PREDICTIVE MODEL FOR HVAC CONTROL
KEY FACTORS AND GLOBAL SENSITIVITY ANALYSIS

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ABSTRACT

In previous studies, the authors combined load shifting and Model-based Predictive Control (MPC) for the optimization of a HVAC system. The precision of the predicted model outputs (primary energy, comfort in temperature and peak heat flow rate) depends on the accuracy of several factors (model parameters, inputs and initial state). Here, in order to allocate properly the development effort (parameter identification, forecast strategy, state measurement/estimation algorithm), the authors aim at ranking these factors by using global sensitivity analysis, precisely the elementary effect method. The simulation results summarize very satisfactorily the direct and indirect effects of around 30 model factors.

INTRODUCTION

The study is concerned with a 14-storey office building of the company Electrabel-ENGIE, which requires comfort in terms of temperature and humidity during the working hours only. The Heating, Ventilation and Air Conditioning (HVAC) system is of the forced-air type with energy recovery; heat and cold are produced by standard energy machines (gas boilers, cooling machines) but also by geothermal heat pumps.

In previous works, (Lepore et al., 2013) and (Lepore et al., 2014), the authors have developed in Matlab a load shifting strategy coupled with Model-based Predictive Control (MPC), which consists essentially in pre-heating or pre-cooling the building during the night and week-end. The strategy aims at decreasing the primary energy (alternatively, the billing cost) while maintaining a reasonable comfort level (here in temperature only). The algorithm has been evaluated in simulation and very important and promising results have been obtained.

However, the performance of the MPC algorithm depends heavily on the precision of the predictive model outputs, especially when the sampling time (1 day) is not small in regards of the predictive time horizon (7 days). The outputs of interest are the primary energy (or the billing cost) and some comfort index, both evaluated over the predictive time horizon. The precision of these outputs is subject to several sources of inaccuracy: in the parameters determined at the identification stage, but also in the inputs (weather forecast, occupancy schedules, etc.) and in the initial conditions (measured/estimated node temperatures) at run time. In order to allocate properly the development effort among parameter identification, input forecast and state estimation algorithm, it appears very important to rank all these factors (parameters, inputs and initial conditions) with respect to their effect on the selected model outputs.

Sensitivity analysis consists in a large set of techniques allowing to quantify these effects. From the early studies which focused more on *local sensitivity analysis*, i.e., around a nominal point (Lomas and Eppel, 1992), several methods that are based on statistics (elementary effect, variance-based, Monte-Carlo, etc...) have emerged. They are commonly designated as *global sensitivity analysis* and are applied in a variety of scientific subjects (Saltelli et al., 2008), among them the building energy analysis (see (Tian, 2013) for a complete review).

In this study, despite the fact that the model is run around the nominal conditions, a local sensitivity analysis reveals not to be satisfactory due to factor coupling (especially between some parameters and inputs) and to model non-linearity, which is illustrated in this article in a didactic case.

The choice of a global sensitivity analysis technique depends on the size of the problem (model complexity, number of factors) and on the desired information (factor ranking, accurate variance analysis). Usually, a compromise is made between information accuracy and computation time. When factor ranking with no refined quantification of the indirect effects (coupling, non-linearity) is the objective, as is here for the time being, the *(first-order) elementary effect method* deserves real interest (Morris, 1991; Saltelli et al., 2008), which is also illustrated in the didactic case. In order to apply the method to the predictive model (with dozens of factors), some specific algorithms (for experiment design and calculation of the sensitivity indicators) exist, which have been initially developed by Morris, later enhanced by other authors (Campolongo et al., 2007). These algorithms are Matlab functions obtained from the Simlab software web site (Morris and Campolongo, 2007).

The authors have investigated the sensitivity of four outputs with respect to the model factors, all outputs being evaluated during a winter week. These outputs are the real energy demand, the comfort, the peak heat flow rate and the primary energy; the sensitivity analysis is performed in two separate steps:

- the first three outputs are concerned with the building model only, 16 factors are considered;
- the primary energy is concerned with the whole

model, 27 factors are considered.

Section *Problem statement* first describes the objective of the study and the model, then illustrates the interest of a global sensitivity analysis. Section *Elementary effect method* presents Morris' method in the didactic case. Section *Results* is devoted to the experimental set-up, to the results obtained and to the discussion pertaining to them. Section *Conclusion* contains the conclusion and the future outlook.

PROBLEM STATEMENT

Predictive model objectives and model accuracy

In the framework of the Model-based Predictive Control (MPC) designed in (Lepore et al., 2014), which is based on dynamic optimization, the model is used at regular sampling times (every day) to evaluate three specific output variables of interest over the predictive time horizon (one week): the energy cost (primary energy, billing cost), the comfort index (percentage of comfortable working hours) and the peak value of the heat/cold flow rate. The energy cost has to be minimized whereas the comfort index and the peak value have to be kept within specified limits (constraints).

Even assuming that the structure selected for the model is reliable, the values of these output variables are imprecise due to inaccuracies not only in the physical parameters (inherent to identification) but also in the inputs (inherent to forecast) and in the initial conditions (inherent to state measurement/estimation). Despite the authors' wish to keep the model simple, the total number of all these *factors* can be up to 40. In order to distribute efficiently the development effort (parameter identification, input forecast and state estimation algorithm), it is relevant to rank the factors in regards of their effect on the output variables. Often this ranking approach leads to consider less than a dozen of important factors while the rest of them are given fixed values (*factor fixing*).

Model description

The system under study is composed of the building itself and of the energy distribution/production systems. The predictive model describes the system at the two levels:

- at the building level: a dynamic, network Resistor-Capacitor (R-C) model with two nodes describes the evolution of the indoor air and internal wall/concrete temperatures when subject to inputs such as the outdoor temperature, the internal and solar gains, etc.; additionally, a Proportional Integral (PI) controller maintains the indoor air temperature at some set-point by adjusting the heat/cold flow rate delivered to the building through the forced-air ventilation system;
- at the energy distribution/production level: a static, however non-trivial model describes how the required heat/cold flow rate is distributed between the heat pump and the standard machines (gas boiler/cooling machine).

Building model

Two differential equations describe the evolution of t_i and t_c, the indoor air and internal wall/concrete node temperatures, respectively (in $^\circ$C);

$$\begin{aligned} C_i \frac{dt_i}{d\tau} &= \left((UA)_{tr} + (UA)_{inf}\right)\left(t_o(\tau) - t_i\right) \\ &\quad + (UA)_{gr}\left(t_{gr} - t_i\right) + (UA)_{ci}\left(t_c - t_i\right) \\ &\quad + \Phi_{vent}(\tau) + \Phi_{ig}(\tau) + \Phi_{hvac}(\tau) \qquad (1) \\ C_c \frac{dt_c}{d\tau} &= (UA)_{ci}\left(t_i - t_c\right) + \Phi_{sg}(\tau) \qquad (2) \end{aligned}$$

where

- $C_i = V_i \rho_{air} c_{air}$ and $C_c = V_c \rho_{con} c_{con}$ are the total indoor and concrete thermal capacities, respectively (in $J.K^{-1}$) (V_i and V_c are the respective volumes, ρ_{air} and ρ_{con} are the respective densities, c_{air} and c_{con} are the respective specific heats); $V_c = (nf - 1).A_{gr}.\delta_c$, nf being the number of floors, A_{if} and δ_c being the surface area and the thickness of the inter-floor dividing wall, respectively;
- $(UA)_{tr} = \sum_i (U_i A_i)$ is the thermal conductance corresponding to outdoor transmission losses (in $W.K^{-1}$), where U_i and A_i are heat transfer coefficients and surface areas, respectively, index i takes the symbolic values win, opa and ro for window, opaque wall and roof, respectively; similarly, $(UA)_{gr} = U_{gr} A_{gr}$ is the thermal conductance corresponding to ground transmission losses;
- $(UA)_{inf} = \alpha_{inf} C_i$ is the thermal conductance corresponding to infiltration losses, α_{inf} is the infiltration rate (in s^{-1});
- $(UA)_{ci} = 4.(nf - 1).A_{if} U_{ci}$ is the thermal conductance between the indoor air node and the internal wall/concrete node, $U_{ci}^{-1} = \delta_c / k_{con} + 1/U_{air}$, k_{con} is the concrete thermal conductivity and U_{air} is some average heat transfer coefficient of the indoor air;
- $t_o(\tau)$ is the time-varying outdoor temperature (in $^\circ$C); t_{gr} is the ground temperature (in $^\circ$C); $\Phi_{ig}(\tau)$ and $\Phi_{sg}(\tau)$ are the time-varying internal ans solar gains, respectively (in W).

$\Phi_{vent}(\tau)$ is the loss heat flow rate due to ventilation (in W):

$$\Phi_{vent}(\tau) = \alpha_{vent}(\tau) C_i \left(1 - \epsilon_{rec}\right)\left(t_o(\tau) - t_i\right) \quad (3)$$

where

- $\alpha_{vent}(\tau)$ is the forced ventilation rate (in s^{-1});
- ϵ_{rec} is the efficiency of the recovery wheel (sensible heat).

$\Phi_{hvac}(\tau)$ is the heat flow rate driven by the PI controller (positive for a heat demand, negative for a cold demand), expressed by a convolution

$$\Phi_{hvac}(\tau) = h_{PI}(\tau) * e_t(\tau) \tag{4}$$

where

- $h_{PI}(\tau)$ is the impulse response of the PI controller; it is completely defined by the controller gain G_{PI} (in $W.K^{-1}$) and by the controller time constant T_{PI} (in s);
- $e_t(\tau) = t_i^* - t_i(\tau)$ is the deviation of the indoor temperature from the set-point t_i^* (in °C).

Table 1 lists the fixed building parameters whereas those considered by the sensitivity analysis are found in Table 6. The indoor air temperature set-points are 22 °C (minimum) in winter and 25 °C (maximum) in summer, which leads to a dead-band of 3 °C.

Table 1: Fixed building parameters

Parameter	Value	Unit
Indoor air		
V_i	126700	m^3
ρ_{air}	1.204	$kg.m^{-3}$
c_{air}	1012	$J.(kg.K)^{-1}$
$U_{air;st}$	8	$W.m^{-2}.K^{-1}$
Internal walls		
nf	14	-
A_{if}	2610	m^2
ρ_{con}	2300	$kg.m^{-3}$
c_{con}	1000	$J.(kg.K)^{-1}$
k_{con}	1.7	$W.(m.K)^{-1}$
Envelope		
A_{gr}	2610	m^2
A_{ro}	2120	m^2
A_{win}	10360	m^2
A_{opa}	7910	m^2

Energy production model

The energy production model lies on a fundamental assumption, i.e., the heat pump, because of its higher performance, has priority to deliver the heat flow rate $\Phi_{hvac}(\tau)$ up to its capacity, the rest being produced by the standard machine. To illustrate it in heating mode (typically during a winter week), one names Φ_{HP} and Φ_{bo} the heat flow rates of the heat pump and of the gas boiler, respectively, and expresses these heat flow rates as follows

$$\Phi_{HP} = min\,(\Phi_{hvac}, \Phi_{HP}^{max}) \tag{5}$$
$$\Phi_{bo} = \Phi_{hvac} - \Phi_{HP} \tag{6}$$

The heat pump model describes the maximum heat flow rate Φ_{HP}^{max} and the electrical compressor power P_{HP} as functions of t_{ev} and t_{co}, the input temperatures of the secondary fluids at the evaporator and at the condenser, respectively

$$\Phi_{HP}^{max} = a_0 + a_1 t_{ev} + a_2 t_{co} \tag{7}$$
$$P_{HP} = b_0 + b_1 t_{ev} + b_2 t_{co} \tag{8}$$

where

- the calculation of t_{co} accounts for the heating coil with efficiency ϵ_{hc};
- the calculation of t_{ev} accounts for the heat exchanger between the heat pump and the geothermal boreholes; thus t_{ev} depends on the exchanger efficiency ϵ_{geo} and on the geothermal fluid temperature t_{geo}.

The energy production parameters are listed in Table 6. t_{geo} is not time-varying and has a nominal value of 6.3 °C.

Sensitivity analysis

As the system is run in nominal conditions, it is straightforward to apply a *local sensitivity* analysis, i.e., to evaluate the effect on the output variable of a small deviation from the nominal value for one factor at a time, all other factors kept fixed. This simple and fast procedure makes sense especially because the model is *linear in (most of) the parameters*. However, it can be very misleading in some cases, as is illustrated hereafter.

For the sake of clarity, one considers only the model of the building with the PI control (Equations 1-4). The output variable of interest is the heat energy delivered to the building over the predictive time horizon; it is noted E_{Hor} and expressed by

$$E_{Hor} = \int_{\tau_0}^{\tau_0+T_{Hor}} \Phi_{hvac}(\tau_1) d\tau_1 \tag{9}$$

where τ_0 is the sampling time and T_{Hor} is the predictive time horizon, here equal to one week.

Case 1

Here static conditions are considered, i.e., the building and the HVAC equipment are run continuously and the inputs take the constant nominal values listed in Table 2.

Table 2: Case 1: Nominal input values

Input	Value	Unit
t_o	0	°C
Φ_{ig}	911	kW
Φ_{sg}	32.7	kW

Assume that only *parameter* $(UA)_{inf}$ is successively increased by 1290, then by $2580\,W.K^{-1}$, that is twice the first value (corresponding to $10\,\%$ and $20\,\%$ of the nominal value, respectively) and that the output E_{Hor} is evaluated correspondingly. It appears from Table 3 that the increase ΔE_{Hor} is first 4.7, then $9.4\,MWh$. The same conclusion is drawn when only *input* Φ_{ig} is increased in the same way (by 91.1 then by $182.2 kW$): ΔE_{Hor} is first -7.7, then $-15.4\,MWh$. These results are not surprising as the relationship between E_{Hor} and each factor, whether parameter or input, is linear and, so far, the local sensitivity analysis would apply successfully. Then another input, the outdoor temperature, is shifted from 0 to $-2\,°\mathrm{C}$. A 10 %-variation is applied to only $(UA)_{inf}$ (1290 $W.K^{-1}$), then only to Φ_{ig} (91.1kW) for $(UA)_{inf}$. Table 3 shows that the deviation in the energy due to input Φ_{ig} is still the same as for $t_o = 0\,°\mathrm{C}$ ($-7.7\,MWh$) whereas the one due to parameter $(UA)_{inf}$ is not (5.2 instead of 4.7 MWh). This happens because infiltration losses depend on the outdoor temperature or, mathematically speaking, due to the multiplicative effect of the outdoor temperature on the infiltration thermal conductance.

Table 3: Case 1 - Results

$t_o = 0\,°\mathrm{C}$	$E_{Hor}(MWh)$	$\Delta E_{Hor}(MWh)$
Nominal	90.9	
$(UA)_{inf} * 1.1$	95.6	4.7
$(UA)_{inf} * 1.2$	100.3	9.4
$\Phi_{ig} * 1.1$	83.2	-7.7
$\Phi_{ig} * 1.2$	75.5	-15.4
$t_o = -2\,°\mathrm{C}$	$E_{Hor}(MWh)$	$\Delta E_{Hor}(MWh)$
Nominal	106.3	
$(UA)_{inf} * 1.1$	111.5	5.2
$\Phi_{ig} * 1.1$	98.6	-7.7

Case 2

The nominal values of the parameters and of the inputs are as in Case 1, except that milder weather conditions are selected ($t_o = 10\,°\mathrm{C}$), thus lower heat flow rates are needed. Moreover, the building and the HVAC equipment are run intermittently as in the real situation. Five days per week, occupancy (hence internal gains) applies from 8.00 a.m. to 6.00 p.m. and the HVAC equipment (hence ventilation and comfort control) apply from 7.00 a.m. to 7.00 p.m. The value of parameter Φ_{ig} is successively increased by 10, 20 and 30 % from its nominal value. It is noted in Table 4 that the increase in the energy delivered E_{Hor} is not proportional to the parameter variations. A detailed analysis reveals that the dead-band from 22 to 25 °C is responsible for this non-linear behaviour in the energy demand (indeed, control is switched off when the indoor temperature is within the dead-band limits).

Table 4: Case 2 - Results

	$E_{Hor}(MWh)$	$\Delta E_{Hor}(MWh)$
Nominal	12.5	
$\Phi_{ig} * 1.1$	9.0	-3.5
$\Phi_{ig} * 1.2$	6.3	-6.2
$\Phi_{ig} * 1.3$	4.4	-8.1

To conclude, two major reasons, i.e. the multiplicative effect of parameters with inputs and the model non-linearity, make the local sensitivity method inadequate for the purpose of the study. The next section presents a global sensitivity method that better fits our needs.

ELEMENTARY EFFECT METHOD

Among several global sensitivity techniques, the *elementary effect method* (Morris, 1991) is not the most powerful nor accurate but has several advantages, typically:

- its principle is close to local sensitivity which makes it easier to understand;
- the interpretation of the output indicators is not very difficult.

Here the major features of the method are presented and illustrated, more details about the base method and the subsequent enhancements can be found in (Morris, 1991; Campolongo et al., 2007; Saltelli et al., 2008), more recently in (Garcia Sanchez et al., 2014). Case 2 of Section *Problem statement - Sensitivity analysis* is considered and the objective is to rank parameters $(UA)_{inf}$, Φ_{ig} and t_o in regards of their effect on energy E_{Hor}.

The first aspect is to apply local sensitivity to the total factor space, when parameters are normalized and some discretization level *p* is chosen. For example, if the factor variations allowed are from -15% to $+15\%$ of their nominal value and $p = 4$, then the normalized factor values are -15, -5, $+5$ and $+15$ % (more exactly, for the outdoor temperature, the variation applies to $(t_i^* - t_o)$). Assume that the factor of interest is $X = (UA)_{inf}$ and that Y denotes the combined factors (Φ_{ig}, t_o), Morris suggests to compute the following EE_{kij} values, called *elementary effects*, as follows

$$EE_{kij} = \frac{E^r_{Hor}(X_k + \Delta, Y_{ij}) - E^r_{Hor}(X_k, Y_{ij})}{\Delta} \tag{10}$$

where:

- $X_k, k = 1, 2$ take values -15 % and -5 %;
- $Y_{ij}, i = 1..p, j = 1..p$ is any combination of (Φ_{ig}, t_o);
- Δ is equal to 0.2 (corresponding to a $+20$ %-increase of the parameter);
- $E^r_{Hor}(.) = E_{Hor}(.)/E^{nom}_{1w}$, and E^{nom}_{1w} is the nominal value of E_{1w} (all factors at nominal values).

Thus, the set of EE_{kij} makes a distribution of $2 * p^2 = 32$ elementary effects, out of which mean μ and standard deviation σ indicators are derived. These indicators for all three parameters are shown in Table 5. It is seen that:

- the sign of μ indicates the average direction of the effect (e.g., the higher Φ_{ig}, the lower E_{Hor});
- the absolute value of μ indicates the overall influence of the parameter (e.g., the outdoor temperature is the more influential);
- ratio $|\sigma/\mu|$ denotes the importance of parameter coupling and/or model non-linearities; the highest values are for $(UA)_{inf}$ (due to the coupling with t_o) and Φ_{ig} (no coupling, however non-linearities); the lowest value is for t_o as the coupled influence with $(UA)_{inf}$ is small compared to its direct influence, among others due to transmission).

Table 5: Elementary effect method: illustration in Case 2

	μ	σ	$\|\sigma/\mu\|$
$(UA)_{inf}$	1.35	0.39	0.29
Φ_{ig}	-2.88	0.69	0.24
t_o	4.00	0.77	0.19

The second aspect regards the experiment design or sampling strategy. When the numbers of factors and discretization levels increase, optimized strategies can avoid a high burden of computation. This method falls into the category of *One At Time (OAT)* sampling techniques. The set of sampling points is built in *trajectories*, each trajectory starts from a random point and contains $k+1$ points, where k is the number of factors considered. From the starting point on towards the ending point, each trajectory ensures that every factor varies just once, however the sequence of direction changes is also random.

The number of levels and the number of trajectories can be chosen by the user. It is read in (Saltelli et al., 2008) that choosing 4 levels and 10 trajectories is a good start, which is applied in this work.

RESULTS

Simulation set-up

The simulation experiments are concerned with a winter week where only heating takes place, which is the 5th week of the year, starting on Monday January, 29 0:00. One experiment involves one output of interest, say Y. The allowed variation range of the factors is $[-15;\ +15]$ % of their nominal value. In order to allow comparisons between the output variables, the elementary effects are not calculated based on Y, rather on $(Y - Y^{nom})/Y^{nom}$, where Y^{nom} is the output variable value when all factors take nominal values. The experiments are divided into two parts.

Building model

The model under study is completely described by Equations (1-4). The output variables considered are the heat demand E_{Hor} defined in Equation (9), but also the comfort index $Comf_{Hor}$ and the peak value of the heat flow rate Φ_{hvac}^{max}, which are defined by

$$Comf_{Hor} = \frac{Nh_{comf}}{Nh_{occ}} \tag{11}$$

where Nh_{occ} is the number of occupancy hours during the T_{Hor} period (typically 50 for a one week-period), Nh_{comf} is the number of comfortable hours (those where $t_i \geq t_i^*$)

and by

$$\Phi_{hvac}^{max} = max\left(\Phi_{hvac}[\tau_0; \tau_0 + T_{Hor}]\right) \tag{12}$$

The parameter list does *not* contain geometrical parameters as it assumed that all dimensions (for surface and volume calculation) are known exactly. The parameter list does contain

- the equivalent thickness of the inter-floor diving wall δ_c; this parameter accounts for the unknown concrete mass (because of the numerous beams, columns and internal walls);
- the heat transfer coefficients U_{win}, U_{opa}, U_{ro} and U_{gr};
- the infiltration rate α_{inf}, the ventilation rate α_{vent} and the heat recovery efficiency ϵ_{rec};
- the PI control parameters G_{PI} and T_{PI}.

The inputs to consider are the outdoor temperature $t_o(\tau)$, the ground temperature t_{gr}, the solar gains $\Phi_{sg}(\tau)$ and the internal gains $\Phi_{ig}(\tau)$:

- the outdoor temperature $t_o(\tau)$ and the solar gains $\Phi_{sg}(\tau)$ are from the *Meteonorm* source (Trnsys, 2006);
- the ground temperature t_{gr} is constant (the nominal value is 8 °C) and the internal gains $\Phi_{ig}(\tau)$ apply in an on-off manner depending on the occupancy hours (the nominal value is $911\ kW$)

For inputs t_{gr} and $\Phi_{ig}(.)$, the variation is applied as for the parameters. However, for time-varying inputs like $t_o(\tau)$ and $\Phi_{sg}(\tau)$, the following rules are applied:

- consider first $\Phi_{sg}(\tau)$: it is assumed that the deviation calculated is a mean value over the week because in reality forecast errors are very small at time τ_0 and grow bigger towards time $\tau_0 + T_{Hor}$; here the forecast error is linear with time, from 0 to 2 times the mean deviation;
- consider now $t_o(\tau)$: a similar approach is adopted, but the deviation is not calculated on $t_o(\tau)$ directly, rather on $(t_i^* - t_o(\tau))$ because heat transmission depends on the temperature gradient.

The initial conditions to consider are $t_i(\tau_0)$ and $t_c(\tau_0)$. As for the outdoor temperature, the variations apply to $t_i^* - t_i(\tau_0)$ and $t_i^* - t_c(\tau_0)$, but this time because of the control aspect.

Whole model

The model under study is completely described by Equations (1-4) and additionally by Equations (5-8). The output variable considered is the primary energy demand PE_{Hor}, which is defined by

$$PE_{Hor} = \int_{\tau_0}^{\tau_0+T_{Hor}} \left(\frac{\Phi_{bo}(\tau_1)}{\eta_{bo}} + \frac{P_{HP}(\tau_1)}{\eta_{pp}}\right) d\tau_1 \tag{13}$$

where

- Φ_{bo} is the heat flow rate delivered by the boiler and η_{bo} the boiler efficiency;
- P_{HP} is the electrical power requested by the heat pump and η_{pp} the power plant efficiency;

One recalls that the calculation of Φ_{bo} and P_{HP} requires the values of parameters ϵ_{hc} and ϵ_{geo}, and of input t_{geo}.

The factors are listed in Table 6 with their corresponding factor number and nominal value.

Table 6: Factor list

Number	Symbol	Value	Unit
01	δ_c	0.332	m
02	U_{win}	1.1	$W.m^{-2}K^{-1}$
03	U_{opa}	0.3	$W.m^{-2}K^{-1}$
04	U_{ro}	0.3	$W.m^{-2}K^{-1}$
05	U_{trg}	0.58	$W.m^{-2}K^{-1}$
06	α_{inf}	0.3	h^{-1}
07	α_{vent}	1.18	h^{-1}
08	ϵ_{rec}	0.6	-
09	G_{PI}	$1.06\ 10^5$	$W.K^{-1}$
10	T_{PI}	216	s
11	t_o	time-varying	°C
12	t_{gr}	8	°C
13	Φ_{ig}	911	kW
14	Φ_{sg}	time-varying	W
15	$t_i(\tau_0)$		°C
16	$t_c(\tau_0)$		°C
17	ϵ_{hc}	0.75	-
18	ϵ_{geo}	0.83	-
19	a_0	884	kW
20	a_1	22.5	$kW.K^{-1}$
21	a_2	-4.4	$kW.K^{-1}$
22	b_0	95	kW
23	b_1	0.2	$kW.K^{-1}$
24	b_2	3.4	$kW.K^{-1}$
25	η_{bo}	0.9	$-$
26	η_{pp}	0.4	$-$
27	t_{geo}	6.3	°C

Simulation Results

The results are presented in interval plot form, one plot for every output variable of interest, ss for example Figure 1. One can see for each factor 3 sensitivity indicators, i.e., the mean, minimum and maximum values (all positive or negative) of the elementary effect distribution. To calculate the minimum and the maximum values, it is assumed that the distribution of the elementary effects is Gaussian, and that the minimum and maximum values are $\mu - 2\sigma$ and $\mu + 2\sigma$.

Building model

Consider Figure 1: it is concerned with $(E_{Hor} - E_{Hor}^{nom})/E_{Hor}^{nom}$, where $E_{Hor}^{nom} = 32.5MWh$. It is noted that:

- the maximum effect is about 0.4, for factor $13 - \Phi_{ig}$; the effect of 9 factors is within 0.04 in magnitude (interesting result for factor fixing);
- beside factor $13-\Phi_{ig}$, factor $11-t_o$ has the major impact, then followed by factors $06 - \alpha_{inf}$, $08 - \epsilon_{rec}$ and $16 - t_c(\tau_0)$ and finally by factors $02 - U_{win}$ and $07 - \alpha_{vent}$;
- the (standard) deviation is generally small compared to the mean value, which indicates weak coupling and non-linearity for the most important parameters; higher deviations are exhibited by the effect of factor $11 - t_o$ (coupled effect with many heat transfer coefficients) and by factor $08 - \epsilon_{rec}$ (coupled effect with factors $11 - t_o$ and $07 - \alpha_{vent}$);

The importance of the internal gains and of the outdoor temperature is not surprising, but let us note the importance of infiltration, of recovery and especially of the concrete temperature estimation.

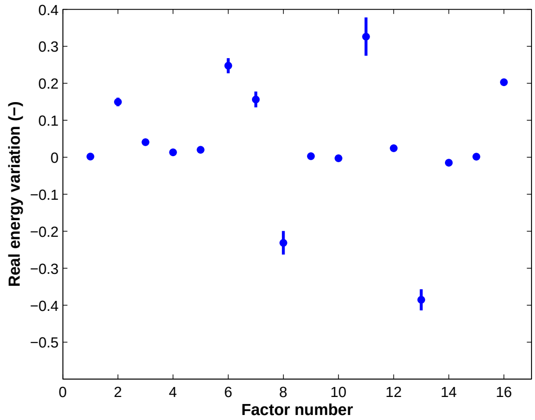

Figure 1: Sensitivity for the delivered energy: mean and estimated variation

Consider Figure 2: it is concerned with $(Comf - Comf^{nom})/Comf^{nom}$, where $Comf^{nom} = 0.88$. It is noted that:

- the overall effect is very small, lower than 0.04; the reason is that the controller ensures comfort, i.e., due to feedback, many of factor effects are attenuated;
- factors $11 - t_o$, $13 - \Phi_{ig}$ and $08 - \epsilon_{rec}$ are the most important.

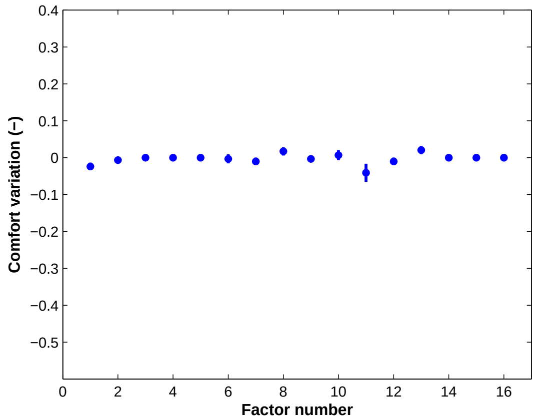

Figure 2: Sensitivity for the comfort index: mean and estimated variation

Consider Figure 3: it is concerned with $(\Phi_{hvac}^{max} - \Phi_{hvac}^{max;nom})/\Phi_{hvac}^{max;nom}$, where $\Phi_{hvac}^{max;nom} = 1.52\ MW$. It is noted that:

- the maximum effect is around 0.15; the most important factors are related to the concrete floor, i.e., factors $16-t_c(\tau_0)$ and $01-\delta_c$; indeed the peak heat flow rate often occurs at the beginning of the week and the concrete floor plays an important role as for the heat demand;

- then come factors $06-\alpha_{inf}$ and $08-\epsilon_{rec}$, which have an effect on the night temperature decrease and on the air temperature at the heating coil inlet, respectively;

- the controller parameters appear as well, i.e., the gain factor $09 - G_{PI}$ but mostly the time constant factor $10 - T_{PI}$.

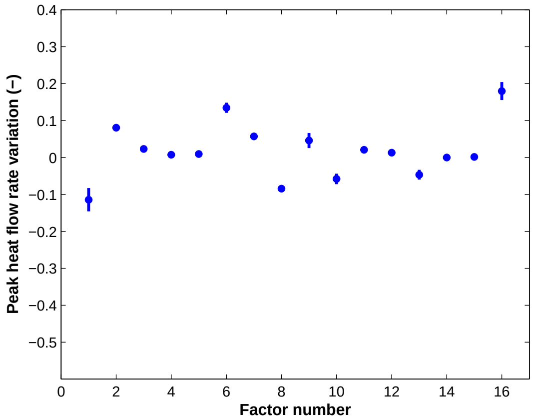

Figure 3: Sensitivity for the peak heat flow rate: mean and estimated variation

Whole model

Consider Figure 4: it is concerned with $(PE_{Hor} - PE^{nom}_{Hor})/PE^{nom}_{Hor}$, where $PE^{nom}_{Hor} = 19.6\,MWh$. Let us remember that this output variable is the one used in the MPC strategy and that it depends on the energy distribution/production systems. It is noted that:

- the maximum effect is around -0.5, which corresponds to factor $13 - \Phi_{ig}$; then a group of factors have an effect of around $+0.3$ in magnitude, i.e., $11-t_o$, $06-\alpha_{inf}$, $08-\epsilon_{rec}$ and $16-t_c(\tau_0)$; these factors are those affecting the delivered energy (this confirms the common idea that saving the primary energy starts with reducing the building needs);

- then appear factor $19 - a_0$ (the capacity of the heat pump) and factor $26 - \eta_{pp}$, the power station efficiency.

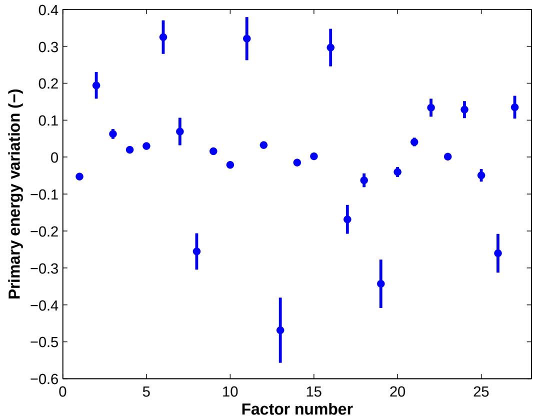

Figure 4: Sensitivity for the primary energy: mean and estimated variation

Discussion

In Model-based Predictive Control, the model outputs of interest are the primary energy, the comfort and the peak heat flow rate and ranking must account of all three output variables. The visual analysis of the plots gives interesting indications as for the linear relationship between the factors and the output variables (small standard deviations), the sensitivity levels of the output variables (comfort is not very sensitive), the factor fixing aspect (the factors with a small effect are numerous) and finally the ranking of the most important factors (internal gains, outdoor temperature, concrete mass and temperature, infiltration, recovery efficiency, heat pump capacity).

CONCLUSION

In this study, the authors have aimed at showing the interest of the (first-order) elementary effect method when the objective is just to rank the more influential factors and to fix the less influential ones. However, the real interest is that the sensitivity analysis applies to a predictive model, where the factors can be at the same time parameters, inputs and initial conditions. First, it is illustrated in a didactic case why a local sensitivity analysis can be misleading, then the elementary effect method is presented and applied to the predictive model to evaluate some output sensitivities to the model factors. The simulation experiments are concerned with the heating mode, i.e., a winter week. Plots representing the mean and maximum deviation of the elementary effects reveal to be very helpful for:

- evaluating the factor coupling and the non-linear aspects;
- evaluating the overall sensitivity levels of the output variables;
- fixing poorly influential factors and ranking the most important factors (for example, it is noted that an accurate estimation of the concrete temperature must be achieved).

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The authors are willing to continue the sensitivity analysis study with the following outlook:

- to derive indicators for other periods of the year (summer, mid-season);
- to refine the numerical interpretation, especially regarding the standard deviation information, hence to make use of higher-order elementary effects for a better detection of coupling and non-linearity phenomena;
- to use other sensitivity analysis methods, e.g., variance analysis, as the number of factors is reasonable.

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