DEVELOPMENT OF A CONTROL-ORIENTED MODEL FOR BOREHOLE DYNAMICS FOR BUILDINGS EQUIPPED WITH GROUND COUPLED HEAT PUMPS

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ABSTRACT
This paper proposes to apply the celebrated Proper Orthogonal Decomposition (POD) approach in order to obtain a short-term control model for the temperature dynamics of the borehole system. The main idea behind POD-based modeling is the extraction of a set of optimal basis functions containing the main characteristics of the system from snapshots of the system dynamics at various time instants. This way, POD allows an accurate description of the heat diffusion process where the POD modes can be used as states of the control model. The obtained control model is used for control of heat extraction and injection.

INTRODUCTION
Energy prices resulting from increasing energy demand and decelerating supply has prompted substantial interest in clean and renewable energy in the last decades. Based on the fact that buildings account for approximately 40% of total energy use in Europe (EPBD, 2010) and similar percentages in the rest of the world, development of energy efficient buildings, heating and cooling technologies together with appropriate control strategies are required.

Ground coupled heat pump (GCHP) systems combined with low-exergy heat emission systems have the potential to reduce the primary energy use related to space heating and cooling by 70% compared to conventional heating and cooling systems. For GCHP systems with vertical borehole heat exchangers (BHE), however, the large investment cost of the borefield represents a major bottleneck. This explains the trend towards compact, hybrid GCHP systems which combine smaller boreholes with supplementary heating or cooling devices (Cullin, 2010). The control of such compact, hybrid GCHP systems constitutes an important and challenging problem both for academic and industrial communities (Scott and Amanda, 2011; Ridder et al., 2011).

Recent studies indicate the influence of the short-time response (Partenay et al., 2011; Yavuzturk and Spitler, 1999) and the control strategies used (Verhelst, 2012) on GCHP performances. For example, in hybrid GCHP systems when maximizing the share of the heat pump or passive cooling covering the heating and cooling loads, the fluid temperature should be respectively at the lower and upper optimal bound. Current rule-based control strategies fail to get optimal system operation within (but close to) the physical constraints. When operating near the temperature constraints, they often result in an on-off cycling, which is detrimental for both the system energy performance and installation life-time. To track a fluid temperature setpoint without cycling, a closed-loop control strategy like Linear Quadratic Regulator (LQR) is appropriate. LQR requires a dynamic model of the system to be controlled. In this case, we need a BHE model to predict the response of the circulating fluid temperature to the injected or extracted heat power.

Most existing BHE models are however intended for simulation purpose and are far too complex to be used in a control framework where a low-order linear state-space model form is desired (Kim et al., 2011). The first borehole control model, developed by Franke (Franke, 1998), was obtained by model reduction of the detailed DST-model in TRNSYS. Verhelst and Helsen (Verhelst and Helsen, 2011) evaluated two comparable methods: first, model reduction of a very simple first principle numerical model—which can be set up by hand and, second, parameter estimation of a resistance-capacitance model using TRNSYS simulation data. Monteyne et al. (Monteyne et al., 2011) evaluated the use of rational transfer functions in the variable \( \sqrt{s} \) to describe the diffusion process, with the model parameters being identified from measurement data. These studies, dealing with the setup of a model predictive control (MPC) framework, aimed at obtaining a BHE model capturing both short and long term BHE dynamics. As a result, there is a tradeoff in these models in terms of accuracy when compared to a stand-alone short-term model or a stand-alone long-term model. Here, the focus is on modeling and control of the short-term BHE dynamics.

In this paper, we propose the Proper Orthogonal Decomposition (POD) method to obtain a reduced-order control model for short term LQR-based circulating fluid temperature control in a borehole. Compared to the standard model reduction techniques, POD allows us to focus on the fast dynamics in which we are interested. Compared to system identification, POD is a computationally less complex and physically more elegant technique, at least if a detailed numerical model is available. The current study can be extended to the more complex problem of borefield control with hybrid GHP systems using more sophisticated control algorithms such as MPC.
TEMPERATURE DYNAMICS MODELING

The borehole filled with grout is shown in Figure 1 where an equivalent diameter approach is used. In the equivalent diameter approach the heat transfer from the U-tube is approximated by the heat transfer from a pipe with a hypothetical diameter through which the heat exchanging fluid circulates (Chiasson, 2007; Javed and Claesson, 2011). The system variables are listed in Table 1 at the end of the paper. The objective is the determination of the circulating fluid mean-value temperature over borehole length, \( T_f(t) \), corresponding to an extracted/rejected heat per length, \( q(t) \), and then the reverse problem of its control.

For both the grout and the soil region the temperature distribution is modeled by

\[
\frac{1}{\alpha(r)} \frac{\partial T}{\partial t} = \frac{1}{\partial r} \left( r \frac{\partial T}{\partial r} \right),
\]

where \( \alpha_g \) and \( \alpha_s \) are grout and soil diffusivity, respectively. Assuming that the undisturbed initial ground temperature was \( T(r, 0) = T_0 \), we can write \( \lim_{r \to r_s} T(r, t) = T_0 \), \( \forall t \). However, assuming that at a sufficiently large radius \( r = r_s \) the effect of heat diffusion is negligible, we can approximate the above condition at infinity as

\[
\left. \frac{\partial T(r, t)}{\partial r} \right|_{r_s} = 0, \forall t.
\]

The next step is to relate the fluid temperature \( T_f(t) \) to the grout temperature \( T_{fg}(t) \) at \( r = r_{fg} \). Assume that a thermal resistance \( R_{fg} \) exists over the pipe periphery between the fluid in the pipe and the near outside grout. This thermal resistance takes into account the conduction resistance through the pipe and the fluid convective resistance (Chiasson, 2007; Javed and Claesson, 2011):

\[
R_{fg} = \frac{1}{2\pi k_p} \ln \left( \frac{r_{pi}}{r_{po}} \right) + \frac{1}{2\pi r_{fg} h_f},
\]

where \( k_p \) is the pipe thermal conductivity, \( r_{pi}, r_{po} \) are the inner and outer pipe radiuses, respectively, and \( h_f \), is the fluid convective heat transfer coefficient. For short-time response analysis and considering for example variable flow rates, the value of \( h_f \) can change significantly between laminar and turbulent states but its effect on the \( T_f \) is small. The boundary condition between the circulating fluid and the grout is

\[
T_f(t) - T_{fg}(t) = R_{fg}q_{fg},
\]

where \( q_{fg} \) is the heat transfer at fluid-grout boundary and \( R_{fg} \) is the corresponding thermal resistance. Finally, the energy balance in the pipe results in

\[
q = \pi r_p^2 \rho_f c_f \frac{dT_f}{dt} + q_{fg},
\]

where \( c_f \) is the fluid specific heat and \( \rho_f \) is its density. Equation (4) is derived by assuming the fluid to be incompressible so that \( \frac{du}{dT} = c_f \) and using the chain rule

\[
\frac{du}{dt} = \frac{du}{dT} \frac{dT}{dt} = c_f \frac{dT_f}{dt},
\]

where \( u \) denotes the specific internal energy. Next, we discretize Equation (1) using \( n_r \) space nodes for the grout region and \( n_s \) nodes for the soil region as shown in Figure 2(a). Integration of Equation (1) over a control volume in Figure 2(c) and over a time step of size \( \Delta t \) gives

\[
\int_t^{t+\Delta t} \left( \int_{\Delta V} \frac{1}{\alpha(r)} \frac{\partial T}{\partial t} \, dV \right) \, dt = \int_t^{t+\Delta t} \left( \int_{\Delta V} \frac{1}{\alpha(r)} \, \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \, dV \right) \, dt. \tag{5}
\]

Next, we will calculate the terms \( S_1 \) and \( S_2 \), starting with \( S_1 \). Changing the order of integration and using backward difference in time derivative approximation we obtain

\[
S_1 = \int_{\Delta V} \left( \int_t^{t+\Delta t} \frac{1}{\alpha(r)} \, \frac{\partial T}{\partial t} \, dt \right) \, dV = \int_{\Delta V} \frac{1}{\alpha(r)} \, (T_P - T_{po}) \, dV, \tag{6}
\]

where \( T_P \) denotes the temperature at time \( t + \Delta t \) and \( T_{po} \) at time \( t \). Using \( dV = 2\pi Hrdr \), Equation (6) becomes

\[
S_1 = \frac{\pi H r_o^2}{\alpha_e} (T_P - T_{po}) - \frac{\pi H r_o^2}{\alpha_w} (T_P - T_{po}), \tag{7}
\]
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\[ S_2 = 2\pi H \int_0^{\Delta t} \left( \frac{\partial}{\partial r} \left( \frac{r}{\partial T} \right) \right) dr \]

For any internal node in Figure 2(b), we have the Equations (9) and (10) repeated below:

\[ a_P T_P = a_W T_W + a_E T_E + a_P^o T_P^o, \]

\[ a_E = \frac{2\Delta t}{\Delta r} r_e, \quad a_P^o = \frac{r_e^2}{\alpha_e}, \quad a_A = \frac{4\Delta t}{\Delta r} r_A. \]

For the last node with east face at \( r_B \) in Figure 2(b), we have

\[ S_1 = 2\pi H \int_0^{\Delta t} \frac{1}{\alpha(r)} \left( T_P - T_P^o \right) r dr = \]

\[ \frac{\pi H}{\alpha_e} r_e^2 (T_P - T_P^o) - \frac{\pi H}{\alpha_A} r_A^2 (T_P - T_P^o). \]
Next, we obtain $S_2$ as

$$S_2 = 2\pi H \int_{t}^{t+\Delta t} \left( \int_{w}^{B} \frac{\partial}{\partial r} \left( \frac{\partial T}{\partial r} \right) \ dr \right) \ dt$$

$$= 2\pi H \int_{t}^{t+\Delta t} \left[ \left( \frac{\partial T}{\partial r} \right)_B - \left( \frac{\partial T}{\partial r} \right)_w \right] \ dt$$

$$= 2\pi H \int_{t}^{t+\Delta t} \left[ \left( \frac{\partial T}{\partial r} \right)_B - \frac{r_w T_{w} - T_{W}}{\Delta r} \right] \ dt$$

$$= \frac{4\pi H \Delta T}{\Delta r} r_B(T_B - T_P) - \frac{2\pi H \Delta T}{\Delta r} w(T_P - T_W).$$

Setting $S_1 = S_2$, we get

$$a_p T_P = a_w T_W + a_E T_E + a_{p0} T_{p0} + a_B T_B,$$  

where

$$a_p = \frac{r_B^2}{\alpha_B} - \frac{r_w^2}{\alpha_w} + \frac{4\Delta T}{\Delta r} r_B, \quad a_E = 0, \quad a_{p0} = \frac{r_B^2}{\alpha_B} - \frac{r_w^2}{\alpha_w} = \frac{4\Delta T}{\Delta r} r_B.$$  

**Case 2:** $T(r_A) = T_{A0}, q(r_B) = 0$

The equations for the first node and for the internal nodes are the same as Equations (11)-(12) and (13)-(14), respectively. For the node with east face at $r_B$, we have

$$S_1 = 2\pi H \int_{w}^{B} \frac{1}{\alpha(r)} (T_P - T_{P0}) \ dr$$

$$= \frac{\pi H}{\alpha_B} r_B^2 (T_P - T_{P0}) - \frac{\pi H}{\alpha_w} r_w^2 (T_P - T_{P0}).$$

Next, we obtain $S_2$ as

$$S_2 = 2\pi H \int_{t}^{t+\Delta t} \left( \int_{w}^{B} \frac{\partial}{\partial r} \left( \frac{\partial T}{\partial r} \right) \ dr \right) \ dt$$

$$= 2\pi H \int_{t}^{t+\Delta t} \left[ \left( \frac{\partial T}{\partial r} \right)_B - \left( \frac{\partial T}{\partial r} \right)_w \right] \ dt$$

$$= 2\pi H \int_{t}^{t+\Delta t} \left[ 0 - \frac{r_w T_{w} - T_{W}}{\Delta r} \right] \ dt$$

$$= - \frac{2\pi H \Delta T}{\Delta r} w(T_P - T_W).$$

Setting $S_1 = S_2$, we get

$$a_p T_P = a_w T_W + a_E T_E + a_{p0} T_{p0},$$  

where

$$a_p = \frac{r_B^2}{\alpha_B} - \frac{r_w^2}{\alpha_w} + \frac{2\Delta T}{\Delta r} r_B, \quad a_E = 0, \quad a_{p0} = \frac{r_B^2}{\alpha_B} - \frac{r_w^2}{\alpha_w}.$$  

**Temperature dynamics for combined grout-soil region**

Using Equations (11)-(12), (13)-(14) and (15)-(16) in Section with $\alpha_e = \alpha_A = \alpha_w = \alpha_B = \alpha_g, r_A = r_g = r_B = r_{gs}, T_A = T_{fg}, r_B = r_{gs}, T_B = T_{gs}, \Delta r = (\Delta r)_g,$ we get

$$x_g(k+1) = A_g x_g(k) + B_g (T_{fg}(k)),$$  

(19)

where $x_g \in [T_{g1}, \ldots, T_{gs}]^T$. In the same way, using Equations (11)-(12), (13)-(14) and (17)-(18) in Section with $\alpha_e = \alpha_A = \alpha_w = \alpha_B = \alpha_s, r_A = r_{gs}, T_A = T_{gs}, r_B = r_s, q_B = 0, \Delta r = (\Delta r)_s,$ we get

$$x_s(k+1) = A_s x_s(k) + B_s T_{gs}(k),$$  

(20)

where $x_s \in [T_{s1}, \ldots, T_{sns}]^T$. From the continuity of heat flux at $r = r_g$, we have

$$-\frac{k_g x_g}{(\Delta r)_g/2} = -\frac{k_g x_s}{(\Delta r)_s/2},$$

which when combined with $T_{gs} = C_{gs0} x_g$ and $T_{s1} = C_{s10} x_s$, where $C_{gs0} = [0 \ 0 \ 1 \ 1 \ 0 \ 0]^T \in \mathbb{R}^n_s, C_{s1} = [1 \ 0 \ 0 \ 0 \ 0 \ 0]^T \in \mathbb{R}^n_s,$ gives

$$T_{gs} = \frac{k_g (\Delta r)_g C_{gs0} x_g + k_s (\Delta r)_s C_{s10} x_s}{k_g (\Delta r)_g + k_s (\Delta r)_s}.$$  

(21)

Using Equation (21) in Equations (19) and (20), partitioning $B_g$ as $B_g = [B_g^1 \ B_g^2]$ and defining $x \in [x_g \ x_s]^T$, we obtain the combined grout-soil temperature dynamics as in (22) at the top of page 5.

**Mean-value temperature prediction of the circulating fluid**

In addition to Equation (3), we can write another expression for $q_{fg}$ as

$$q_{fg} = -\frac{1}{\Delta T} \frac{A dT}{dr} \bigg|_{r=r_{fg}} = -4\pi k_g r_{fg} \frac{T_{gs} - T_{fg}}{(\Delta r)_g}.$$  

(23)

Using Equations (3) and (23), we obtain

$$T_{fg} = \frac{1}{1 + k_{fg}} T_f + \frac{k_{fg}}{1 + k_{fg}} T_{gi}$$

$$= \frac{1}{1 + k_{fg}} T_f + \frac{k_{fg}}{1 + k_{fg}} C_{g1} x,$$  

(24)

where the temperature at the first grout node, $T_{gi}$, is expressed as $T_{gi} = C_{g1} x$ with $C_{g1} = [1 \ 0 \ 0 \ 0 \ 0 \ 0]^T \in \mathbb{R}^{n_s+n_s}$ and

$$k_{fg} = 4\pi k_g R_{fg} r_{fg}.$$

Using Equation (24), Equation (22) becomes

$$x(k+1) = \left[ A + \frac{k_{fg}}{1 + k_{fg}} BC_{g1} \right] x(k) + \frac{1}{1 + k_{fg}} BT_f.$$  

(25)

Now, using Equation (24) in Equation (3) we get

$$q_{fg} = \frac{k_{fg}}{1 + k_{fg}} \left( T_f - C_{g1} x \right).$$  

(26)
Using expression Equation (26) and discretizing (4) (with the same time discretization step size \( \Delta t \)) we get
\[
T_f(k+1) = \left[ 1 - \frac{\Delta t k_f g}{(1 + k_f g) R_f g (2\pi r_p^2 \rho_f c_f)} \right] T_f(k) + \frac{\Delta t k_f g}{(1 + k_f g) R_f g (2\pi r_p^2 \rho_f c_f)} C_{g1} x + \frac{\Delta t}{2\pi r_p^2 \rho_f c_f} u, \tag{27}
\]
where \( u = q \). Finally, letting \( x_{sys} = [T_f \ x]^T \), we can write the combined system of Equation (27) and (25) as in Equation (28) at the top of page 5.

**POD-BASED MODEL REDUCTION**

In this section we will give a summary of the model reduction problem by POD and its solution. The interested reader is referred to (Holmes et al., 1996) for details. Let \( \mathcal{H} \) be an \( N \)-dimensional Hilbert space with an orthonormal basis \( \{ f_i \}_{i \in I} \) where \( I = \{1, 2, \ldots, N\} \). Let \( \mathcal{H}_1 \) and \( \mathcal{H}_2 \) be a complementary orthogonal decomposition of \( \mathcal{H} \) with the corresponding basis functions \( \{ f_i \}_{i \in I_1} \) and \( \{ f_i \}_{i \in I_2} \), respectively. Any element \( f \in \mathcal{H} \) can be written as
\[
f = \sum_{i \in I_1} a_i f_i + \sum_{i \in I_2} a_i f_i = f_1 + f_2, \tag{29}
\]
where the coefficients \( a_i = \langle f, f_i \rangle \) are the Fourier coefficients. This decomposition has the following properties:
- \( ||f - f_1||^2 = ||f_2||^2 = \sum_{i \in I_2} a_i^2 \),
- \( ||f - f_2||^2 = ||f_1||^2 = \sum_{i \in I_1} a_i^2 \).

We define the misfit function as
\[
\mathcal{C}_{s0}(f_1) = \frac{1}{2} \left( ||f - f_1||^2 + ||f_2||^2 \right),
\]
where \( ||\cdot|| \) is the inner product-induced norm. The geometric meaning of the last two properties is that \( f_1 \) is the best approximation of \( f \) in \( \mathcal{H}_1 \) and \( f_2 \) is that of \( f \) in \( \mathcal{H}_2 \). The objective is the “optimal” projection of \( f \) onto \( \mathcal{H}_1 \). For that, first an order complexity function \( c_0 : \mathcal{H} \rightarrow \mathbb{Z}^+ \) is defined as the number of nonzero Fourier coefficients in Equation (29). Next, we define the misfit function \( d_{mf} : \mathcal{H} \times \mathcal{H}_1 \rightarrow \mathbb{R}^+ \) as
\[
d_{mf}(f, f_1) \triangleq ||f - f_1||^2, \tag{29}
\]
Then, the optimal projection problem can be stated as follows: given \( f \in \mathcal{H} \) of complexity order \( c_0(f) \), find \( f_1 \in \mathcal{H}_1 \) with complexity order \( c_0(f_1) \leq c_0(f) \) such that \( d_{mf}(f, f_1) \) is minimal. The solution of this problem is as follows. If the Fourier coefficients \( \{a_1, a_2, \ldots, a_N\} \) are ordered as
\[
a_1^2 \geq a_2^2 \geq \cdots \geq a_{c_0(f_1)}^2 \geq \cdots \geq a_N^2,
\]
then the optimal projection \( f_1 \) onto \( \mathcal{H}_1 \) is given as
\[
f_1 = \sum_{j=1}^{c_0(f_1)} a_j f_j, \tag{30}
\]
where hat denotes the projection and
\[
d_{mf}(f, f_1) = \sum_{j=c_0(f_1)+1}^{N} a_j^2.
\]

So far we considered the optimal projection of a single vector \( f \in \mathcal{H} \) onto \( \mathcal{H}_1 \). Next, we will consider the extended problem of optimal projection of a set of
vectors onto \( \mathcal{H}_1 \). To deal with this kind of problem, let
\[
F_{\text{snap}} = \begin{bmatrix} f(t_1) & f(t_2) & \cdots & f(t_n) \end{bmatrix},
\]
where \( f(t_i) \in \mathcal{H}, t_i \in \mathbb{T}, i = 1 : n \) and \( \mathbb{T} \) is a time interval. For each of \( f(t_i) \), we can write
\[
f(t_i) = \sum_{j=1}^{N} a_j(t_i)f_j,
\]
where \( a_j(t_i) = \langle f(t_i), f_j \rangle \). \( F_{\text{snap}} \) is a snapshot matrix whose columns are values of \( f \) at a sequence of time points. The complexity function in this extended case is defined as
\[
c_0(F_{\text{snap}}) = \max_{t_i, t_j} \left\{ a_j(t_i) \neq 0; 1 \leq j \leq N \right\}.
\]
The problem of optimal projection of \( F_{\text{snap}} \) onto \( \mathcal{H}_1 \) can be stated as follows: given \( F_{\text{snap}} \) of complexity order \( c_0(F_{\text{snap}}) \), find \( F_{\text{snap}}^1 \) with complexity order \( c_0(F_{\text{snap}}^1) \leq c_0(F_{\text{snap}}) \) such that \( d_{\text{off}}(F_{\text{snap}}, F_{\text{snap}}^1) \) is minimal. The solution of this problem is as follows. If the Fourier coefficients \( \{a_j(t)\} \) are averaged over time sequences \( t_i \) and the averages are ordered as
\[
a_1^2 \geq a_2^2 \geq \cdots \geq a_{c_0(F_{\text{snap}})}^2 \geq \cdots \geq a_{N}^2,
\]
where the bar denotes the average, then the optimal projection of \( F_{\text{snap}} \) onto \( \mathcal{H}_1 \) is given as
\[
\hat{F}_{\text{snap}}(t_i) = \sum_{j=1}^{c_0(F_{\text{snap}}^1)} a_{i,j} f_j, \quad 1 \leq i \leq n,
\]
where
\[
d_{\text{off}}(F_{\text{snap}}, F_{\text{snap}}^1) = \sum_{j=c_0(F_{\text{snap}})+1}^{N} a_j^2.
\]
From the results up to this point we see that the optimal projection problems are solved by ordering the Fourier coefficients or their averages. However, Fourier coefficients depend on the chosen basis functions. Therefore, the next problem is the determination of the optimal orthonormal basis (optimal in the sense of minimizing the misfit between \( F_{\text{snap}} \) and \( F_{\text{snap}}^1 \) among all other candidate bases) \( \{f_j\} \) from a snapshot matrix \( F_{\text{snap}} \) and then order the averages of Fourier coefficients to find the optimal projection for a given order. The solution steps of this optimal basis problem are as follows (See for example, (Holmes et al., 1996)):}

**step 1:** Construct the snapshot matrix from observations at a sequence of time points:
\[
F_{\text{snap}} = \begin{bmatrix} f(t_1) & f(t_2) & \cdots & f(t_n) \end{bmatrix}.
\]

**step 2:** From \( F_{\text{snap}} \) construct the correlation matrix \( C_{\text{cor}} \):
\[
C_{\text{cor}} = \frac{1}{N} F_{\text{snap}} F_{\text{snap}}^T.
\]

**step 3:** Solve the eigen-value problem
\[
C_{\text{cor}} \Phi = \Phi \Lambda,
\]
where \( \Lambda = \text{diag}\left( \lambda_1, \lambda_2, \ldots, \lambda_N \right) \geq 0 \) and \( \Phi \) is an orthogonal matrix.

**step 4:** Let \( \phi_i \) be the \( i \)-th column of \( \Phi, i \in \mathbb{I} \). Then, \( \{\phi_i\}_{i \in \mathbb{I}} \) is the optimal basis, which is called the POD basis in the literature.

**Reduced-order Modeling of Borehole Dynamics Using POD Basis**

Letting \( \mathcal{H} \) be the \( n_g + n_s + 1 \)-dimensional Euclidean space \( \mathbb{R}^{n_g + n_s + 1} \) containing \( x_{\text{sys}}(t_k) \), we will determine a POD basis from a set of values of \( x_{\text{sys}} \) and then use it to obtain a reduced-order model for the typically large-scale system in Equation (28). The associated snapshot matrix is \( F_{\text{snap}} = \begin{bmatrix} x_{\text{sys}}(t_1) & x_{\text{sys}}(t_2) & \cdots & x_{\text{sys}}(t_n) \end{bmatrix} \). Next, \( N \) eigen-values are ordered and then the following sequence is constructed:
\[
P_r = \frac{\sum_{i=1}^{r} \lambda_i}{\sum_{i=1}^{N} \lambda_i}, \quad r = 1, \ldots, N.
\]

Since eigen-values are positive, we have \( 0 < P_1 \leq P_2 \leq \cdots \leq P_N = 1 \). The truncation degree \( r \) is chosen based on the value of \( P_r \) and it should be large enough to capture the system dynamics. Typically \( r \) is chosen such that \( P_r \geq 0.99 \) (Holmes et al., 1996).

The reduced-order model is obtained using the state transformation \( x_{\text{sys}} = \Phi_r x_{\text{sys}} \), where \( r \) is the truncation order in Equation (33), \( \Phi_r \in \mathbb{R}^{(n_g+n_s+1) \times r} \) represents the first \( r \) columns of \( \Phi \) and \( x_{\text{sys}} \) is the reduced-order system state vector. The reduced-order system is given as
\[
x_{\text{sys}}(k + 1) = A_{\text{sys}} x_{\text{sys}}(k) + B_{\text{sys}} w(k), \quad (34a)
\]
\[
y = C_{\text{sys}} x_{\text{sys}}(k) \quad (34b)
\]

where \( A_{\text{sys}} = (\Phi_r^T \Phi_r)^{-1} \Phi_r^T A_{\text{sys}} \Phi_r \), \( B_{\text{sys}} = (\Phi_r^T \Phi_r)^{-1} \Phi_r^T B_{\text{sys}} \) and \( C_{\text{sys}} = \Phi_r C_{\text{sys}} \).

**Reduced-order Model Validation**

Next, we apply the POD approach to obtain a reduced-order model for the overall system presented by Equation (28) and then show its validation. To that end, we divide the grout region into 20 nodes and the soil region into 500 nodes. Other assumed parameter or
property values are given in Table 1. The time discretization step size is 1 second. The system in Equation (28) is excited by a multi-sine random power input in the range of \([-500, 500]\) W/m and the simulation period is 200 hours. Snapshots are created at every 500 seconds. The initial temperature for the field is assumed to be 10°C. The eigen-value plot of $C_{cor}$ is given in Figure 3 and based on it the selected order of the reduced-order model is 5.

For validation another multi-sine random power input in the range of \([-500, 500]\) W/m is created and the circulating fluid mean-value temperatures are compared in Figure 4. As seen, the performance of the reduced-order model is very satisfactory. The thermal property values and the pipe size-related parameter values in Table 1 were taken from (7) where an equivalent analytical form of the large-scale model given by Equation (28) was derived in the Laplace-domain and experimentally validated.

**CONCLUSIONS AND FUTURE WORK**

In this paper, a reduced-order control model for short-term prediction of circulating-fluid mean-value temperature in borehole heat exchangers was derived using a POD model order reduction technique. The main advantage of POD is that it is derived based on the governing mathematical equation of the system dynamics instead of using an input-output framework. As a result, it is possible to obtain a very low-order model with a good performance, as illustrated here. The borehole system was decomposed into 521 nodes or states where the first state corresponded to circulating fluid mean-value temperature. Hence, the order of the large-scale system describing the heat transfer dynamics was 521. The reduced-order model was fifth order, giving almost the same performance as the original one. Next, the reduced-order model was used to control the circulating fluid mean-value temperature set point values for $T_f$, respectively. The circulating fluid mean-value temperature set point is 3°C for the heat extraction mode and 15°C for heat rejection mode. The initial value of the circulating fluid mean-value temperature is taken equal to the undisturbed ground temperature, 10°C. The high performance of controller is observed from the figures.
by taking the snapshots over the corresponding time-periods but it is computationally efficient mainly for short-term periods since the simulation time step-size is 1 second. Therefore, it was used for mean-value temperature control of circulating fluid in short-term periods. The future extensions of the paper will be in the following directions:

- the large-scale model including both short-and long-term process dynamics will be obtained with a less number of nodes by considering a nonuniform grid size where the grid size increases from the grout region towards the far away soil region.
- the heat transfer in the vertical direction will be taken into account to obtain a 2-D large-scale model and the corresponding reduced-order model to be used for control purposes.
- by considering a set of borehole lengths, first a corresponding set of reduced-order 2-D borehole models and then a single model parameterized in terms of borehole length will be obtained from these set of reduced-order models. This parameterized model can be used to determine the optimal borehole length for a given purpose.
- the above concepts will be extended to borefield and to hybrid GHPs.

NOMENCLATURE

Table 1: Borehole system variables

<table>
<thead>
<tr>
<th>Var.</th>
<th>Descrip.</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>borehole length (m)</td>
<td></td>
</tr>
<tr>
<td>ρg</td>
<td>grout density (kg/m³)</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>specific heat (J/kgK)</td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>conductivity (W/mK)</td>
<td></td>
</tr>
<tr>
<td>αg</td>
<td>grout diffus. (m²/s)</td>
<td></td>
</tr>
<tr>
<td>αs</td>
<td>soil diffus. (m²/s)</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>heat transfer in vertical direction</td>
<td></td>
</tr>
<tr>
<td>n_grt</td>
<td>of grout (soil) nodes</td>
<td>20 (500)</td>
</tr>
<tr>
<td>(Δr)_g</td>
<td>discr. step size (grout) (m)</td>
<td>0.014</td>
</tr>
<tr>
<td>(Δr)_s</td>
<td>discr. step size (soil) (m)</td>
<td>0.019</td>
</tr>
<tr>
<td>Δh</td>
<td>time discr. step size (sec.)</td>
<td>1</td>
</tr>
<tr>
<td>c_f</td>
<td>fluid specific heat (J/kgK)</td>
<td>4182</td>
</tr>
<tr>
<td>h_f</td>
<td>fluid conv. coeff. (W/(m²K))</td>
<td>1700</td>
</tr>
<tr>
<td>ρ_f</td>
<td>fluid density (kg/(m³))</td>
<td>1000</td>
</tr>
</tbody>
</table>

REFERENCES


