplitude. The ITAE index for the bad control II case, is varying drastically. From the time t=0s to t=1000s, the increasing time weight leads to the increase of ITAE index. Between the time t=1000s and t=2250s, the decreasing error values lead to the decrease of the ITAE values. Between the time t=2250s and t=3000s, the control errors are almost constant, together with the increasing time weights, lead to the slowly increasing ITAE values. After the t=3000s, the control errors are almost constant about 0.2 Fahrenheit, together with increasing time weights, which leads to slowly increasing ITAE index values.

Subplot (d): The ITSE index, based on ISE index, is similar to the ITAE index but with the square of control errors and time included. For the good control case, the ITAE index is almost constant close to zero. This is because the control errors are almost zeros. For the bad control I case, the ITAE index is oscillating with increased amplitude. This is due to the oscillating control errors and increasing time weights. For bad control II case, the ITSE index f is similar to the ITAE index. The increasing control errors and time weights lead to the increasing ITSE index between the time t=0s and t=750s. The decreasing control errors and increasing time weights lead to decreasing ITSE indices between the time t=750s to t=2250s. After t=2250s, the smaller constant control errors lead to smaller ITSE index values.

Based on above analysis, we can further divide the integral based indices into two groups. The first group is IAE and ISE indices. The ISE index weights the square of the error while the IAE index only considers the absolute value of control errors. The second group is the ITAE and ITSE indices. Due to the time weight included in both the ITAE and ITSE indices, the initial value of ITAE and ITSE indices are zero. After the initial stage, the ITAE and ITSE indices are increasing.

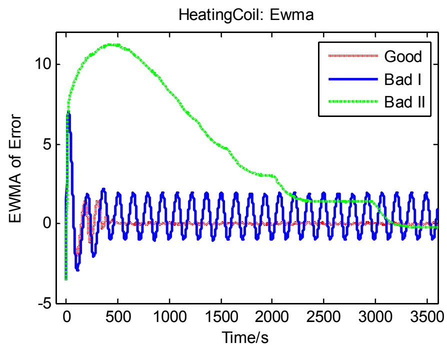

Figure 7 EWMA of Error Index for Heating Coil

The EWMA of error index is calculating the control errors along the control outputs but with weighted smoothing factor. In Figure 7, the EWMA of Error value for good control case is constant with the value close to zero. This is because the error is almost zero. The EWMA of Error for the bad control I case, is oscillating synchronous with the control outputs. This is because the control errors are oscillating. It is a good index for detecting the oscillated control loops. Between the time t=0s and t=2250s, the control errors are decreasing rapidly, lead to the decreasing EWMA of Error. Between the time t=2250s and t=3000s, the index reduces to almost constant with a value close to 1. After t=3000s, the index value is decreasing from 1 to 0, where the control outputs are close to the setpoint.

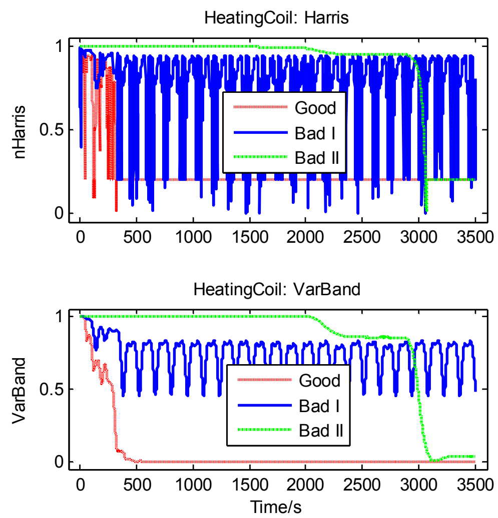

Figure 8 Variance Based Index for Heating Coil

As shown in Figure 8, the Harris index can identify the heating coil control loop performance for the three control scenarios. For the good control case, the Harris index is almost constant of 0.2 after the settling time. For the bad control I case, the Harris index is oscillating from zero to one, which corresponds to the control outputs oscillations. The Harris index is one for the bad control II case from t=0s to t=2000s. For the period from t=2000s to t=3000s, the Harris index is about 0.95. After the time t=3000s, the Harris index is about 0 where the control errors are close to zero. For the period from t=3100s to t=3600s, the Harris index is increasing to 0.2 when the control outputs are further deviating from the set point.

The VarBand index can also identify the heating coil control loop performance. For the good control case, the VarBand index is almost zero after the settling time. For the bad control I case, the VarBand index is oscillating with a lower bound of 0.5 and an upper bound of 0.8. The oscillation is synchronous with the control outputs. For the bad control II case, the VarBand index remains one from the time t=0s to t=2000s due to the bigger control errors comparing to the predefined error band (as shown in Table 2). From the time t=2000s to t=3000s, when the control errors start to become smaller, the VarBand index is about 0.85. After t=3000s, the VarBand index is decreasing rapidly to zero after the control output getting closer to the setpoint.

<u>b. Cooling Coil Control</u>

From Figure 9 to Figure 11, the control loop assessments using proposed indices are illustrated for three control scenarios of the cooling coil control.

Comparing the integral based indices between Figure 6 and Figure 9, we can see the assessment indices for cooling coil control are similar to those of heating coil control. The ITAE index and ITSE index are also showing similar behaviours.

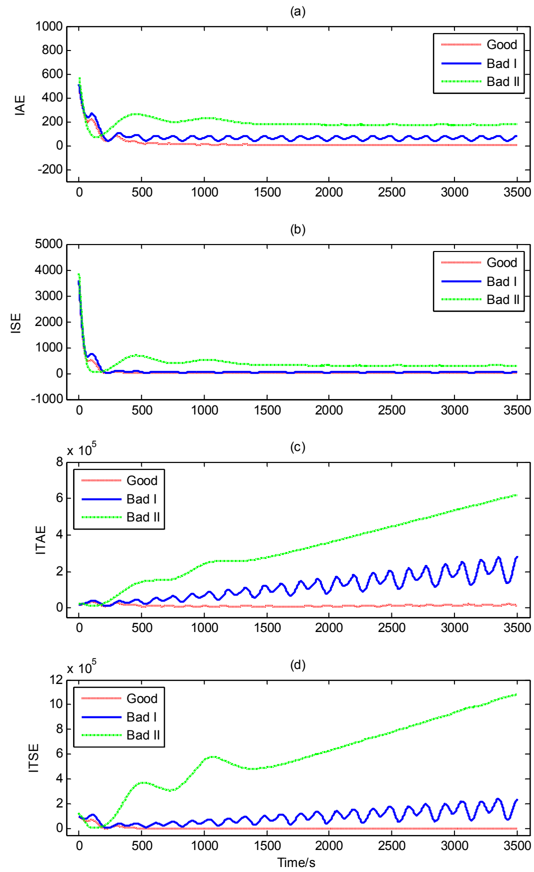

Figure 9 Integral Based Index for Cooling Coil

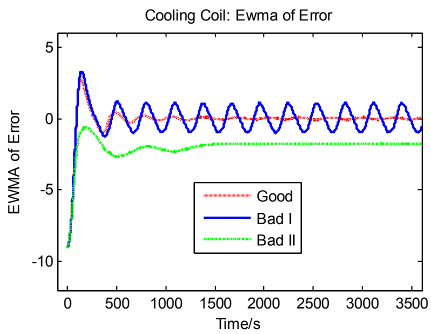

Figure 10 EWMA of Error Index for Cooling Coil

The EWMA of Error for cooling coil control is similar to that of heating coil control. The EWMA of Error is almost zero for the good control case. The EWMA of Error is oscillating for the bad control I case where the oscillations are existing. The EWMA of Error index is negative for the bad II control which is indicating the control outputs are below the setpoints. The cooling coil control case study also demonstrates that the EWMA of Error is a good choice for oscillation detection and identification of the setpoint tracking as well.

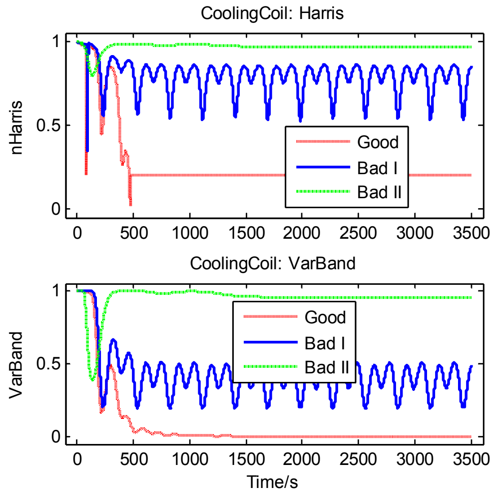

Figure 11 Variance Based Index for Cooling Coil

From the Harris index plot in Figure 11, we can see the Harris index can identify the cooling coil control loop performance as well. For the good control case, the outputs have a little bit of oscillation. The Harris index is around 0.2. For the bad control I case, the Harris index is oscillating from 0.55 to 0.9 indicating a bigger oscillation. For the bad control II case, the Harris index is constant with a value of 1 indicating the failure of the control.

The VarBand index can also identify the cooling coil control loop performance as shown in Figure 11. For the good control case, the VarBand index is about 0. For the bad control I case, the VarBand index is oscillating from 0.2 to 0.5 indicating the oscillation of the control loop. For the bad control II case, the VarBand index is around 1 with lower variations close to 0.95 after the settling time.

Using simulation data from Modelica-based models, we evaluated the seven indices for HVAC control loops. From above evaluations, the integral based indices can distinguish the control loop performance and indicate the oscillation of the control loops. Similarly, the EWMA of Error has the same potential. The integral based indices and the EWMA of Error are good choices for the setpoint tracking. However, no upper bound is the major issue of integral based indices and the EWMA of Error. Both the Harris index and VarBand index are able to identify the control loop performance. The merit of Harris index and VarBand index is they both have a lower bound and a upper bound in the range of [0, 1], which can be further classified into different scales, for example, excellent, good, fair, bad and failure. Thus, the simulation control data provides a strong support for the performance indices assessment. Evaluations are desirable for real control loops which are more complicated than simulation control loops. This is shown in the next part follows.

2)Control Loop Assessments for Real Data

From the case study using simulation data, we can see the Harris and VarBand index are better choices due to the explicit upper and lower bound, comparing to the integral based indices and the EWMA of Error. Thus, we further test the Harris and VarBand indices using the real control data from Figure 5.

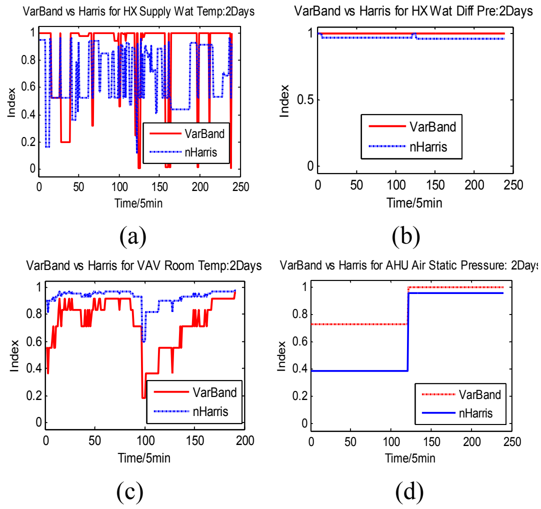

Figure 12 Assessment for Real Control Loops

From Figure 12 (a), we can see Harris (blue dash dot line) and VarBand (red solid line) index are almost synchronous for the assessment of hot water supply temperature control. The VarBand index values are one for bad control periods and close to 0.2 for good control periods. The Harris index is about 0.9 for bad control periods and 0.5 for good control periods.

Figure 12 (b) illustrates the assessment of the hot water differential pressure loop control. Both the VarBand index and the Harris index are about 1 around the 5th data point and the 125th data point. For the other periods, the Harris index is almost one based on the noise variance of 0.1 psi^2, and the VarBand index is about 0.9 based on the predefined error band of 0.1 psi.

Figure 12 (c) demonstrates the room air temperature control loop behaviours using Harris index and VarBand index. From the 0th to the 20th data point, the Harris index is increasing from 0.8 to 0.95, and the VarBand index is increasing from 0.4 to 0.9. Between the 20th data and the 100th data point, the Harris index remains 0.85 with variations from 0.8 to 0.9, and the VarBand index remains as 0.95. Between the 100th data point to the 200th data point, the Harris index increases from 0.6 to 0.95, and the VarBand index increases from 0.2 to 0.95.

Figure 12 (d) demonstrates the AHU air static pressure control loop behaviors using Harris index and VarBand index. Between the 0th to the 125th data point, the Harris index remains a constant of 0.4 and the VarBand index remains a constant of 0.75. From the 125th data point to the 200th data point, the Harris index remains as 0.95 and the VarBand index remains as 1.

The evaluations using data from real control loops show that the Harris index and VarBand index can both qualify the control loop performance.

CONCLUSIONS AND FUTURE WORK

Based on above evaluations, all the seven indices are able to identify the control loop performance to some extent. The integral based indices and the EWMA of Error are good choices for the setpoint tracking. However, they are not bounded. They are usually coupled with other indices for the control loop performance. Under such considerations, we determine two candidates with bounded values: Harris index and VarBand.

Nevertheless, further improvements are still worthy. The Harris index applied the approach of Box and Jenkins (1976), which are computational expensive. More computation efficient algorithms are still expected for Harris index calculation. The predefined error bands for VarBand are subjective, where there is a need for normalized error bands that are independent of control loops. Future research on the explicit performance scales (excellent, good, fair, bad, failure) are still needed for the two chosen indices. Another aspect worth consideration is the feasibility of the online implementation in Direct Digital Control (DDC) control for HVAC applications, which require the less memory and less complicated calculation. Finally, the assessment index will be useful for control loop tuning by characterizing the control loop performance.

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