

COMPARISON OF CONTROL OPTIMIZATION APPROACHES FOR LOW-EXERGY HEATING AND COOLING SYSTEMS

Dominik Wystrcil¹, Doreen Kalz¹

¹Fraunhofer Institute for Solar Energy Systems ISE, Freiburg, Germany

dominik.wystrcil@ise.fraunhofer.de, doreen.kalz@ise.fraunhofer.de

Tel. +49 761 4588-5125

ABSTRACT

The paper focuses on control optimization of non-linear system models where the differential-algebraic equations are formulated in Modelica. Two optimization approaches utilizing (i) direct search methods and (ii) derivative-based methods are compared. The approaches are applied to a thermo-hydraulic low-exergy heating and cooling system model in form of a borehole heat exchanger, a heat pump, a hydraulic distribution system, thermally activated building systems (TABS) and a simplified building model. The optimization approaches are compared concerning the found optimal solutions, the optimization performance and the requirements to the formulation of the non-linear optimal control problem.

INTRODUCTION

Over the past years, there is an increasing interest in design and control optimization of energy systems in buildings. For this, dynamic system models are used that often have a non-linear dynamic system behavior. This paper focuses on the performance evaluation of optimization methods for non-linear optimal control problems (NOCP). Many different approaches exist for the solution of non-linear optimization problems. Detailed reviews on optimization algorithms and their application to different types of system models are presented in Shaikh et al. 2014, Evins 2013, Nguyen et al. 2014, Machairas et al. 2014. Most of the applied optimization algorithms can be divided into two different concepts: (i) direct search methods and (ii) derivative-based methods.

Direct search methods and meta-heuristics

These methods do not require derivatives of the objective function or constraints. A simulation program numerically calculates the final value of the objective function that results from a given set of optimization variables. The optimization variables are iteratively adapted based on information from previous simulations.

Different types of algorithms exist for the iterative adaption of the optimization variables like e.g. pattern search, Nelder-Mead method, evolutionary algorithms, and particle swarm optimization. These

methods are implemented in many freely and commercially available software distributions as listed for example in Rios and Sahinidis 2013.

Derivative-based optimization

The principle of these methods is to start from a given initial point of the objective function and to go stepwise through the search space by determining a (descent) search direction and step size until a given termination criteria is satisfied. For the determination of the search direction, derivatives of the objective function and constraints like gradients and Hessians are used.

Many optimization algorithms for non-linear unconstrained and constrained optimization problems exist that differ in the choice of search direction and step size, e.g. Newton's method, sequential quadratic programming and interior points method. Software like Matlab, AMPL or IPOPT can be used.

Objectives

The aim of this paper is to present state of the art methods for solving non-linear optimal control problems shown on a hydronic space heating and cooling system that uses low-exergy environmental heat sources and sinks like e.g. surface-near geothermal energy. For the heat distribution thermally activated building systems (TABS) are used, with which low supply water temperatures in heating mode and high supply water temperatures in cooling mode can be achieved. This offers the opportunity to reduce the energy demand for the heat transformation in a heat pump or chiller. However, due to the low temperature gradients between supply water and room higher volume flow rates are necessary compared to e.g. high-temperature heat distribution systems like radiators that lead to an increased energy demand for the circulating pumps. Therefore, for these systems a thermo-hydraulic optimization of the applied control strategies is important. The goal is to achieve minimized total energy demand for heat pump and circulating pumps at which the thermal comfort requirements are still met. The formulation of the differential-algebraic equations (DAE) is realized in Modelica, an object-oriented, equation-based programming language for modelling complex physical systems (Modelica 2014).

In the reviewed publications in Shaikh et al. 2014, Evins 2013, Nguyen et al. 2014 and Machairas et al. 2014 mainly direct search methods are utilized. Direct search methods offer the advantage that they are easy applicable with existing simulation software like e.g. TRNSYS, EnergyPlus and Dymola. However, due to the large number of iterative simulations that are usually necessary these methods converge only slowly to the optimal solution. Therefore, the number of free optimization parameters is often limited. Usually, for control optimization (trajectory optimization) many optimization parameters have to be taken into account. Derivative-based methods offer the opportunity of a very fast convergence but have higher requirements to the system model and the objective function formulation. To the author's knowledge, no derivative-based non-linear control optimizations have been conducted for whole thermo-hydraulic low-exergy systems taking the environmental heat source/sink, transformation, hydraulic distribution and radiant slabs into account.

For the comparison of the optimization approaches, the following two methods are chosen:

- (i) Direct search methods: Particle Swarm Optimization (PSO) and Generalized Pattern Search (GPS). These methods are often used in recent publications and are seen to have a fast convergence speed within the group of direct search methods.
- (ii) Derivative-based methods: Interior Point Method (IPM). This method is well suited for non-linear optimization and an existing framework for the coupling to Modelica-models is implemented in OpenModelica (Ruge et al. 2014).

OPTIMIZATION (DIRECT SEARCH)

The freely available tool GenOpt is used for the optimization approach using direct search algorithms. The modelling and simulation of the system model is realized with Dymola, a simulation environment for Modelica. As a prerequisite, GenOpt needs an executable of the system model that is generated using Dymola. The executable can be used with different sets of optimization variables that are represented by a time series within a text-file. GenOpt can be seen as a supervisory program that uses the executable as "objective function evaluator" for a given set optimization variables. The final value of the objective function is written after the simulation to a text-file. GenOpt performs iteratively simulations and adaptations of the optimization variables in the text-file according to the information that are given from previous simulations and the changes of the final objective function values, respectively.

Different algorithms are available for the adaptation of the optimization variables. In this paper, the

implemented hybrid algorithm of GenOpt is used. Firstly, particle swarm optimization (PSO) is applied to identify a good initial guess as starting point. Secondly, generalized pattern search (GPS) is used with the found solution from PSO as starting point. The implemented algorithms are summarized in the following. More details can be found in the user manual (Wetter 2011).

Particle swarm optimization

Particle swarm optimization is a population-based method first proposed by Kennedy and Eberhart 1995. A set of potential solutions is defined in the search space, where each potential solution represents one particle. Initially, the particles are spread uniformly in the search space that is restricted to user-defined bounds like e.g. minimal and maximal values of the optimization variables. For each particle, the objective function value is determined at its current initial position. After one iteration (called generation), containing a user-defined number of particles, particle update equations are applied that update the velocity (direction vector in the search space) of each particle. The velocities of the particles are updated taking into account the particle's own objective function value from previous iterations and objective function values from the globally best particle.

After a user-defined number of generations, the globally best solution of all particles from the current and all previous generations represents the found optimal set of the optimization variables.

Different variations of PSO introducing inertia weights or constriction coefficients of the particles exist and are implemented in GenOpt. The used hybrid algorithm uses the PSO-variant with constriction coefficients. The following user-defined parameters influence the performance of the algorithm. The used parameter values in the simulation study are specified:

- number of particles = 128
- number of generations = 20
- cognitive acceleration = 2.8
- social acceleration = 1.3
- maximal velocity gain = 4
- constriction gain = 0.5

Generalized pattern search

Generalized pattern search algorithms are based on the definition of a mesh in the search space \mathbb{R}^n , where n represents the number of optimization variables. The mesh spacing is a user-defined parameter. Beginning from an initial guess representing a certain grid point, neighboring grid points are evaluated concerning their objective function values. Can the objective function be reduced, the exploration of neighboring grid points starts again from the updated grid point. If no reduction of the objective function value is yielded, the mesh spacing is reduced. Different rules and

implementations in GenOpt exist for the exploration of neighboring grid points. In the used hybrid algorithm, GPS according to Hooke and Jeeves 1961 is used. The following user-defined parameters influence the performance of the algorithm. The used parameter values in the simulation study are specified:

- mesh size divider = 2
- initial mesh size exponent = 0
- mesh size exponent increment = 1
- number of step reduction = 1

OPTIMIZATION (DERIVATIVE-BASED)

For the derivative-based optimization, a framework implemented in OpenModelica is used (Ruge et al. 2014). In the framework, first the OCP is discretized using a direct collocation scheme to transform the infinite dimensional optimization problem to a finite dimensional problem. The solution of the resulting (discretized) optimal control problem is then realized using the free software IPOPT (COIN-OR 2015). Within IPOPT the interior point method is applied.

Direct collocation

The transformation of the OCP to a finite-dimensional problem is realized by a discretization of the control trajectory on the one hand and the dynamic system equations on the other hand. For this, an equidistant time grid is introduced. The control trajectory is approximated by functions that depend on a finite number of control parameters. For example polynomials of degree 0 can be used resulting in piecewise constant control parameters on each discretization interval. In this paper, 3 free control parameters are chosen in one discretization interval, one on each time grid interval of the collocation scheme and additionally on two intermediate steps. Then, a linear interpolation between consecutive grid points is used.

Additionally, a finite number of equality constraints replace the dynamic system model. Therefore, intermediate grid points in between the collocation discretization scheme are introduced. Polynomials are used to approximate the system model equations on the pre-defined time grid. By this, no integration of the dynamic system model by an external solver is necessary. Instead, the discretized finite number of equality constraints are taken into account by the used optimization solver. The discretization of the system equations can be seen as the use of an implicit Runge-Kutta integration with a pre-defined step size. The direct collocation leads to the following (discretized) optimal control problem:

$$\min_{\hat{x}, \hat{u}} \int(\hat{x}, \hat{u}) = E(\hat{x}(t_f), t_f) + \sum_{k=0}^{N-1} l^k(\hat{x}^k, \hat{x}^{k+1}, \psi^k(\hat{u})) \quad (1)$$

$$\text{s.t. } c_k(\hat{x}^k, \hat{x}^{k+1}, \psi^k) = 0, \quad (2)$$

$$\hat{x}(t^0) = x_0 \quad (3)$$

$$h(\hat{x}^k, \psi^k(\hat{u})) \leq 0 \quad (4)$$

Where \hat{x} and \hat{u} are the discretized controls and states. $\psi(\hat{u})$ represents the function dependent on the discretized controls. Equation (2) represents the discretized system equations, equation (3) the initial conditions and equation (4) the inequality constraints.

Interior point method

The NOCP in equation (1)-(4) can be solved using the interior point method that is described shortly in the following. Detailed information can be found in the user documentation (COIN-OR 2015).

Preliminary, all inequality constraints in equation (4) are replaced by equality constraints and newly introduced slack variables s so that the remaining inequalities have the form $s, x, u \geq 0$. After this, the original NOCP is reformulated to a dual NOCP where barrier functions are introduced that penalize s, x and u that are close to 0, i.e. the initial point for solving the NOCP is forced to be (deep) inside the feasible set of solutions. The dual NOCP is then solved using Newton-type algorithms until certain termination criteria are fulfilled. For this, derivatives of the objective function and the constraints are obtained by algorithmic differentiation from the free software ADOL-C. Once a solution is found the penalization of s, x and u that are close to 0 is reduced. It can be shown that the solution of the dual NOCP converges to the solution of the original NOCP for sufficiently small barrier function penalizations.

MOVING HORIZON OPTIMIZATION

The goal is the optimization of longer periods like an entire month, heating and cooling seasons or years. The resulting NOCP would have a large number of optimization variables that could possibly not be solved within reasonable time scales. Therefore, for both optimization methods, direct search methods as well as derivative-based methods, moving horizon optimizations are performed. The methodology was previously applied e.g. in Corbin et al. 2013. Figure 1 shows schematically the approach. The whole optimization period is discretized into an equidistant time grid of e.g. days. Then the individual NOCPs are solved sequentially and the found solutions are joined to a global solution for the whole optimization period. The global solution can be seen as an approximation of the solution that could be found by solving the NOCP for the whole optimization period at once.

The underlying assumption for the success of this approach is that made control actions at a certain time lose impact on the dynamic system behavior after a certain period. In the system that is investigated here thermally activated building systems (TABS) are used for the heat distribution that typically have time constants of about 10 to 20

hours. Therefore, in the moving horizon approach used in the simulation study, optimizations are conducted with a planning horizon of 48 hours, but only the found optimal control inputs of the first 24 hours are applied on the system model for validation of the results (execution horizon).

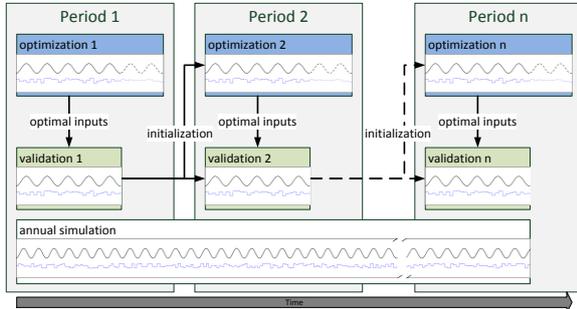


Figure 1 Scheme of the moving horizon optimization approach. Room temperatures shown schematically as black lines and control inputs as blue lines.

For the validation of the moving horizon approach, the system model is optimized with the described derivative-based optimization method. One solution is obtained by applying the moving horizon optimization for a period of 30 days by sequential optimizations of one day, the other by solving the total optimal control problem at once for 30 days. The objective function represents the minimization of the energy consumption, whereby comfort violations with room temperatures below 21 °C are penalized with a quadratic term.

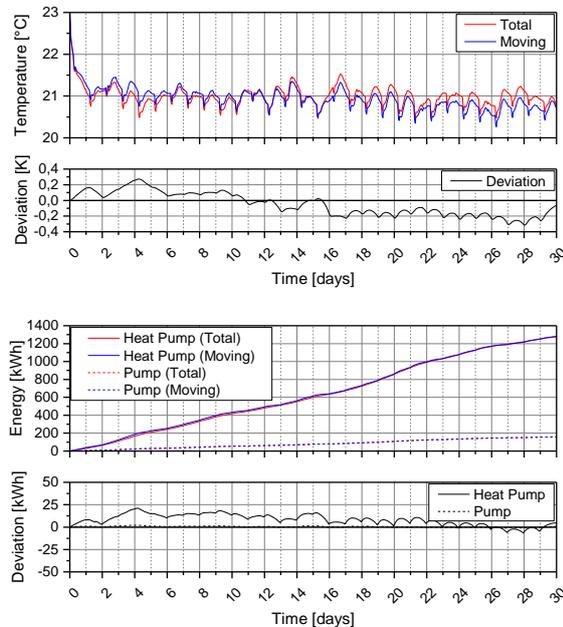


Figure 2 Experimental validation of the moving horizon approach with the investigated system model

Figure 2 shows that both found optimal solutions lead to almost the same trajectories for the room temperatures and the consumed energy of the heat

pump compressor and the circulating pump. The deviation of the room temperatures are in the range of ± 0.2 K. The deviation of the total consumed energy for the heat pump and the circulating pump after the period of 30 days is below 5 kWh_{el}.

MODELING AND SIMULATION

The optimization approaches are applied to an thermo-hydraulic system model of a low-exergy heating and cooling systems as shown in Figure 3 that is implemented in Modelica. The model represents a whole energy plant consisting of the environmental heat source, heat generation and distribution systems as well as a simplified grey-box building model. In the following model details, the definition and implementation of the optimal control problem and the boundary conditions for the simulation case study are described.

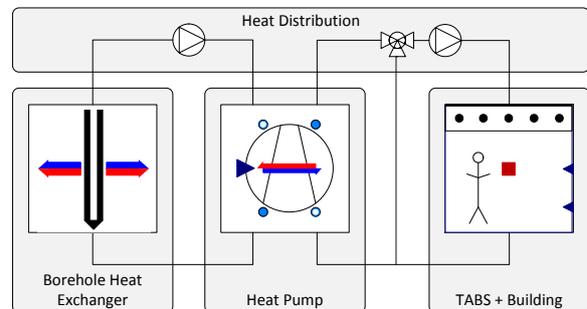


Figure 3 Scheme of the thermo-hydraulic low-exergy system model

Dynamic system model

As environmental heat source, an borehole heat exchanger is modelled. The model is based on a finite element approach using the borehole resistance from a thermal response test from an existing installation. The heat transfer between fluid and borehole wall is quantified by the given borehole resistance and the temperature difference between fluid and borehole wall. The surrounding ground is discretized into five cylindrical layers each containing a thermal capacity. The heat transfer between the layers is modelled by using form factors for cylindrical heat conduction in the ground.

The heat pump model is based on the Carnot efficiency according to equation (5) for an ideal process for heat transformation from a heat source at certain temperature T_{src} to a heat sink at a certain higher temperature T_{snk} in Kelvin. The efficiency factor η_{eff} is assumed as a quadratic function dependent on the temperature difference between sink and source and is fitted to measurement data from Pärish et al. 2014. By this, the power consumption of the heat pump compressor results from equation (6).

$$\eta_c = 1 - \frac{T_{src}}{T_{snk}} \quad (5)$$

$$P_{el} = u \cdot \eta_c \frac{\dot{Q}}{\eta_{eff} \cdot \eta_{pl}} \quad (6)$$

A speed controlled heat pump is assumed with the input value u and a part load efficiency factor η_{pl} .

For the modelling of the circulating pumps, affinity laws for the pump speed, the volume flow rate, the pressure head and the energy consumption according to equations (7)-(9) are used. Characteristic curves for the pressure head and the energy consumption dependent on the volume flow rate are obtained by linear fits to measurement data from manufacturer data sheets.

$$\frac{\dot{V}_1}{\dot{V}_{nom}} = \frac{n_1}{n_{nom}} \quad (7)$$

$$\frac{\Delta p_1}{\Delta p_{nom}} = \left(\frac{n_1}{n_{nom}} \right)^2 \quad (8)$$

$$\frac{P_1}{P_{nom}} = \left(\frac{n_1}{n_{nom}} \right)^3 \quad (9)$$

A simplified hydraulic resistance in the hydronic circuit is taken into account by a quadratic function for the pressure drop dependent on the volume flow rate. The quadratic function is parametrized according to a given set of volume flow rate and pressure drop under nominal conditions.

The model for the thermally activated building system (TABS) is based on Weber and Jóhannesson 2005. Herein, a simplified RC-network is presented that reduces the 3-dimensional heat transfer from the fluid to the room to a 1-dimensional heat transfer. Therefore, a heat transfer between the fluid and a fictitious layer temperature is introduced and then a heat conduction through a flat wall between the fictitious layer and the TABS surfaces is assumed.

For the room model, a grey-box approach based on a simplified RC-network proposed by the international norm ISO 13790 is used, where two thermal capacities for the room's thermal mass and the room's air are introduced. Five thermal resistances between the two nodes are used and parametrized according to the norm. The coupled room and TABS model is validated with measurement data from an existing building (Wystrcil and Kalz 2014).

The whole system model contains 19 differential states and 220 time-varying variables. The implementations of the component models are all twice continuously differentiable and compatible to the compiler of OpenModelica.

Optimal control problem

The following objective function is used for the optimizations:

$$J(u) = w_E \cdot \left(E_{el_{hp}}(t_f)^2 + E_{el_{pump}}(t_f)^2 \right) + w_\theta \cdot \int \max(\vartheta_{set} - \vartheta_{room}, 0)^2 dt \quad (10)$$

where w_E and w_θ are weighting factors for the multi-objective optimization, E_{el} the energy consumption of the heat pump (hp) and the circulating pump at the final times t_f and ϑ_{set} and ϑ_{room} the setpoint and room temperatures. Furthermore, upper and lower bounds for the input parameters u are defined as for example the minimum and maximum speed the heat pump compressor and pump.

Using the optimization approach with direct search methods, the objective function is implemented within the Modelica system model. The final value of the objective function is exported via a text-file at the end of a simulation. In the optimization approach using OpenModelica the NOCP is formulated in Optimica, an language extension to Modelica, where one can define Mayer- and Langrange-terms in the objective function by using variables from the system model. Furthermore, constraints for the system states and controls can be set.

Simulation case study

In the simulation case study, optimizations for a period of 30 days in January are conducted. Ambient conditions like ambient temperatures and solar irradiation are taken from Test Reference Years from the German Meteorological Service for the region of Essen, Germany. Internal gains are assumed according to the German norm DIN V 18599-10 for an office building.

The free control inputs in the simulation study are the normalized speed of the heat pump compressor and the circulating pump. The discretization of the control inputs is in the OpenModelica framework implemented by the direct collocation scheme where the number of intervals is user-defined. In the optimization approach using GenOpt the control discretization is realized by creating text-files with the control inputs that contain values for the control inputs at certain time steps. Within a simulation in Dymola a linear interpolation is conducted between given time steps. In the simulation study, the number of free control inputs is set to 13 for the planning horizon of 48 hours, and the first 7 are applied to the execution horizon of 24 hours.

RESULTS AND DISCUSSION

In the following optimizations for the described non-linear optimal control problem are conducted using the two optimization approaches. The found optimal solutions, the solution times needed and the

restrictions to the NOCP are compared and discussed.

Both optimization approaches have in common that there is no analytic proof that a global optimum is found if a non-convex optimization problem is present. Presumably, particle swarm optimization could be advantageous to find a global optimum due to a more homogeneous covering of the search space by using many (initial) particles.

Found optimal solutions

Firstly, the objective function values for the achieved thermal comfort and the energy consumption for the heat pump compressor and the circulating pumps are compared. Figure 4 shows the cumulative probabilities for the room temperatures during the analyzed optimization period. It can be seen that the resulting room temperatures are very similar. In both solutions only 5 % of the hourly room temperatures fall below 20.5 °C. The occurring room temperatures below the defined set point of 21 °C can be explained by the chosen weight factors $w_E = w_\theta = 0.5$ in equation (10). Room temperatures below the set point result in a quadratic penalty that is lower for small deviations for example compared to a linear penalization but result in lower energy consumption.

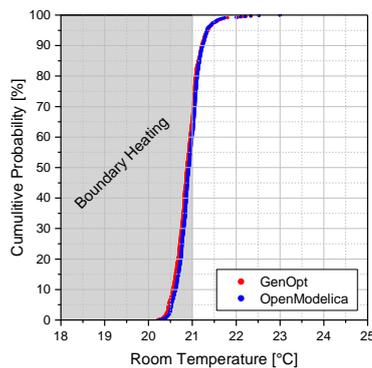


Figure 4 Comparison of cumulative probability of room temperatures using GenOpt and OpenModelica

The energy consumption for the optimal solutions is shown in Figure 5. The deviations of both optimization approaches are also small during the

whole optimization period. The final total energy consumption of the heat pump compressor after the whole period of 30 days is 1280 kWh using GenOpt and 1271 kWh using OpenModelica, i.e. a relative deviation of about 0.7 %. The comparison of energy consumption for the pumps shows 150 kWh using GenOpt and 158 kWh using OpenModelica that means a deviation of about 5 %.

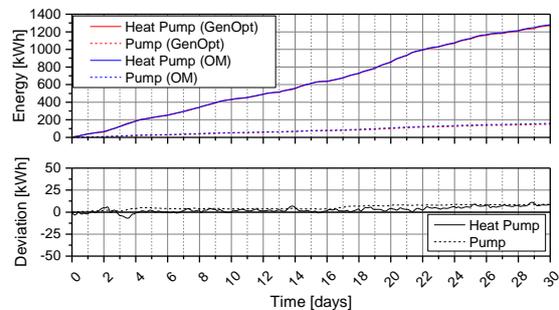


Figure 5 Comparison of energy consumption using GenOpt and OpenModelica

Furthermore, the identified optimal control inputs are compared. In Figure 6 the time series of the mass flow rates and the resulting supply water temperatures for the whole optimization period are shown. Similar progressions can be seen with higher fluctuations of the found optimal solution using GenOpt. The deviations are mostly in the range of ± 0.7 kg/s and ± 0.5 K, respectively. For both, the mass flow rate and the supply water temperature, the found optimal solution using OpenModelica is smoother.

For a direct comparison, the daily mean values for the optimal mass flow rates and supply water temperatures identified by the two optimization approaches are shown in Figure 7. A perfect match of both found optimal solutions would result in a linear relation leading to points on the angle bisector in the diagrams. The daily mean mass flow rates as well as the supply water temperatures show a good correlation with Pearson's correlation coefficients of 92 % for the mass flow rates and 99 % for the supply water temperatures.

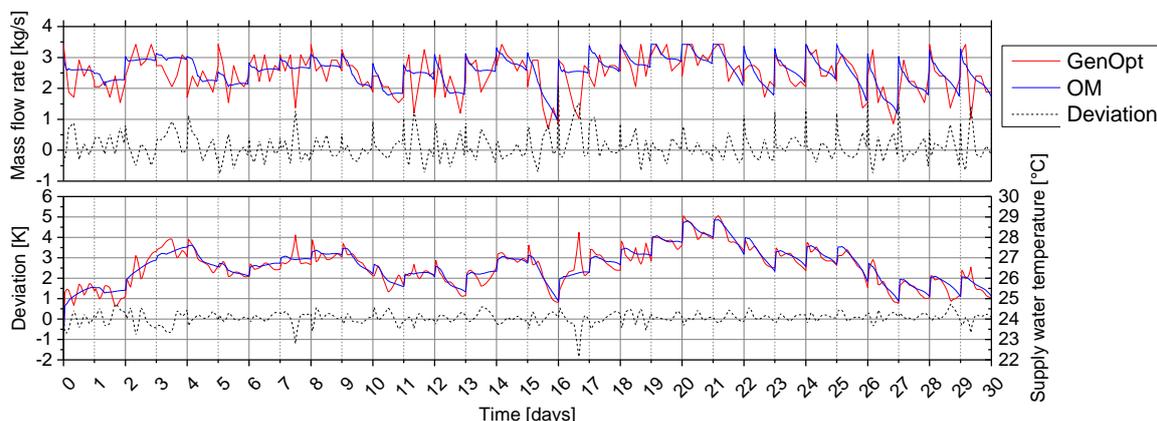


Figure 6 Identified optimal control inputs using GenOpt and OpenModelica as time series.

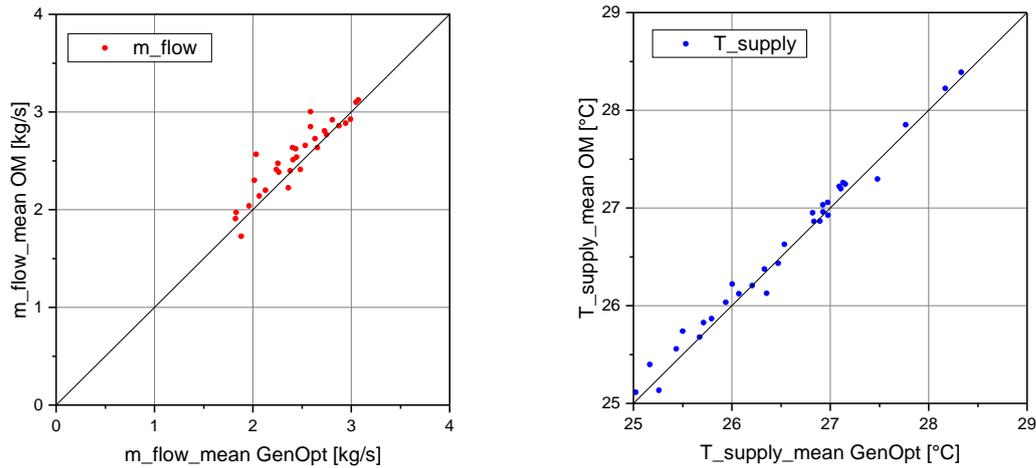


Figure 7 Comparison of the daily mean values of the identified optimal mass flow rates and supply water temperatures from GenOpt and OpenModelica

Solution times

In this section the times needed to numerically solve the NOCP are compared. The optimizations are conducted on a PC with an Intel® Core™ i7 quad-core CPU with 3.4 GHz each. Figure 8 shows the solution times using GenOpt and OpenModelica. GenOpt needs approximately 12,029 s and OpenModelica 37 s, i.e. the solution using the derivative-based optimization approach within the OpenModelica framework is about 325-times faster than using the direct search methods particle swarm optimization and generalized pattern search in GenOpt.

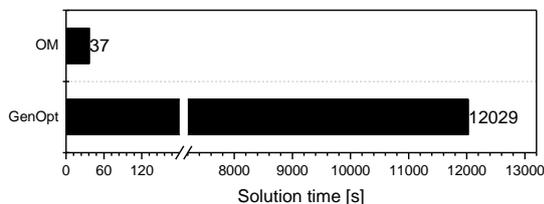


Figure 8 Comparison of the solution times needed using GenOpt and OpenModelica

In the following the number of simulations and function evaluations, respectively, for the solution of the NOCP are compared. The optimization approach using direct search methods results in approximately 2,500 simulation executions per planning horizon of which about 1,800 are conducted by the PSO using parallelization and about 700 by the GPS.

In comparison, the optimization framework in OpenModelica using IPOPT for the solution of the discretized NOCP needs approximately 13 iterations in each planning horizon to converge to an optimal solution resulting in about the same number evaluations of the objective function, the gradient of the objective function and Hessians of the Lagrangian function.

Despite the much better performance of the derivative-based approach in this simulation study, the convergence speed of the direct search methods can be further improved by optimizing the algorithm parameters of the PSO and GPS that are listed above. The higher performance of the derivative-based approach can be used to solve bigger NOCPs e.g. for using longer planning and executions horizons in the moving horizon approach.

Requirements to the optimal control problem formulation

For using the described derivative-based optimization approaches, some requirements to the system model, the objective function and the constraints must be met. Firstly, the model equations, the objective function and the constraints must be twice continuously differentiable with regard to the optimization variables. Secondly, the system models have to be compatible to the compiler of OpenModelica. Until now, the OpenModelica compiler does not support every available Modelica implementation. Thirdly, sufficiently small discretization intervals must be used in the direct collocation scheme to obtain a good approximation of the optimal solution. Due to the substitution of the system model equations by discretized equality constraints in the direct collocation scheme, the found solution might not be satisfactory when using other solvers for simulation/validation using step size control (e.g. Dassl).

Advantages of direct search methods are that, firstly, any model that is suitable for simulation can be used for optimization as well. Therefore, existing model libraries that were built for simulation purposes can be utilized. Secondly, discontinuous system models and/or discrete optimization variables can be treated using direct search methods. Thirdly, the implementation effort for performing optimizations is relatively small and can be realized with any

simulation environment that uses text-files for the import of parameters and the export of results.

CONCLUSION

In the paper, the demonstration and comparison of two optimization approaches for non-linear optimal control problems was presented. A low-exergy heating and cooling system model was used for the comparison of the optimization approaches. The formulation of the differential-algebraic equations is realized in the modelling language Modelica. The first optimization approach utilizes derivative-free direct search methods, namely particle swarm optimization and generalized pattern search, implemented in the freely available tool GenOpt. In the second optimization approach, derivative-based optimizations are performed using derivative information from the algorithmic differentiation tool ADOL-C. A tool chain implemented within the OpenModelica framework is applied. It has been shown that both optimization approaches lead to comparable optimal solutions concerning the identified optimal control inputs as well as achieved room temperatures and energy consumption. The derivative-based optimization approach resulted in 325-times faster convergence speed compared to the approach using the direct search methods. Nevertheless, the direct search methods could offer advantages concerning the use of existing model libraries that are built for simulation purposes. Furthermore, the objective function and the system model do not have to be continuously differentiable and the approach can also be applied to optimal control problems with discrete optimization variables.

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