ABSTRACT

A mathematical model to simulate an unsaturated highly capacitive porous medium on roof (sand) is considered in order to predict its effects on building passive cooling. The present methodology is based on the theory of Philip and De Vries, using the thermophysical properties for different types of porous medium (building envelope) with different chemical composition and porous size distribution. The governing equations were discretized using the finite-volume method for describing the physical phenomena of heat and mass transfer in unsaturated porous medium. The robust MultiTriDiagonal-Matrix Algorithm (MTDMA) was used to solve this strongly-coupled problem, allowing the use of large time steps for long-term simulations. In order to precisely predict room air temperature and humidity, a lumped transient approach for a building room is considered. Comparisons of the sand layer effect in terms of heat fluxes through a horizontal roof, indoor conditions and thermal comfort (PMV – Predicted Mean Vote) are presented.

INTRODUCTION

During the last decades several computational codes for building simulation have been developed for different purposes. Most building simulation models address a specific application area of the building physics field, as for example, prediction of indoor climate in association with heating and cooling loads, prediction of solar gains through windows and other passive solar components, air infiltration phenomena, HVAC system behavior, air flow patterns and ventilation efficiency, among others. However, simplifications on the building envelope model may provide unrealistic results. Combined analysis of heat and moisture transport through porous building elements is barely explored in the literature due to many difficulties such as modeling complexity, computer run time, numerical convergence and highly moisture-dependent properties. Besides, the moisture in the envelope of buildings implies an additional mechanism of transport absorbing or releasing latent heat of vaporization, affecting the hygrothermal building performance or causing mold growth (Santos and Mendes, 2009a, 2009b, 2009c). In this context, taking into account the energy savings and the thermal comfort benefits, a layer of moist porous medium, such as humid sand layer on the roof can counterbalance the heating of fervent solar radiation and outdoor air temperature, which can produce passive cooling effect, mainly for low-rise buildings with large roof area (Meng and Hu, 2005). In the same context, Liu et al. (1995) showed that the free evaporative cooling in or on an unsaturated porous packed bed that works as a part of roofs or walls in buildings is a highly promising project due to its engineering background and theoretical interests.

Green roof technology could be used to reduce the urban heat island effect and to reduce the cooling loads (Onmura et al., 2000; Sailor, 2008), however, some disadvantages have been presented. According to Wanphen and Nagano (2009), green roof installation cost is expensive and the maintenance of the system is required regularly at additional cost. Furthermore, in terms of building structure, an enhanced structural support is required, to ensure sufficient capacity to resist weight load under the soil and vegetation.

In this way, to analyse more precisely the porous media layer effects in the roof, a mathematical description for predicting envelope building hygrothermal dynamics is required and complex, due to non-linearities and interdependence among several variables. The parametric uncertainties in the modeling, simulation time steps, external climate, building schedules, hygrothermal properties and moisture storage and transport also contribute to increase this complexity. In this case, for ensuring numerical stability in the present model, the linearized set of equations was obtained by using the finite-volume method and the MultiTriDiagonal-Matrix Algorithm (Mendes et al., 2002) to solve a 1-D model to describe the physical phenomena of heat and mass transfer in the building envelope, including the sand layer. The code has been conceived to be numerically robust with a fast-simulating procedure.

The heat and moisture transfer in envelope building was based on the theory of Philip and De Vries (1957), which is one of the most disseminated and
accepted mathematical formulation for studying heat and moisture transfer through porous media, considering both vapor diffusion and capillary migration. In this paper, the weather data of Brasilia-Brazil were utilized for computing external boundary conditions and comparisons in terms of heat fluxes through a horizontal roof, indoor conditions and thermal comfort (PMV – Predicted Mean Vote) are presented.

MATHEMATICAL MODEL

The physical problem is divided into two domains: building envelope (walls and roof) and room air. At the external surfaces, the heat transfer due to short-wave radiation and heat and mass convection were considered as boundary conditions and the long-wave radiation losses were taken into account only at the roof. At the internal surfaces, besides the convection heat and mass transfer, long-wave radiation heat exchange between the surfaces was also considered.

Building Envelope Domain

The governing equations, based on the theory of Philip and De Vries (1957), to model heat and mass transfer through porous media, are given by Eqs. (1) and (2). The energy conservation equation is written in the form

\[ \rho_0 c_m (T, \theta) \frac{\partial T}{\partial t} = \nabla \cdot \left( \lambda(T, \theta) \nabla T \right) - L(T) \nabla j_v, \]

and the mass conservation equation as

\[ \frac{\partial \theta}{\partial t} = -\nabla \cdot \left( \frac{j}{\rho_i} \right), \]

where \( \rho_0 \) is the solid matrix density (kg/m³), \( c_m \), the mean specific heat (J/kg K), \( T \), temperature (°C), \( \lambda \), thermal conductivity (W/m K), \( L \), latent heat of vaporization (J/kg), \( \theta \), volumetric moisture content (m³/m³), \( j_v \), vapor flow (kg/m² s), \( j \) total flow (kg/m² s) and \( \rho_i \) the water density (kg/m³).

The total 1-D moisture flow (\( j \)) - given by summing the vapor flow (\( j_v \)) and the liquid flow (\( j_l \)) - can be described as

\[ j = \left( D_{jl} (T, \theta) \frac{\partial T}{\partial x} + D_{jl} (T, \theta) \frac{\partial \theta}{\partial x} \right), \]

with \( D_{jl} = D_{jT} + D_{j\theta} \) and \( D_{jl} = D_{jl} + D_{j\theta} \). where \( D_{jl} \) is the liquid phase transport coefficient associated to a temperature gradient (m²/s K), \( D_{jT} \), vapor phase transport coefficient associated to a temperature gradient (m²/s K), \( D_{j\theta} \), liquid phase transport coefficient associated to a moisture content gradient (m²/s), \( D_{jl} \). vapor phase transport coefficient associated to a moisture content gradient (m³/s). \( D_T \), mass transport coefficient associated to a temperature gradient (m²/s K) and \( D_{\theta} \), mass transport coefficient associated to a moisture content gradient (m³/s).

The internal boundary conditions (\( x=0 \)) for temperature can be mathematically expressed as:

\[ \left( \lambda(T, \theta) \frac{\partial T}{\partial x} \right)_{x=0} + \left(L(T) \frac{\partial j_v}{\partial x} \right)_{x=0} = h(T_{int} - T_{ext}) + \sum_{i=1}^{m} f \varepsilon \sigma (T_{surf}^4 - T^4) + L(T) h_m \left( \rho_{v,\text{int}} - \rho_{v,\text{ext}} \right), \]

and the external surface (\( x=L \)), as:

\[ \left( \lambda(T, \theta) \frac{\partial T}{\partial x} \right)_{x=L} + \left(L(T) \frac{\partial j_v}{\partial x} \right)_{x=L} = h(T_{int} - T_{ext}) + \alpha q_{\text{ext}} + L(T) h_m \left( \rho_{v,\text{int}} - \rho_{v,\text{ext}} \right) - e R_w, \]

where \( h(T_{int,ext} - T_{x=0,L}) \) represents the heat exchanged by convection with the internal and external air. \( \alpha q_{\text{ext}} \) (W/m²) is the absorbed short-wave radiation and \( L(T) h_m \left( \rho_{v,\text{int,ext}} - \rho_{v,\text{ext}} \right) \), the phase-change energy term. The long-wave radiation loss (horizontal roof) is defined as \( R_w \) (W/m²) and \( e \) is the surface emissivity. The solar absorptivity is represented by \( \alpha \) and the mass convection coefficient by \( h_m \), which is related to \( h \) by the Lewis’ relation.

Similarly, the mass balance at the internal surface is written as

\[ \frac{D_l (T, \theta) \frac{\partial \theta}{\partial x} + D_{jl} (T, \theta) \frac{\partial \theta}{\partial x}}{\rho_l} = h_m \left( \rho_{v,\text{int}} - \rho_{v,\text{ext}} \right), \]

and for the external surface, as

\[ \frac{D_l (T, \theta) \frac{\partial \theta}{\partial x} + D_{jl} (T, \theta) \frac{\partial \theta}{\partial x}}{\rho_l} = h_m \left( \rho_{v,\text{ext}} - \rho_{v,\text{int}} \right), \]

For the floor, at \( x = L \), an impermeable and adiabatic condition was adopted. Equations (6) and (7) show a vapor concentration difference, \( \Delta \rho_v \), on their right-hand side. This difference is between the porous surface and air and is normally determined by using the values of
previous iterations for temperature and moisture content, generating additional numerical instability. Due to the instability created by this source term, the solution of the linear set of discretized equations normally requires the use of very small time steps, which can be exceedingly time consuming especially in long-term soil simulations; in some research cases, a time period of several decades has to be simulated, taking into account the three-dimensional heat and moisture transfer through a very refined grid.

In order to increase the simulation time step, Mendes et al. (2002) presented a procedure to calculate the vapor flow, independently of previous values of temperature and moisture content. In this way, the term $(\Delta \rho_v)$ was linearized as a linear combination of temperature and moisture content, viz.,

$$
(\rho_{v,\infty} - \rho_v(s)) = M_1(T_{\infty} - T(s)) + M_2(\theta_{\infty} - \theta(s)) + M_3,
$$

where

$$
M_1 = A \frac{M}{R} \phi;
$$

$$
M_2 = \frac{M}{R} \left( \frac{P_r(s)}{T(s)} \right)^{\text{prev}} \left( \frac{\partial \phi}{\partial \theta(s)} \right)^{\text{prev}};
$$

$$
M_3 = \frac{M}{R} \left[ \frac{P_r(s)}{T(s)} \right]^{\text{prev}} \left[ \phi_v(R(T_{\infty}) - R(T^{\text{prev}}(s))) \right].
$$

In the equations above, the index (s) represents the surface on contact with air and (\infty) the air far from that surface, $R$ is a residual function of $\left( \frac{P_r}{T} \right)$. $P_r$, saturated pressure (Pa), $\phi$, universal gas constant (J/kgmol K), $M$, molecular mass (kg/kmol), $\phi$, relative humidity, prev, previous iteration and $A$ is the straight-line coefficient from the approximation $\left( \frac{P_r}{T} \right) = AT + B$.

**Internal Air Domain**

The present work uses a dynamic model for analysis of hygrothermal behavior of a room without HVAC system or infiltration/exfiltration. Thus, a lumped formulation for both temperature and water vapor is adopted. Equation (9) describes the energy conservation equation applied to a control volume that involves the room air, which is submitted to loads of conduction, convection, short- wave solar radiation, inter-surface long-wave radiation and infiltration:

$$
\dot{E}_r = \rho_{\text{int}} c_{\text{int}} V_{\text{int}} \frac{dT_{\text{int}}}{dt},
$$

where:

$\dot{E}_r$ is the energy flow that crosses the room (W), $\rho_{\text{int}}$, the room air density (kg/m$^3$), $c_{\text{int}}$, the room air specific heat (J/kg-K), the internal air volume (m$^3$) and $T_{\text{int}}$, the room air temperature (°C).

The sensible heat flow rate released by the building envelope is calculated as

$$
Q_s(t) = h_{\text{int}} A[T_{x=0}(t) - T_{\text{int}}(t)]
$$

and the latent heat flow rate is calculated as

$$
Q_v(t) = L(T_{x=0}(t)) h_{\text{int}} A[\rho_{v,\infty}(t) - \rho_v(t)].
$$

In Eq. (10) $A$ represents the area of the internal surface (m$^2$), $h_{\text{int}}$ the internal convection heat transfer coefficients (W/m$^2$ K), $T_{x=0}(t)$ the temperature at the internal surface of the building (°C) and $T_{\text{int}}(t)$ the room air temperature (°C). In Eq. (11), $L$, the vaporization latent heat (J/kg), $h_{\text{int}}$, the internal mass convection coefficient (m/s), $\rho_{v,\infty}$, the room air vapor concentration (kg/m$^3$) and $\rho_v(t)$ the vapor concentration of the internal surface (kg/m$^3$). The temperature and vapor density are calculated by the combined heat and moisture transfer model by using the values of temperature, moisture content and sorption isotherm.

In terms of water vapor mass balance, the lumped formulation becomes:

$$
h_0 A \left( W_{v,\infty} - W_{\text{int}} \right) = \rho_{\text{int}} V_{\text{int}} \frac{dW_{\text{int}}}{dt},
$$

where $h_0$ is the mass transfer coefficient (kg/m$^2$ s), $W_{v,\infty}$, the humidity ratio of the internal surface (kg water/kg dry air), $W_{\text{int}}$, the room humidity ratio (kg water/kg dry air), $\rho_{\text{int}}$, the room air density (kg dry air/m$^3$) and $V_{\text{int}}$ the room volume (m$^3$).

**SIMULATION PROCEDURE**

For the simulations, a 25-m$^2$ single-zone building as shown in Fig. 1 has been used. Openings were not considered to increase the importance of the roof load on the building thermal performance. For the conduction loads, 10-cm brick were considered.
The roof has been represented by 3 layers: sand (10 cm), sealing layer and brick (10 cm), as shown in Fig. 2.

The differential equations of energy and mass conservation for each node of the roof were discretized by using the finite-volume method (Patankar, 1980), with a central difference scheme, a uniform grid and a fully-implicit approach. The solution of the set of algebraic equations was obtained by using the MTDMA (Multi-TriDiagonal-Matrix Algorithm – Mendes et al., 2002). In the roof model, a regular mesh of 2 mm was utilized for each control volume.

In this work, highly temperature and moisture dependent properties were gathered from Oliveira et al. (1993). The brick properties were obtained from Perrin (1985). The basic dry-basis material properties are shown in Tab. 1.

In the internal region, a constant convection heat transfer coefficient of 3 W/m²K was considered. The external region were submitted to the TRY (Test Reference Year) weather data for the city of Brasilia-Brazil (South latitude of -15.47°), with a constant convective heat transfer coefficient of 12 W/m²K and a solar absorptivity of 0.5 for both sand and brick layers.

Figures 3 and 4 show the values of temperature, relative humidity and total solar radiation in January (1st – 7th) from TRY (Test Reference Year) weather data for the city of Brasilia-Brazil.

RESULTS

In order to predict the sand layer effect on building passive cooling, the Brasilia TRY weather data file has been chosen and simulations have been carried out from Jan. 1st to Jan. 7th, using a 30-sec simulation time step. The results presented in this section Figs. 5-9 are related to January 7th, in order to reduce the initial conditions effects.

Simulations in the roof were performed with or without a sand layer. It can be noticed, in Fig. 5, temperature differences close to 4°C for peak values at the internal surface of the roof. The cooling effect was observed during daylight period, between 10 am and 6 pm, with different behaviour in the night period. As observed in Fig. 6, the small difference on the external surface temperature (with or without sand layer) shows that the internal surface

Table 1: Dry-basis properties of the sand and brick.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \rho_0 ) (kg/m³)</th>
<th>( c_m ) (J/kg K)</th>
<th>porosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>1650</td>
<td>800</td>
<td>0.38</td>
</tr>
<tr>
<td>Brick</td>
<td>1900</td>
<td>920</td>
<td>0.29</td>
</tr>
</tbody>
</table>
temperature is dominantly affected by the sand layer inertia rather than evaporative cooling.

Figure 5: Temperature at the internal surface of the roof on January 7th.

Figure 6: Temperature and relative humidity at the external surface of the roof on January 7th.

Figure 6 also shows that evaporation at the external surface during the daylight period is counterbalanced by solar radiation. An interesting physical aspect noticed in Fig. 6 is the relative humidity at the sand surface. As the external surface temperature decreases below the dew point due to the negative net radiation balance at the surface, the sand becomes saturated at the surface for a 9-h period and then dries out rapidly thanks to the high short-wave radiation on the horizontal surface during day time. As the Luikov number is low (Dantas et al., 2002) and the sand is very thermal capacitive, this effect does not have much influence on the temperature.

Figure 7 shows the room air temperature and relative humidity temporal evolution, where a variation of 1°C on the temperature and 4% on the relative humidity in the peak values is noticed. This effect is attributed to the reduction of sensible heat flux at the ceiling internal surface when the sand layer is considered, as illustrated in Fig. 8. Although the room air temperature and relative humidity variations are small, they cause a higher effect on the PMV index. Figure 9 shows an improvement on the thermal comfort index (clo = 0.66, met = 0.79, air velocity = 0.01 m/s) during all day long. It should be mentioned that no internal gain has been considered in the simulation, in order to avoid misinterpretations from the effect of the coupled heat and moisture transfer through the roof.

Figure 7: Room air temperature and relative humidity on January 7th.

Figure 8: Sensible and latent heat flux at the internal surface of the roof on January 7th.

Figure 9: Predicted mean vote (PMV) on January 7th.
Although this section shows that the sand layer produces a better indoor thermal comfort, this effect could be increased with a layer of a material composed of larger porous, i.e., volcanic ash and siliceous shale. Waphen and Nagano (2009) investigated the hygrothermal performance of several non-porous and porous potential roofing materials and it was observed that under the conditions of no solar radiation, unchanging temperature and relative humidity, a high amount of latent heat could be released from finer materials such as silica sand and porous particles with smaller sizes due to capillary action. On the other hand, when a climatic situation was analysed, particles at the surface layer of large sized samples tended to dry faster due the ventilation and solar radiation penetration in large gaps, enhancing the evaporation.

Some parameters such as the emissivity did not demonstrate a significant impact on the results (not shown). However, there is a significant uncertainty degree on the hygrothermal properties that needs to be carefully examined, which is part of an international study in the frame of the International Energy Agency Annex 55.

CONCLUSION

A mathematical model to simulate an unsaturated highly capacitive porous medium on roof (sand) was proposed to predict its effects on building passive cooling. Comparisons of the sand layer effect in terms of heat fluxes through a horizontal roof, indoor conditions and PMV index were presented. It has been shown that the sand layer inertia effect is dominant when compared to the one from evaporative cooling. Although the room air temperature and relative humidity variation has been small with a sand layer, it caused a better PMV index during all day long. This effect is attributed by sensible heat flux decreasing at the internal surface and it could be increased in low-rise buildings with large roof area.

For further work, sensitivity and uncertainty analyses should be carried out on the hygrothermal properties, besides a profound investigation on the validity and limitations of conventional heat and moisture transfer models to be used to simulate a complex phenomenon in non-consolidated porous media such as sand.

ACKNOWLEDGEMENT

The authors thank the Brazilian Research Council (CNPq) of the Secretary for Science and Technology of Brazil and ELETROBRAS (Centrais Eletricas Brasileiras) for support of this work.

REFERENCES


