UNCERTAINTY ANALYSIS AND PARAMETER ESTIMATION OF HVAC SYSTEMS IN BUILDING ENERGY MODELS

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ABSTRACT
Building performance simulation has the potential to quantitatively evaluate design alternatives and various energy conservation measures for retrofit projects. However before design strategies can be evaluated, accurate modeling of existing conditions is crucial. This paper extends current model calibration practice by presenting a probabilistic method for estimating uncertain parameters in HVAC systems for whole building energy modeling. Using Markov Chain Monte Carlo (MCMC) methods, probabilistic estimates of the parameters in two HVAC models were generated for use in EnergyPlus. Demonstrated through a case study, the proposed methodology provides predictions that more accurately match observed data than base case models that are developed using default values, typical assumptions and rules of thumb.

INTRODUCTION
A calibrated building energy model has the potential to guide building retrofit designs by providing a means to evaluate various energy conservation measures. Current practice in model calibration is supported by methods established within ASHRAE Guideline 14 (ASHRAE, 2002) or the International Performance Measurement and Verification Protocol (IPMVP) (EVO, 2012). These calibration processes often involve tuning model parameters until errors between model predictions and observations satisfy a certain threshold with respect to whole building energy consumption. One caveat is that the calibrated model may not be representative of actual building performance since various combinations of inputs can still produce reasonable matching results. In particular, HVAC systems are commonly neglected because of the large number of input parameters required by existing simulation tools. In reality, information on these parameters are often not available, and assumptions are made using rules of thumb or predetermined default values. As a result, such models often come with various uncertainties. These uncertainties are usually ignored and do not translate to the calibrated model, making its reliability questionable.

Uncertainty modeling in building performance simulation is uncommon but is not new. Macdonald and Strachan (2001) reviewed uncertainties in the thermophysical properties of construction materials, and incorporated them into the building simulation tool ESP-r using Monte Carlo Analysis. Using regression based methods, Sun et al. (2014) provided a framework to quantify uncertainties in microclimate variables when weather data is obtained from a nearby meteorological station. Eisenhower et al. (2012) modeled 1009 parameters as uncertain in an EnergyPlus model by varying them ±20% of their nominal value. Together with sensitivity analysis, they provided insights to how uncertainty in input parameters may affect model outputs. Using Bayesian calibration, Heo et al. (2012) provided a methodology to determine the posterior distributions of uncertain parameters in a normative energy model given observed utility data.

This paper extends current approaches by applying Bayesian calibration to HVAC systems. The HVAC models considered are similar to those used in the EnergyPlus building energy simulation program but the method can be extended to other deterministic HVAC models. Using data from building management systems, the proposed method uses Markov Chain Monte Carlo (MCMC) methods to provide probabilistic estimates of the input parameters. It is demonstrated that these estimates provide higher accuracy over using default values, typical assumptions and rules of thumb. In addition, the proposed method provides the decision analyst with valuable information on the distribution of each parameter. Since the approach provides probabilistic predictions instead of the usual point estimates with no measure of uncertainties, these predictions offer decision-makers greater assurance when considering retrofit design alternatives.

METHOD

Bayesian calibration
An Bayesian approach was employed for the calibration of HVAC models in energy simulation models. PyMC (Patil et al., 2010), an open source Python package and the Metropolis-Hastings algorithm, a specific MCMC method was used for the calibration process. The procedure begins with the quantification of uncertainty in model parameters using prior probability distributions. This is followed by an application of Bayes’s Theorem to produce an updated posterior probability distribution for each parameter through a process of random-walks. This updating process is driven by the log-likelihood of the model parameters given the observations (Equation 1). This likelihood function is determined assuming Gaussian noise ($\epsilon \sim \mathcal{N}(0, \sigma^2_i)$)

\[
\log \frac{p(y_i | \theta)}{P(\theta)} = \frac{1}{2} \left( \frac{y_i - \theta}{\sigma_i} \right)^2 + \frac{1}{2} \log \frac{1}{\sigma_i^2} - \frac{1}{2} \log 2\pi
\]
with $\sigma_e$ modeled as a random variable.

$$\ell(\theta | y_1, ..., y_n) = \log \prod_{i=1}^{n} N(y_i; f(x_i, \theta), \sigma_e^2)$$  \hspace{1cm} (1)$$

Observations at each measured condition $x_i$ are denoted by $y_i$; $f(x_i, \theta)$ denotes the HVAC model’s output computed at measured condition $x_i$ and calibration parameters $\theta$, $\theta$ can be a single or vector of parameters. Overall, the proposed method can be summarized by the following steps:

1. Define the prior probability distributions of the input parameters.
2. Arbitrarily select a valid initial starting point $\theta_0$.
3. Suppose $\theta_0$, $\theta_1$, ..., $\theta_i$ have been generated. To generate $\theta_{i+1}$, first generate a candidate value $\theta_{i+1}$ with probability $q$ from the proposal distribution $q$. The candidate value in this case refers to the input parameter(s) to the deterministic HVAC model.

$$C \sim q(c|\theta_i)$$ \hspace{1cm} (2)$$

4. Perform computation of the deterministic HVAC model output using the generated candidate value as input.
5. Evaluate the likelihood of the computed output given the measured data (Equation 1) and compute $r$, the probability of transitioning to the new candidate value.

$$r = \min \left\{ \frac{p(c) q(\theta|c)}{p(\theta) q(\theta|c)} , 1 \right\}$$ \hspace{1cm} (3)$$

6. Accept and set $\theta_{i+1}$ to the new candidate value with probability $r$ or stay at the same point with probability $1 - r$.

$$\theta_{i+1} = \begin{cases} C & \text{with probability } r \\ \theta_i & \text{with probability } 1 - r \end{cases}$$ \hspace{1cm} (4)$$

7. Repeat steps (3) to (6) until convergence. The Gelman-Rubin statistic (Gelman and Rubin, 1992) was used to assess convergence. This diagnostic method uses multiple chains to check for convergence, and is based on the concept that if multiple chains have converged, there should be little variability between and within the chains. A Gelman-Rubin statistic below 1.2 was considered as acceptable (Braak, 2006).

The generated sequence of values can then be used to approximate the posterior distributions of the input parameters. Since the initial starting values might bias the generated sequence, the first 10,000 samples were discarded.

**Case study**

The Bayesian calibration methodology is illustrated with a case study. The building analyzed is an actual ten story office building located in Pennsylvania, U.S.A. The HVAC system is a dual duct system where both warm and cold air are separately ducted and mixed at each terminal unit to achieve the desired temperature. Cooling is supplied with a water cooled chiller while heating is supplied through gas boilers. Using measurements from the building management system, we apply Bayesian calibration to two EnergyPlus HVAC models (Boiler:HotWater and Chiller:Electric:EIR). Details on each deterministic model can be found in the EnergyPlus source code and Engineering Reference (UIUC and LBNL, 2014b).

Building data collection took place from May 1st 2013 to December 31st 2014. Separate sets of data were used for the training and testing of each model. Data from May 1st 2013 to April 30th 2014 was used for training while a separate data set (May 1st 2014 to December 31st 2014) was used for testing. Missing and erroneous data were removed. Examples of erroneous data includes negative chiller or boiler efficiency, boiler efficiency greater than 1 and negative chiller or boiler power.

**Boiler:HotWater**

This EnergyPlus boiler model calculates the performance of the boiler based on a nominal thermal efficiency (UIUC and LBNL, 2014a). In addition, the model allows the inclusion of a cubic curve (Equation 5) to provide a more accurate representation of its efficiency. This curve models the boiler’s efficiency as a function of its loading or part-load ratio ($PLR$). The output of Equation 5 ($BoilerEfficiencyCurveOutput$) is multiplied by the boiler’s nominal thermal efficiency to give the boiler’s efficiency at different part load conditions. Equation 6 is then used to determine the fuel used by the boiler, after accounting for changes in its thermal efficiency due to the load (UIUC and LBNL, 2014a).

$$\text{BoilerEfficiencyCurveOutput} = \begin{cases} \text{Nominal capacity} & \text{if } PLR \leq 0.2 \\ \text{Minimum Part load ratio} & \text{if } 0.2 < PLR < 0.4 \\ \text{Coefficients of Equation 5} & \text{if } 0.4 \leq PLR \leq 0.7 \\ \text{Nominal Thermal efficiency} & \text{if } PLR > 0.7 \end{cases}$$ \hspace{1cm} (5)$$

$$\text{FuelUsed} = \frac{\text{BoilerEfficiencyCurveOutput}}{100} \times \text{Nominal Thermal efficiency}$$ \hspace{1cm} (6)$$

Two sets of measurements from the installed boiler are required as inputs for the calibration of this EnergyPlus Boiler:HotWater model. They are: (1) hourly energy consumption of the boiler as observations $y_i$ and (2) the corresponding boiler load $x_i$. Table 1 summarizes the input, output and parameters calibrated by the Bayesian calibration procedure described in the method above.

<table>
<thead>
<tr>
<th>Parameters calibrated for boiler model.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Table 1</strong></td>
</tr>
<tr>
<td>Observations $y_i$: Boiler energy consumption</td>
</tr>
<tr>
<td>Conditions $x_i$: Boiler load</td>
</tr>
<tr>
<td>Parameters $\theta$:</td>
</tr>
<tr>
<td>Nominal capacity</td>
</tr>
<tr>
<td>Nominal Thermal efficiency</td>
</tr>
<tr>
<td>Minimum Part load ratio</td>
</tr>
<tr>
<td>Coefficients of Equation 5</td>
</tr>
</tbody>
</table>

Two sets of measurements from the installed boiler are required as inputs for the calibration of this EnergyPlus Boiler:HotWater model. They are: (1) hourly energy consumption of the boiler as observations $y_i$, and (2) the corresponding boiler load $x_i$. Table 1 summarizes the input, output and parameters calibrated by the Bayesian calibration procedure described in the method above.
\[
\text{BoilerEfficiencyCurveOutput} = a_0 + b_0(\text{PLR}) + c_0(\text{PLR})^2 + d_0(\text{PLR})^3 \quad (5)
\]

\[
\text{FuelUsed} = \frac{\text{BoilerLoad}}{(\text{NominalThermalEfficiency}) \cdot \text{BoilerEfficiencyCurveOutput}} \quad (6)
\]

\[
\text{CapFTemp} = a_1 + b_1(T_{cw,l}) + c_1(T_{cw,l})^2 + d_1(T_{cond,e}) + e_1(T_{cond,e})^2 + f_1(T_{cw,l})(T_{cond,e}) \quad (7)
\]

\[
\text{EIRFTemp} = a_2 + b_2(T_{cw,l}) + c_2(T_{cw,l})^2 + d_2(T_{cond,e}) + e_2(T_{cond,e})^2 + f_2(T_{cw,l})(T_{cond,e}) \quad (8)
\]

\[
\text{EIRFPLR} = a_3 + b_3(\text{PLR}) + c_3(\text{PLR})^2 \quad (9)
\]

**Chiller:Electric:EIR**

This EnergyPlus chiller model uses performance information at reference conditions along with three performance curves to determine the chiller’s performance at off-reference conditions (UIUC and LBNL, 2014a). The three performance curves are:

- Cooling Capacity Function of Temperature Curve (CapFTemp) (Equation 7)
- Energy Input to Cooling Output Ratio Function of Temperature Curve (EIRFTemp) (Equation 8)
- Energy Input to Cooling Output Ratio Function of Part Load Ratio Curve (EIRFPLR) (Equation 9)

Equations 7 and 8 are biquadratic curves with two independent variables, namely the leaving chilled water temperature \((T_{cw,l})\) and the entering condenser water temperature \((T_{cond,e})\). The output of Equation 7 (CapFTemp) is multiplied by the reference capacity to give the full-load cooling capacity at different operating conditions. The output of Equation 8 (EIRFTemp) is multiplied by the reference EIR, where EIR is defined as the inverse of the Coefficient of Performance (COP). This gives the full-load EIR at different operating conditions. Equation 9 is a quadratic curve that parameterizes the variation of chiller input power ratio as a function of the part-load ratio \((\text{PLR})\) (UIUC and LBNL, 2014a). The output of this curve (EIRFPLR) is multiplied by the reference EIR and the EIRFTemp (Equation 8) to give the EIR at specific temperatures and part-load ratios. By way of explanation, this computes the efficiency of the chiller at a specific temperature and part-load ratio.

Two sets of measurements from the installed chillers are required as inputs for the calibration of this EnergyPlus Chiller:Electric:EIR model. They are:

1. Hourly energy consumption of the chiller as observations \(y_i\), and
2. Their corresponding conditions \(x_i\) which include the hourly averages of leaving chilled water temperature, entering condenser water temperature and the chiller load.

Table 2 summarizes the inputs, output and parameters that are calibrated. Maximum part load ratio is set to 1.0.

**DISCUSSION AND RESULT ANALYSIS**

**Prior and posterior distributions**

In Bayesian statistics, uncertain parameters are assigned prior probability distributions based on a degree of belief. These could be derived from prior knowledge such as experiments, existing databases, expert knowledge, etc. Using MCMC, the theoretical posterior probability distribution of each calibration parameter is approximated and a histogram plot generated to get a visual representation of its distribution. To assess convergence, the Gelman-Rubin statistic (Gelman and Rubin, 1992) was computed for every calibration parameter (Tables 3 and 4). Table 3 lists the prior probabilities assigned to each calibration parameter for the Boiler:HotWater model. Nominal capacity was assigned a normal \(N(\mu, \sigma^2)\) prior with parameters \(\mu = \text{boiler plate capacity}\) and \(\sigma = 5\% \text{ of the boiler plate capacity}\). Nominal thermal efficiency was assigned a normal prior with parameters \(\mu = \text{boiler plate efficiency}\) and \(\sigma = 5\% \text{ of the boiler plate efficiency}\). Minimum PLR was assigned a uniform \(U(a, b)\) prior with lower bound \(a = 0\) and upper bound \(b = 1\). The bounds were selected to cover the valid range for PLR. The coefficients of Equation 5 were assigned flat priors (uniformly distributed from negative infinity to positive infinity) since no prior information was available.
This means that all possible values of these parameters are equally likely \textit{a priori}.

Figure 1 shows the posterior distributions of the calibration parameters for the Boiler:HotWater model. From the histogram plots, the following can be observed:

- Posterior distribution of the nominal thermal efficiency is between 0.8 and 1.
- Posterior distribution of the minimum PLR is towards the lower bound (0 to 0.3) of the range specified (0 to 1) which is logical.
- Figure 2 shows the mean value of the normalized efficiency ($\text{NominalThermalEfficiency \times BoilerEfficiencyCurveOutput}$) for different values of PLR in the range of 0 to 1. The bounds represent the variability in the boiler’s efficiency computed from different sets of coefficients ($a_0, a_1, a_2$ and $a_3$) and nominal thermal efficiency. The plot (Figure 2) provides insights to the boiler’s performance and can be useful in monitoring the boiler’s performance at different operating conditions.
- Model error ($\sigma_{\epsilon}$) is approximately normal with parameters $\mu \approx 60$ kW and $\sigma \approx 5$ kW.

Table 4 lists the prior probabilities assigned to each calibration parameter for the Chiller:Electric:EIR model. Reference capacity was assigned a normal prior with parameters $\mu =$ chiller plate capacity and $\sigma = 5\%$ of the chiller plate capacity. Reference COP was assigned a $U(0, 10)$ prior. The bounds were selected to cover the valid range for a Chiller’s typical COP. The coefficients of Equations 7, 8 and 9 were assigned a uniform prior probability distribution. The lower and upper bounds were determined using the data set (Chillers.idf) that comes with EnergyPlus. The minimum and maximum values for each coefficient in the data set were translated into the lower and upper bounds of each uniform prior by rounding down the minimum values and rounding up the maximum values respectively. Both the minimum PLR and minimum unloading ratio were assigned a $U(0, 1)$ prior. The bounds were selected to cover the valid range of both calibration parameters. It should be noted that no prior and Gelman-Rubin statistic were presented for coefficients $a_1, a_2$ and $a_3$ (Table 4). This is because these parameters are computed as a constraint where the curves (Equations 7, 8 and 9) output a value of 1.0 at reference temperatures (UIUC and LBNL, 2014b).

Figure 3 shows the posterior distribution of the calibration parameters for the Chiller:Electric:EIR model. From the histogram plots, the following can be observed:

- At reference conditions, the posterior distribution of the chiller’s capacity is approximately normal with parameters $\mu \approx 3100$ kW and $\sigma \approx 160$ kW.
- At reference conditions, the chiller has a COP of between 3 and 8.5 with mean of approximately 4.
- Model error ($\sigma_{\epsilon}$) is approximately normal with parameters $\mu \approx 15$ kW and $\sigma \approx 1$ kW.

Figure 1 Posterior distribution of calibration parameters for Boiler:HotWater model.

Figure 2 Normalized thermal efficiency of boiler at different part load conditions.
Table 3  
Prior probabilities and Gelman-Rubin Statistic of calibration parameters for Boiler:HotWater model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Posterior Summary Statistics (mean,median,min,max)</th>
<th>Gelman-Rubin Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Capacity</td>
<td>$N(1519,76^2)$</td>
<td>(1495,1495,1118,1803)</td>
<td>0.99990</td>
</tr>
<tr>
<td>Nominal Thermal Efficiency</td>
<td>$N(0.912,0.0456^2)$</td>
<td>(0.912,0.911,0.830,1.000)</td>
<td>1.00011</td>
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<tr>
<td>Minimum PLR</td>
<td>$U(-\infty,\infty)$</td>
<td>(0.749,0.750,0.351,0.987)</td>
<td>0.99999</td>
</tr>
<tr>
<td>$a_0$</td>
<td>$U(-\infty,\infty)$</td>
<td>(0.808,0.815,0.000,2.489)</td>
<td>0.99999</td>
</tr>
<tr>
<td>$b_0$</td>
<td>$U(-\infty,\infty)$</td>
<td>(-1.138,-1.121,-3.498,0.343)</td>
<td>1.00004</td>
</tr>
<tr>
<td>$c_0$</td>
<td>$U(-\infty,\infty)$</td>
<td>(0.556,0.540,-0.241,1.838)</td>
<td>1.00011</td>
</tr>
<tr>
<td>$d_0$</td>
<td>$U(0,200)$</td>
<td>(60.957,60.718,46.038,87.578)</td>
<td>0.99998</td>
</tr>
</tbody>
</table>

Table 4  
Prior probabilities and Gelman-Rubin Statistic of calibration parameters for Chiller:Electric:EIR model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Posterior Summary Statistics (mean,median,min,max)</th>
<th>Gelman-Rubin Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference Capacity</td>
<td>$N(3165,158^2)$</td>
<td>(3141,3141,2581,3703)</td>
<td>1.00004</td>
</tr>
<tr>
<td>Reference COP</td>
<td>$U(0,10)$</td>
<td>(4.354,4.254,3.097,8.492)</td>
<td>1.14125</td>
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<tr>
<td>$a_1$</td>
<td>$U(-1,1)$</td>
<td>(-0.025,-0.045,-0.999,1.000)</td>
<td>1.07159</td>
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<tr>
<td>$b_1$</td>
<td>$U(-0.1,0.1)$</td>
<td>(0.060,0.063,-0.011,0.100)</td>
<td>1.09317</td>
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<tr>
<td>$c_1$</td>
<td>$U(-0.1,1)$</td>
<td>(0.204,0.214,-0.447,0.915)</td>
<td>1.00109</td>
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<tr>
<td>$d_1$</td>
<td>$U(-0.1,1)$</td>
<td>(-0.004,-0.004,-0.022,0.0210)</td>
<td>1.00111</td>
</tr>
<tr>
<td>$f_1$</td>
<td>$U(-0.1,0.1)$</td>
<td>(-0.028,-0.028,-0.094,0.021)</td>
<td>1.07570</td>
</tr>
<tr>
<td>$a_2$</td>
<td>$U(-1,1)$</td>
<td>(0.741,0.709,0.414,1.822)</td>
<td>1.11659</td>
</tr>
<tr>
<td>$b_2$</td>
<td>$U(-0.1,0.1)$</td>
<td>(0.254,0.286,-0.823,0.581)</td>
<td>1.11965</td>
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<tr>
<td>$c_2$</td>
<td>$U(-0.1,1)$</td>
<td>(-0.017,-0.020,-0.166,0.156)</td>
<td>1.00448</td>
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<tr>
<td>$d_2$</td>
<td>$U(-0.1,1)$</td>
<td>(-0.0004,-0.0004,-0.004,0.005)</td>
<td>1.00012</td>
</tr>
<tr>
<td>$e_2$</td>
<td>$U(-0.1,0.1)$</td>
<td>(-0.0007,0.0000,-0.012,0.007)</td>
<td>1.12804</td>
</tr>
<tr>
<td>$f_2$</td>
<td>$U(-0.1,0.1)$</td>
<td>(0.741,0.709,0.414,1.822)</td>
<td>1.11106</td>
</tr>
<tr>
<td>$a_3$</td>
<td>$U(-1,1)$</td>
<td>(0.005,0.004,0.0000,0.035)</td>
<td>-</td>
</tr>
<tr>
<td>$b_3$</td>
<td>$U(-5,5)$</td>
<td>(0.254,0.286,-0.823,0.581)</td>
<td>1.11659</td>
</tr>
<tr>
<td>$c_3$</td>
<td>$U(-5,5)$</td>
<td>(0.003,0.003,0.0000,0.026)</td>
<td>1.01753</td>
</tr>
<tr>
<td>Minimum PLR</td>
<td>$U(0,1)$</td>
<td>(0.006,0.005,0.0000,0.052)</td>
<td>1.06040</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>$U(0,200)$</td>
<td>(14.610,14.545,10.762,19.422)</td>
<td>1.00468</td>
</tr>
</tbody>
</table>
Model validation

This calibration procedure not only provides insights from observations, but also gives a better estimation of the calibration parameters by changing their prior distributions based on their likelihood given the observed data. As a result, the resulting posterior distribution of each calibration parameter helps to establish a better understanding of each parameter and improves the predictive power of the energy model. To validate the model, a separate dataset (May 1st 2014 to December 31st 2014) was used for testing. We compared the predictions with the observed values at an hourly resolution. Periods of time with missing data were removed for validation purposes.

Figure 4 shows how predictions from the calibrated Boiler:HotWater model compares with a base case model that was built upon parameters derived from conventional methods in building energy modeling. This base case model is derived by setting the boiler’s nominal capacity (1519 kW) and thermal efficiency (0.826) as specified by the boiler plate. Minimum PLR is set to 0 (default value in EnergyPlus). For the base case model, the coefficients in Equation 5 were determined using least squares regression. From Figure 4, it can be observed that the calibrated model matches the hourly actual energy consumption more accurately as compared to conventional modeling procedures (base case model). A similar observation can be made in Figure 5, which illustrates the comparison of predictions from the calibrated Chiller:Electric:EIR model with the base case model. The chiller base case model is derived by setting the chiller’s nominal capacity (3165 kW) as specified by the chiller plate. All other parameters were set to default values for a generic centrifugal chiller provided by DOE-2.1E, and can be found in the chiller data set (chillers.idf) that comes with EnergyPlus. Unlike the Boiler:HotWater model, least square regression is not used for determining the coefficients of the performance curves due to the lack of data at full load conditions.

Comparing Figures 4 and 5, there is greater uncertainty around the chiller model than the boiler model. This could be due to the greater dispersion in measured chiller energy use as indicated by a larger coefficient of variation \( (CV = \text{ratio of standard deviation } \sigma \text{ to the mean } \mu) \) of 0.74 as compared to boiler energy use \( (CV = 0.33) \). It is also interesting to observe large uncertainties in predictions made by the chiller model for short periods of time during June and July (Figure 5). This is because the operating conditions during these periods fall outside the range used to train the model. A closer look at these operating conditions reveals the entering condenser temperature \( (T_{\text{cond,e}}) \) to exceed those found in the training data. Using Equa-
tions 7 and 8 during these periods would therefore lead to unsupported model extrapolations since the training data cannot support inferences for these $T_{\text{cond,e}}$. In this model, values of $T_{\text{cond,e}}$ greater than the maximum allowable will be replaced by the maximum. This results in predictions that are close to 0 kWh and the larger uncertainties observed in Figure 5.

To quantify the improvements in accuracy, we compared the coefficient of variation of the root mean square error (CVRMSE). Equation 10 shows the formulation of the CVRMSE (ASHRAE, 2002).

$$\text{CVRMSE} = 100 \times \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n - 1}} \bar{y}$$

(10)

where $y_i$ = observed value at hour $i$; $\hat{y}_i$ = predicted value at hour $i$; $\bar{y}$ = mean energy consumption of $n$ observations; and $n$ = number data points.

Table 5 shows the CVRMSE of the calibrated and the base case models. Since the calibration process provides probabilistic estimates, the mean value of the estimates was used in the calculation of its CVRMSE. The CVRMSE of the calibrated models are significantly lower than that of the base case models (Table 5). Furthermore, models calibrated using the proposed Bayesian approach outweigh deterministic models that provide point estimates with no measure of uncertainty. The provision of probabilistic predictions also offer greater assurance to decision makers.

### Table 5

<table>
<thead>
<tr>
<th>HVAC model</th>
<th>Type of model</th>
<th>CVRMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boiler:HotWater</td>
<td>Calibrated model</td>
<td>12.9%</td>
</tr>
<tr>
<td></td>
<td>Base case model</td>
<td>28.4%</td>
</tr>
<tr>
<td>Chiller:Electric:EIR</td>
<td>Calibrated model</td>
<td>23.5%</td>
</tr>
<tr>
<td></td>
<td>Base case model</td>
<td>40.1%</td>
</tr>
</tbody>
</table>

Figure 4: Comparison of measured boiler gas consumption against expected values from calibrated probabilistic model (top plot) and base case model (bottom plot).

Figure 5: Comparison of measured chiller energy consumption against expected values from calibrated probabilistic model (top plot) and base case model (bottom plot).
CONCLUSION

This paper has shown how HVAC models in building energy simulation tools can be calibrated using a Bayesian approach. Using the proposed method, probabilistic estimates of model parameters were generated based on their likelihood given the measured data. The resultant posterior probability distribution not only provides a better understanding of the model parameters but also improves the predictive power of the building energy model. Using two HVAC component models, the case study demonstrates that this method provides predictions that more accurately match observed data. The CVRMSE of the calibrated models and the baseline models (built upon parameters derived from default values and conventional methods in building energy modeling) differ by about 50%. Furthermore, by providing probabilistic predictions instead of the usual point estimates with no measure of uncertainties, decision makers can be provided with more assurance when evaluating alternative designs or operations.

To improve the applicability of the proposed methodology in practice, future work includes:

- More case studies to affirm the findings presented in this study. This includes the extension of the current method to other building systems and envelope parameters.
- The proposed calibration method may not be computationally efficient in large scale applications where a large number of parameters needs to be calibrated at the same time. Hence, it is important that a simplified, computationally efficient algorithm in Bayesian calibration be developed for applications in building energy models.
- The use of a Bayesian approach for calibration requires the construction of appropriate prior probability distributions and a retrospective evaluation of the model’s fitness. Knowledge of the underlying problem and subjective judgement are therefore required to specify both the likelihood and prior parts of the model. Hence, the development of a database consisting of examples and probability specifications would help provide guidelines on these issues and help alleviate inconsistencies and the need to correct modeling specifications retrospectively.

REFERENCES


