ABSTRACT
The smart grid paradigm implies flexible demand and/or energy storage in order to cope with the stochastic behaviour of renewable energy sources. The building stock is often put forward as a potential supplier of flexibility services through demand side management (DSM) and distributed energy storage, partly as thermal energy. This paper presents a bottom-up approach for the quantification of this flexibility service in the form of cost functions. The cost functions are computed from the solution of optimal control problems with low-order models. The method is generic and can be applied to heating, ventilation and air-conditioning (HVAC) services, thermal energy storage (TES) and local electricity production.

INTRODUCTION
Due to the increased penetration of renewable energy systems with stochastic production characteristics, the need for flexible production, flexible demand and energy storage increases. Simultaneously the electricity system evolves from a centralized to a distributed architecture with small-scale distributed generation (DG), distributed storage (DS) and controllable loads, often referred to as distributed energy resources (DER) (Chicco and Mancarella 2009). This evolution, in combination with advanced ICT and control systems may lead to so-called smart grids in which highly distributed loads are involved in power system control actions (Callaway and Hiskens 2011). The benefits of increased responsiveness of the loads are described by Kirschen (2003) and Strbac (2008).

Buildings have a high energy demand and therefore they play a key role in the roll-out of these smart grids. In fact, these buildings can offer flexibility as a service to the energy markets through DSM. Typically flexibility is referred to as the possibility to adapt the electricity consumption profile, to deviate from a certain electricity consumption profile or to shift a consumption profile in time. Therefore, flexibility is a requirement for DSM. DSM can be defined from a utility perspective as “the planning and implementation of those electric utility activities designed to influence customer uses of electricity in ways that will produce desired changes in the utility’s load shape” (Gellings 1985).

The first section in this paper will give an overview of how this concept of flexibility has been treated in the literature. We will show that there is no common metric or indicator to quantify the ‘amount’ of flexibility a building can offer.

In the second section, a methodology is developed to quantify the flexibility potential for individual buildings. The methodology leads to the construction of cost curves in order to estimate the real cost of using the available flexibility. Moreover, the cost curves can be easily aggregated in order to quantify the potential flexibility service of a group of buildings. This allows comparing the cost of DSM in buildings with alternatives like electrical energy storage or providing load-following capacity centrally (redispatch or ancillary services).

Finally, the methodology is illustrated by application to an office building with a cooling plant. The cost curves are computed for two different cold emission systems: concrete core activation (CCA) and chilled ceilings.

The aim of this work is to enable a quantitative comparison of the ‘amount’ of flexibility and corresponding cost between different buildings and groups of buildings. The metric can thus be used in the design process or when selecting a set of buildings to include in a DSM scheme. It is not the intention to develop a methodology for operational decision making or real-time operation.

LITERATURE REVIEW
The impact of DSM can be assessed for a single building or a multitude of buildings. For a single building generally a detailed model of the specific system or building is made to study the possible demand shift by simulation or optimisation. The model typically takes electricity price profiles as input, and cannot take into account the different feedback mechanisms between load and centralized production. Many authors have studied load shifting control strategies in single building simulation or optimisation. The reason for load shifting is reduction of peak power, consumption, emissions or costs. They rarely define flexibility nor present a general methodology to assess the potential for
different buildings. Six et al. (2011) simply defined the flexibility of an appliance as the number of hours the operation can be delayed. In the EU FP7 Adress project (Belhomme et al., 2009), an hourly flexibility index is calculated proportionally to the hourly load. The hourly load is computed according to the probability of use of the considered appliance during the day.

To assess DSM for multiple buildings, a top-down and a bottom-up approach can be used. The top-down approach mostly starts from the electricity generation park and models the demand by load curves. In these studies, the flexibility of the buildings is defined as the elasticity of the demand as a function of the electricity price (De Jonghe 2011). In this approach, the detailed (thermal) dynamics of the buildings are completely neglected, and very general assumptions about the demand elasticity have to be made. It is therefore an input to the model, and not a result. A bottom-up approach starts from very simple and generalized models of the buildings and solves a unit commitment or (distributed) optimal control problem in order to optimise the operation of the full system (Bruninx et al. 2012). This approach can be agent-based in order to apply it for operational optimization in the context of energy markets and smart grids (McArthur et al. 2007; Kok et al. 2008; Zhou, Chan, and Chow 2009). These methods assess the impact of load shifting on the total system, but do not quantify the amount nor cost of the flexibility of real buildings. Finally, the analogy between multiple buildings and virtual power plants (VPP’s) can be made. The flexibility service that a VPP can offer is described by different authors (Braun and Strauss 2008; Pudjianto, Ramsay, and Strbac 2007; Bel et al. 2007).

A calculation method to quantify the flexibility service that a specific single building, group of buildings, VPP or a DER system can offer, was not found in the scientific literature. This paper elaborates a generic method that enables aggregation of the flexibility of different DER systems and applies it to a specific case of cooling in an office building.

**QUANTIFICATION OF SYSTEM FLEXIBILITY**

**Definitions**

In the consulted scientific literature, flexibility is referred to as the possibility to ‘change’ (more specific: adapt, deviate, shift) the electricity consumption profile. This indicates the need for a reference scenario. We call the reference electricity consumption the ‘business as usual’ (BAU) scenario.

To generalise the methodology, we will speak of a DER system or simply system instead of building.

We suppose that the flexibility of a system is a function of time. A given system may allow load shifting at some times while not allowing any deviations from the BAU scenario at other times. Flexibility can then be defined as the possibility to deviate the electricity consumption from the BAU consumption at a certain point in time and during a certain time span. Flexibility can be determined at the level of a DER system or at the level of a VPP (consisting of different DER systems). Furthermore, we suppose that the BAU scenario represents a control strategy that minimizes the operational cost for the system owner. For simplicity, we will consider that the operational cost is based only on electricity consumption or production and a time dependent electricity tariff. This means that all deviations from the BAU scenario will increase the operational cost. The difference in cost with the BAU scenario is the corresponding cost for offering the flexibility service.

**Approach**

The first step in the quantification process is to create an appropriate system model. This system model should be as simple as possible although still accurate in predicting the system’s behaviour.

Making use of this system model, at least three optimal control problems (OCP’s) need to be solved to determine the flexibility at a certain point in time and during a certain time span.

Firstly, the BAU OCP is solved in order to minimize total costs over a horizon of e.g. 24 hours. Thermal comfort requirements are set as inequality constraints (hard constraints). Secondly, in a similar OCP a different objective function is defined, forcing the solver to deviate from the lowest cost solution by using the available flexibility in the system in order to minimize the electricity consumption during the specified time span. The solution of this OCP will on the one hand return the amount of electricity that can be shifted out of the considered time span and on the other hand the cost that follows from this load shifting. This process is repeated with a maximization of the electricity consumption during the specified time span in order to estimate how much electricity could be consumed additionally during the time span and at which cost. These three optimizations can be used to build up a cost curve as illustrated in Figure 1. Finally (and optionally), intermediate points on the cost curve can be obtained by solving additional OCP’s that force the solution to intermediate power levels. The following paragraphs elaborate the approach in more detail.

**BAU scenario**

The aim of the BAU OCP formulation is to optimize the system operation with respect to the electricity cost. The OCP formulation, represented by equation (1), is evaluated in the optimization interval [t₀, t_end]. In this study, we consider an optimisation interval of 24 hours.
In this equation $e_l [\text{€}/J]$ is the (time dependent) electricity tariff and $P_{el} [W]$ is the power consumption. System equations and constraints $g(x)$ and $h(x)$ have to be added to formulate the full OCP. In the remainder of this study they are identical and are omitted from the equations. The only difference between the OCP’s discussed is the objective function.

The solution of this OCP is the appropriate time varying control variable $u(t)$ that minimizes the electricity cost over the next 24 hours.

**Flexibility range**

We want to know the flexibility of the system during the time span $[t_0^{\text{flex}}, t_{end}^{\text{flex}}]$. This flexibility is characterized by two values that determine the limits of the flexibility range: the maximal decrease and the maximal increase in electricity consumption compared to the BAU scenario during the investigated time span. Two OCP formulations, represented by equations (2) and (3), determine the minimal respectively maximal electricity consumption during the time span $[t_0^{\text{flex}}, t_{end}^{\text{flex}}]$ while maintaining the aim to minimize the electricity cost during the whole optimization interval $[t_0, t_{end}]$.

\[
\begin{align*}
\min_{u(t)} \left\{ \int_{t_0}^{t_{end}} (1 - K) c_{el}(t) P_{el}(t)dt + K \int_{t_0}^{t_{end}^{\text{flex}}} P_{el}(t)dt \right\} \quad (2)\\
\min_{u(t)} \left\{ \int_{t_0}^{t_{end}} (1 - K) c_{el}(t) P_{el}(t)dt - K \int_{t_0}^{t_{end}^{\text{flex}}} P_{el}(t)dt \right\} \quad (3)
\end{align*}
\]

The objective function in equations (2) and (3) represents a weighted sum of two conflicting objectives, consequently a weighting factor $K [-]$ is introduced.

From the solution of each of these three OCP formulations the value of the electricity consumption $E_{el}$ during the flexibility interval can be retrieved using equation (4).

\[
E_{el} = \int_{t_0}^{t_{end}^{\text{flex}}} P_{el}(t)dt
\]

The flexibility range is characterized by the maximal decrease $\Delta E_1 [\text{kWh}]$ and the maximal increase $\Delta E_2 [\text{kWh}]$ in electricity consumption compared to the BAU consumption in the flexibility interval as given in equations (5) and (6).

\[
\begin{align*}
\Delta E_1 &= E_{el,\text{min}} - E_{el,\text{BAU}} \leq 0 \quad (5)\\
\Delta E_2 &= E_{el,\text{max}} - E_{el,\text{BAU}} \geq 0 \quad (6)
\end{align*}
\]

By convention, with positive flexibility we designate an increase in consumption compared to BAU, negative flexibility designates a decrease.

Similarly, we can obtain the resulting cost $C_{el} [\text{€}]$ and compute the difference with the BAU scenario as shown in equations (7) – (9). The cost for using the flexibility will always be higher than the BAU cost.

\[
\begin{align*}
C_{el} &= \int_{t_0}^{t_{end}} c_{el}(t) P_{el}(t)dt \quad (7)\\
\Delta C_{\Delta E_1} &= C_{el,\text{min}} - C_{el,\text{BAU}} \quad (8)\\
\Delta C_{\Delta E_2} &= C_{el,\text{max}} - C_{el,\text{BAU}} \quad (9)
\end{align*}
\]

**Cost function**

The flexibility range can be graphically represented in a cost function, shown in Figure 1. The costs function shows the flexibility in the electricity consumption $\Delta E [\text{kWh}]$ on the horizontal axis and
the corresponding additional cost $\Delta C_{AE}$ (absolute as well as relative compared to the BAU cost) on the vertical axis. The point associated with the BAU control strategy is located in the origin. The two points $(\Delta E_1, \Delta C_{AE_1})$ and $(\Delta E_1, \Delta C_{AE_1})$ show how much the electricity consumption can decrease respectively increase compared to BAU, and at which cost.

It can be anticipated that intermediate deviations from the BAU scenario will not lie on a straight line between the origin and the extremes. Moreover, it is likely that there are consumption deviations $\Delta E_1$ or $\Delta E_1$ that cannot be reached due to discrete power levels in the control (like on/off instead of modulation). Therefore, it may be useful to investigate intermediate flexibility points inside the range of $[\Delta E_1, \Delta E_1]$. They can be identified by solving the OCP formulation represented by equation (10).

$$\min_{u(t)} \left[ (1 - K) \int_{t_0}^{t_{end}} c_{el}(t) P_{el}(t) + K \left( \int_{t_0}^{t_{end}} P_{el}(t) dt - E_{el,track}^* \right)^2 \right]$$

In this equation $E_{el,track}^*$ is the desired value of the electricity consumption. The objective function in equation (10) represents a weighted sum of two conflicting objectives. The aim of the first objective is to minimize the electricity cost during the entire optimization interval. The aim of the second objective is to force the actual electricity consumption during the flexibility interval to a desired value of the electricity consumption $E_{el,track}^*$.

To reach this second objective the square of the difference between the actual and the desired electricity consumption is used. The subscript ‘track’ denotes the results of equation (10).

**Aggregation**

Costs functions have the property that they represent the amount of flexibility and associated costs in a clear way. An additional benefit is that they can be easily aggregated to represent the flexibility of a system that is composed of different subsystems, like a VPP. As an example, the aggregation of two different cost functions is shown in Figure 2. As can be seen from the figure, the aggregation is based on a piecewise sum of line segments, sorted according to their slope. This methodology can easily be scaled up to aggregation of multiple cost curves, and in different levels. It is clear that an aggregated system can provide more flexibility than a single system. Also, by aggregating costs functions it is often possible that a certain amount of flexibility can be offered at a lower cost than any of the subsystems could offer.

**CASE STUDY**

**Models**

The cost functions for a cooled office building are computed for two different cold emission systems: concrete core activation (CCA) and a chilled ceiling (CC). In a first step, the models are implemented in Modelica. They are identical for both cases, except for the cold emission system. The building is a simple single state model with heat losses to the surroundings, internal gains according to a fixed profile and an input for the cooling power. This input is coupled to the cold emission system and is an indirect consequence of the operation of the chiller to be optimized, $u(t)$. In case of the CCA, the embedded pipe model of Koschenz and Lehmann (2000) is used in combination with a 7-state RC model of the floor. Both the upper- and lower side of the CCA are connected to the room model. In case of the CC, the embedded pipe model is connected to a single state RC model of the ceiling which is connected to the room model. Cold is produced in a modulating chiller with a fixed EER of 3.0. The chiller and the embedded pipe in the cold emission system have a dynamic pipe model representing their thermal capacity and associated thermal inertia. Both a fictive sinusoidal electricity price and ambient temperature are assumed.

The OCP’s are formulated by use of Optimica, a Modelica language extension that is embedded in JModelica (Åkesson et al. 2009). The Modelica model is converted in equality constraints for the optimal control problem by the Optimica compiler. Additional inequality constraints are added to force the room temperature $T$ to lie within the comfort boundaries $23.5 \, \text{°C} \leq T \leq 25.5 \, \text{°C}$. The OCP calculates the modulating operation of the chiller $u(t)$ in order to satisfy the comfort boundaries while minimizing the objective function. The OCP’s are solved with the collocation method and 48 collocation elements (each with a single collocation...
point) in order to obtain a discrete time step of 0.5 hour.

**Results**

To understand the system and flexibility service that can be offered by each of these cases we first analyse the BAU scenario. Figure 3 and Figure 4 show the room and cold emission temperatures and the chiller load for respectively the CCA and CC case for a period of one day. We see that the room temperature stays within the imposed boundaries and that the system tries to operate the chiller as much as possible during periods of cheap electricity. We can also clearly see the difference in thermal inertia between the two cases.

Next, we quantify the flexibility that can be offered for different time spans starting at 6h00 and with durations of 1, 2 and 4 hours. The results are shown in Figure 5.

**DISCUSSION**

**Cost curves**

From the cost curves presented in Figure 5, we can observe different trends. Firstly, both systems can offer flexibility by shifting the operation of the chiller. Depending on the BAU operation of the chiller during the investigated time span, this flexibility can be positive, negative or both. The cost of the flexibility is in the same order of magnitude for both systems. This is logical as the cases are identical, except for the cold emission system. Secondly, the flexibility range (the difference between the maximum positive and minimum negative flexibility) is limited by thermal comfort constraints in the building. For example for the longer time span of 4 hours, the chilled ceiling system is unable to reduce the electricity consumption as much as the concrete core activation because of the lower thermal inertia.

**Specific cost of flexibility and price elasticity**

For every point on the cost curve we can obtain the specific cost of the flexibility as:

$$c_{sp} = \frac{\Delta C}{|\Delta E|}$$

(11)

The specific cost $c_{sp}$ could for instance be expressed in €/kWh and can be compared to typical alternatives for the flexibility service of the DER system, like battery storage or the operation of upward reserves (in case of negative flexibility). For the studied case, the specific flexibility cost is in the range of 0.5 – 3 €/kWh.

The inverse of the specific cost $c_{sp}$ can be normalised and thus be interpreted as the marginal price elasticity $\varepsilon$ of the system. This indicator expresses the change in load that would occur following a change in electricity price. Consequently, the proposed method can be used in combination with unit commitment problem formulations for which the demand side is often modelled as a fixed profile with a certain price elasticity. Often, general assumptions about the price elasticity are made, while the sensitivity of the results to these assumptions can’t be neglected (De Jonghe 2011).

**Models**

The computation of the cost curves requires a model of the DER system. This requirement may be seen as a very severe constraint to the practical application of the proposed methodology. However, the required model may be obtained as a grey box model from a system identification procedure based on monitoring data of the considered DER system. Moreover, the same model can be used for operational purposes. For example, in the chilled ceiling case discussed above, replacing the initial rule-based control of the chiller by the cost-optimal BAU scenario results in cost savings of 24%. This is because the BAU scenario already implicitly uses the available flexibility in the system in order to take profit of the variable electricity price through the day. Finally, the same models can be used for operational purposes in a smart grid context. This will probably not be based on the proposed cost curves. Although the effect of load distribution (effect of action on time $t$ will change the load on other time steps) is taken into account in the cost, the resulting load outside the investigated time span is not visible in the cost curve. A real operational smart grid therefore needs to use other techniques in order to optimise the loads in a VPP, like dual decomposition and decentralised optimisation (McArthur et al. 2007; Kok et al. 2008; Zhou, Chan, and Chow 2009). These techniques require a local model for optimisation of the local DER system based on (fictive) price levels communicated by the aggregator. These optimisations are almost identical to the OCP’s required to obtain the cost curves in the current study.
Figure 3 – BAU scenario for the office building with concrete core activation (CCA)

Figure 4 – BAU scenario for the office building with chilled ceilings (CC)

Figure 5 – Cost curves for concrete core activation (CCA) and chilled ceiling (CC) case study, starting at 6h00 and for three different time spans (Period).
CONCLUSION

This paper presents a methodology to assess the amount of flexibility and the corresponding cost for a DER system. The method is applied to a case study of office cooling with concrete core activation and chilled ceilings.

Flexibility is defined as the possibility to deviate the electricity consumption from the business as usual (BAU) consumption at a certain point in time and during a certain time span. The BAU scenario corresponds to a control strategy that minimizes the electricity cost.

The method is based on the solution of at least three optimal control problems (OCP’s) with an appropriate system model. This results in two values characterizing the flexibility range of the DER system, i.e. the maximal decrease $\Delta E_1$ and the maximal increase $\Delta E_2$ in electricity consumption from the BAU consumption in the flexibility interval. Simultaneously, the corresponding cost of the load shifting is obtained. The results are represented by a costs function.

Aggregation of the flexibility of different DER systems that are grouped by an aggregator (e.g. in a VPP), results in the total flexibility and cost for the aggregator at a certain point in time and during a certain time span. This aggregation has an important advantage; a VPP can provide more flexibility than a single DER system, at a lower or equal cost.

The cost curves can be used to assess how much load shifting really costs. This enables comparing DSM solutions with other technologies like electrical energy storage or peak power generation. From the specific cost, an equivalent price elasticity can be derived which can be used as input in unit commitment problems. However, as Bruninx et al. (2012) show, price elasticity may not be the right concept for quantifying flexibility in thermal systems, so this approach has to be used with care. The method can also be used during the design of e.g. a building in order to take into account possible flexibility services that the building could offer on the energy markets. For operational decision-making, the cost curves are not sufficient, but the underlying models will serve in a multi-agent framework.

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