IDENTIFICATION OF EQUIVALENT THERMAL RC NETWORK MODELS BASED ON STEP RESPONSE AND GENETIC ALGORITHMS

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ABSTRACT
When heat conduction is considered, assessment of simplified RC (resistances and capacities) model parameters is a great challenge due to thermal mass. This article presents a new method for identifying the parameters of equivalent thermal RC network models by illustrating the case of 1D conduction in a wall. The parameters of the RC model are identified by the optimization method of genetic algorithms. The performance criterion is based on the sum of errors related to surface temperature responses, due to steps imposed on the ambient temperatures on either side of the wall. The originality of the method is to consider a constant time step on the logarithmic time. This is particularly important to ensure the validity of the RC model for high pulses when the frequency analysis is considered.

INTRODUCTION
The electrical analogy method makes the parallel between heat transfer and electric current (Saulnier, 1985). Thus, the equation of heat balance is equivalent to the conservation of electric current at the corresponding node. Regarding simplified RC models, the parameters can not always be calculated directly from the thermo-physical characteristics of materials and geometrical quantities. It is then necessary to identify these parameters from a reference model (detailed) or experimental data. Regarding the first approach, different methods of identification are possible. For the case of heat conduction through a wall, the quadrupole method allows to determine the reference transfer matrix (Roux, 1985) (Berges, 1986) (Lagonnotte, 1999). The transfer matrix of the RC model is then identified through Taylor series of each term of the reference matrix. The disadvantage of this method is related to the larger number of equations as parameters (resistances and capacities); it is thus necessary to focus on certain terms. The quadrupole method has been applied in the case of 3R2C and 3R4C models based on a comparison of models through a frequency analysis (Fraisse, 2002). It has been demonstrated that the presence of thermal capacity at wall model surface improves the model behavior in the range of «high frequencies ».

In the case of homogeneous layers in walls, considering for each layer a symmetrical distribution of resistance and capacity values can easily identify a simplified model. Davies highlights some weights that can better reproduce the heat flux outputs (Davies, 2003).

Another method of parameters identification of a RC model of multilayer walls is based on a chart approach comparing the RC model to the actual wall. It consists in representing from one side of the wall to the opposite side the product of resistances by capacities as a function of the sum of resistances. Very basic, this graphic method is inefficient.

The identification of a building model with three resistances and two capacities (Laret, 1981) has been developed through genetic algorithms (GA). They have the advantage of being less sensitive to local minima than traditional approaches (Fraisse, 1999). The building inertia and the thermal losses coefficient have been determined from experimental values. In this study, it would have been obviously possible to use another optimization method such as recursive least-squares method that are less costly in computation time (Coley, 1992). These two examples of identification made from experimental data may also be applied to simulation results provided by a reference model.

This article presents a new method of parameter identification when heat conduction is considered, for any equivalent thermal RC network models. The 1D conduction in a wall is illustrated. Nevertheless, the same method can be transposed to more complex configurations like thermal bridges (2D conduction).

STEP RESPONSE BASED IDENTIFICATION

Principle
The method is illustrated in the case of 1D heat conduction in a wall. Our RC model parameter identification approach is based on the step response of a reference model. The method that is used to identify resistances and capacities is genetic algorithms. This method is relevant to define the model because of its ability to explore the set of possible parameters. It thus allows approaching the global optimum.
The performance criterion for identifying the RC parameters is based on the sum of errors on the two outputs in wall surface temperature \( T_w \) (left and right side). The inputs are imposed steps on ambient temperature \( (T_a) \) on each side of the wall. Two simulations providing four step responses by model (RC and reference) are thus considered in order to calculate the overall error \( E \). This error \( E \) is evaluated for each time step between time \( t_i \) (\( \geq \) initial time defined as \( t = 0 \) s) and a final time \( t_f \) when surface temperatures have reached the steady state. As an example, the error to an input on the ambient temperature at the left side \( T_a_{\text{left}} \), with an output on the opposite surface \( T_w_{\text{right}} \) is (figure 1):

\[
E_{T_w_{\text{right}}} = \sum_{t_1}^{t_f} |T_{w_{\text{RC}}}(t) - T_{w_{\text{ref}}}(t)|_{\text{right}}
\]  

Where: \( T_{w_{\text{ref}}} \) is the surface reference temperature

\( T_{w_{\text{RC}}} \) is the surface RC model temperature

The overall error \( E \) is the sum of the four errors:

\[
E = E_{T_w_{\text{left}1}} + E_{T_w_{\text{right}1}} + E_{T_w_{\text{left}2}} + E_{T_w_{\text{right}2}}
\]  

The originality of the method is to consider a constant time step on the logarithmic time scale (figure 1). It allows to consider the full dynamic response of the wall, especially for « short » time. This is particularly important to reproduce the fastest dynamical phenomena. In the example discussed later, we will show that only the first time constant of the reference model is higher that one hour. It is thus necessary to have enough reference values during the early hours in order to take into account the following time constants. This is particularly important to ensure the validity of the RC model for high frequencies. It will be shown in the following through frequency analysis.

**Figure 1 Principle of the error calculation for the identification (left input \( T_a_{\text{left}} \) and right output \( T_w_{\text{right}} \))**

### The genetic algorithms

- **Origin**

The genetic algorithms (GA) have been developed by John Holland at the beginning of the sixties, at Michigan University. One of his collaborators, David E. Goldberg, was entirely devoted to this study and his works with those of Holland (Goldberg, 1994). The GA are inspired by mechanisms of natural selection and genetics.

The GA eliminate some problems inherent in other identification methods. The enumerative methods (all solutions are tested) are inefficient when the investigated domain is large. The random methods (tested solutions are randomly selected) are exploring in a “blind” way the whole domain and could not find the best solution. The traditional methods often require the use of derivatives, which is limiting their scope. The GA have considerable strength and flexibility as they manipulate a binary encoding parameters (and not the parameters themselves) and they explore the domain of search by a population of solutions (and not a single solution). The main disadvantage of the GA is their slow convergence because of the principle of evolution of an individual during the optimization process.

A review of utilization of genetic algorithms in heat transfer problems is proposed by Gosselin et al. (Gosselin, 2009). It highlights the growing interest for the genetic algorithms (GA) in the last 20 years in the design of systems or for inverse problems.

- **Operating principle**

The GA first of all require coding of parameters to identify, usually on binary form. All the set of parameters which are thus coded and placed end to end are a “chromosome”. Part of chromosome defining a parameter is called “gene”. Genes are made up of binary values (bits) corresponding to the value of the parameter. The GA are working on a population of individuals (chromosomes) which are simultaneously tested. Each individual, corresponding to a value assigned to each parameter is evaluated through a fitness function. This mirrors the performance of the individual facing the considered problem. The objective is to obtain an individual with the best performance. The initial population can be randomly selected. This population evolves from one generation to another through simple operations that involve only copies and exchange of pieces of chromosomes. This is the breeding that uses the operator selection, crossover and mutation.

The selection is to retain a limited number of individuals based on their ability. An individual is more likely to be represented that it has a high performance. The crossover operation is performed from selected individuals. A random draw is made when two parents are selected. If the value of the draw is greater than the crossover probability \( P_c \) (user set), parents are recombined by crossing. Otherwise, they are identically reproduced. Finally, the mutation is to change by random selection the value of one or more bits of children coming from the stage of crossing. The primary endpoint of the algorithm is the mutation probability \( P_m \). A draw is
made for each child and for each bit. The bit value is changed if the obtained value is less than \( P_m \).

In summary, the main parameters involved in the GA are the crossover probability \( (P_c) \), the mutation probability \( (P_m) \) and the size of the population.

**APPLICATION EXAMPLE**

**Studied case**

We’ve studied a 1m² wall, composed of e=16cm wood. The thermo-physical properties for this material are the thermal conductivity \( \lambda=0.18 \) W/(m.K), the density \( \rho=600kg/m³ \) and the specific heat capacity \( C_p=1200J/(kg.K) \).

The boundary conditions are of the third type. The left and right heat transfer coefficients are respectively \( h_{left}=20W/(m².K) \) and \( h_{right}=9W/(m².K) \).

The reference model output is obtained for a 500 layer spatial discretization. The system of differential equations which is defined by each temperature node balance is solved between initial time \( (t=0) \) and final time \( (t=t_f) \) thanks to the EES solver (Engineering Equation Solver) developed at the Lawrence Berkeley National Laboratory. Concerning the electrical analogy model, four resistances \( R_{1-4} \) and five capacities \( C_{1-5} \) has been considered in a first step. Two of the capacities are located at the wall surfaces (numbered from the left to the right). The sum of resistance and capacities always check the steady conditions and the total capacity accumulation. In this example:

\[
\sum_{i=1}^{4} R_i = \frac{e}{\lambda} \quad (3)
\]

and

\[
\sum_{i=1}^{5} C_i = \rho \cdot C_p \cdot e \quad (4)
\]

As mentioned, the simulation results are obtained with a constant time step on a logarithmic scale. The \( t_N \) time related to the \( N^{th} \) time step is calculated as follows:

\[
t_{N=1...400} = \exp(x_N) \quad (3)
\]

With: \( x_N = x_{N-1} + 0.035 \) and \( x_0=0 \)

The increment on \( x_N \) (value of 0.035) has been chosen to fix the \( t_N \) simulation value about 12 days, given the number of chosen time steps (400). It is of course useless to prolong the process of identification when the steady state is reached. The error on each individual is calculated between \( t_1 = 77s \) \( (N=125) \) and \( t_2 = 1 161 241s \) \( (N=400) \). The simulation results from the reference model are read from the file created previously from the simulation on ESS solver.

Regarding the optimization, probability of crossover and mutation are respectively \( P_c=0.8 \) and \( P_m=0.2 \). The population is composed of 50 individuals. The initial population is randomly generated with the constraint on the resistances and the capacities mentioned above. The number of generation is fixed at 100. The optimization algorithm has been programmed in Fortran. It identifies nine parameters of the RC model. The evolution of the parameters and the performance function (global error \( E \)) of the best individual are presented in figure 2. Each identified surface capacity approximately represents 3% the wall capacity. This corresponds to the order of magnitude seen in the passage from a 3R2C (2 internal capacities) model to a 3R4C model with two additional surface capacities (Fraisse, 2002). The resistances and the capacities that are obtained are relatively symmetrical (table 1). Even if the wall is a monolayer, the surface heat exchange coefficients are different which explains the asymmetry. Regarding the optimizations, one can notice that the performance function and the parameters do not vary much from 60 generations.

![Figure 2 Variation of the performance function and parameters during iterations](image)

The values from the parameter identification are shown in Table 1. The computation time is about 3 seconds for PC with a 3192MHz processor.

**Table 1 Results from optimization**

<table>
<thead>
<tr>
<th>Identification of the 4R5C model</th>
<th>( R_1 = 0.0774 ) m².K/W</th>
<th>( C_1 = 3 411 ) J/(m².K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_2 = 0.4078 ) m².K/W</td>
<td>( C_2 = 21 687 ) J/(m².K)</td>
<td></td>
</tr>
<tr>
<td>( R_3 = 0.3297 ) m².K/W</td>
<td>( C_3 = 66 690 ) J/(m².K)</td>
<td></td>
</tr>
<tr>
<td>( R_4 = 0.0740 ) m².K/W</td>
<td>( C_4 = 20 235 ) J/(m².K)</td>
<td></td>
</tr>
<tr>
<td>( C_5 = 3 176 ) J/(m².K)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The time varying of the reference model outputs (Ref, thick line) and of the 4R5C (dotted line) is given in figure 3. These two outputs (\( Tw_{left} \) and \( Tw_{right} \)) are related to each input (\( Ta_{left} \) and \( Ta_{right} \)). The used parameters are those of table 1. The simplified model fits well with the reference model regardless of the couple input/output behavior. The good model accuracy in the range of time under one hour has to be particularly highlighted. In the example, a five term capacity model accurately reproduces the dynamic behavior of the reference model on the entire simulation.
The accuracy of a given analog model RC also increase on a considered period \([t_1 – t_2]\) when \(t_1\) increases. However, the results obtained for time under \(t_1\) are likely not to be satisfactory. In the other way, reduction of the value of \(t_1\) can promote the validity of the model response for short time at the expense of its accuracy on the overall response.

RESULTS COMPARISON

Figure 4 shows the time constants associated with the reference model and the 4R5C model obtained with two types of optimization. The first one corresponds to the approach outlined above and the second differs only by the choice of the time step which is constant \((\Delta t=100\ seconds)\). For this last case, learning occurs between \(t=0s\) and \(t=40000s\) (steady state reached at 99.3% of the maximum variation). The first two time constants (the highest) of the 4R5C model are closer to the reference when a constant time step is considered.

Figure 5 shows the input/output couples localized on the same side of the wall. From 2500 seconds, the model identified with a constant time step follows perfectly the reference. The reason is linked to good approximation of the first two time constants (for the higher the discrepancy is lower than 5%) which leads to more accurate modeling for “long” times. Nevertheless, in spite of time constants more far from the reference (figure 4), the results obtained on the temperatures with the variable time step are generally lower compared to the reference (figure 5). This implies that a variable time step makes it easier to “aggregate” the time constants of the reference model. The configuration with an input on the left side is providing generally less satisfactory than for the right side because of the heat transfer coefficient values.

The wall model validity for “short” times is essential in order to reproduce the thermal behavior of a building submitted to fast inputs (radiation, control of the room temperature …). The validity of the model (constant or variable time step) is checked through the frequency analysis. The objective of the frequency analysis is to study the answer of the system submitted to sinusoidal input. The output of a linear system that is submitted to a sinusoidal input is sinusoidal with the same pulse as the input but with different magnitude and phase. On of the advantages of the harmonic analysis is to assess the frequency range of inputs for which a model is valid. The validity limits of the model are \(a\ priori\) determined by the input frequencies of which one desires to restore the effects.

Figures 6 and 7 allow confirming the interest of variable time step identification. Even if an identification achieved with a constant time step \((\Delta t=100s)\) is leading to a model that strictly follows the reference amplitude till a pulse \(W=4.10^{-4}\ \text{rad/s},\) then the discrepancy increases much more compared to the variable time step which follows the reference regardless of the angular frequency. Concerning the magnitude, the discrepancy with the constant time step occurs at a much lower angular frequency \((W=7.10^{-4}\ \text{rad/s corresponding to a period of 2h30min})\) than for a variable time step \((4.10^{-3}\ \text{rad/s corresponding to 26 minutes})\). The RC model obtained through this last approach is preferable when fast phenomena are taken into account as for thermal behavior of buildings.

The frequency analysis results are also compared between the quadrupole method developed for the case of the 3R4C models (Fraisse, 2002) and that proposed in this paper. Figures 8 and 9 represent a Bode diagram for both magnitude and phase that corresponds to an output at the right side of the wall \(Tw\_right\), when a sinusoidal input is applied on the ambient temperature on the same side \(Ta\_right\). The 3R4C models are identified by the quadrupole...
method and with the step response (GA) whereas the 4R5C model is simply identified by the step response (GA).

The comparison between the two 3R4C models shows an improvement for both magnitude and phase for the new method. Nevertheless, the 4R5C model provides much more interesting results. It follows as we have already seen (figures 6 and 7) the magnitude of the reference model whatever the angular frequency \( \omega \), whereas the phase deviates at \( 4.10^{-3} \) rad/s.

**Figure 6** Bode diagram of magnitude (input and output at the right side)

**Figure 7** Bode diagram of phase (input and output at the right side)

**Figure 8** Bode diagram (magnitude)

**CONCLUSION**

A new method for parameter identification of RC models has been illustrated through a simple example of 1D heat conduction in a homogeneous wall. Our approach is based on the error related to the step output between the reference and the RC models. The optimization is achieved through genetic algorithms. This method is particularly relevant to define the optimal model because of its ability to explore all the set of parameters in order to approach the global optimum. The initial population was defined in a totally random way ensuring that each individual checks both steady state and total capacity accumulation conditions.

The advantage of using a variable time step to indentify the “fast” dynamical behavior of the RC model has been demonstrated. This is important to increase the range of validity of the RC model through the frequency analysis.

It is of course possible to identify RC models for much more complex configurations (multilayer walls, 2D and 3D thermal bridges…). Despite the complexity of physical phenomena to consider, the RC models are always as easy to formulate. Only results of reference models are required in order to apply the developed identification method.

The identification of RC models from experimental results is possible in the same way for laboratory conditions in order to accurately control the inputs. It is however much more difficult from in-situ data. All the inputs will have indeed to be considered while the identification approach must be adapted to the input characteristics.

**REFERENCES**


Coley DA., Penman JM. 1992. Second order system identification in the thermal response of real...


