

PROGRESS IN SIMULATION OF A THERMAL PROBE: MODELLING THE PROBE TO SAMPLE CONDUCTANCE

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ABSTRACT

This paper describes the latest developments in the simulation of a thermal probe apparatus, building on earlier work as reported by de Wilde et al. (2007). The work focuses on researching the role of the probe to sample conductance H . Results obtained with the existing model are proven to be inconclusive, and necessitate the change to flexible general engineering finite element software.

KEYWORDS

Building material properties, thermal probe, transient simulation, experiments, tool selection

INTRODUCTION

The rationale for developing a thermal probe apparatus is that this technique allows the measurement of thermal conductivity (λ) and thermal diffusivity (α) in existing buildings, under actual use conditions. This is important (1) for vernacular buildings, where actual construction details often are unavailable, (2) for buildings that employ innovative materials with unknown properties, and (3) for buildings that make use of materials whose properties are highly dependent on the construction process, for example cob or rammed earth. Furthermore, it allows account to be taken of actual conditions in use such as moisture content, an accepted influence upon the thermal properties of building materials in use (Salmon et al., 2002). An existing thermal probe is shown in figure 1.

However, the thermal probe itself remains a technique under development; see for instance Pilkington et al. (2008), an article that links the development of the thermal probe technique with in situ measurements undertaken on buildings. Although sufficient accuracy and good repeatability of the measurement of thermal properties of materials like masonry have been demonstrated, studies of other less dense materials, such as insulating materials,

have proved less satisfactory. One way to better understand the barriers to further development of the thermal probe technique is the use of simulation.

The general strategy followed for investigating the different factors that influence experimental results starts by modelling an ideal infinitely thin and long line heat source in an infinite homogenous block of material, which is the underpinning theory for the thermal probe technique. Note that this, essentially, is a two-dimensional case. From this starting point a step-by-step process is followed that introduces a thermal probe around the heater, which then will progressively evolve to represent a probe inserted into a material sample, in three dimensions, and taking into account issues like the probe construction and the thermal contact resistance between the heater and the sample.

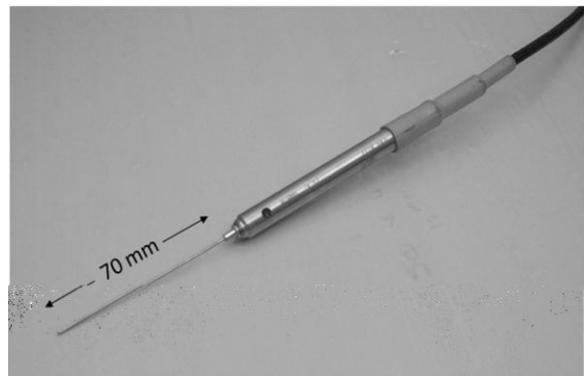


Figure 1: Image of a thermal probe

Work on the first stage, the infinite line source model, has been reported by de Wilde et al. (2007). This work did not consider any effects related to the presence of a real probe. Upon further refinement and verification of the model, this work was used to validate the data analysis routines that are used with the actual thermal probe, as developed by Goodhew and Griffiths (2004). For a description of this validation work, see de Wilde et al. (2008).

In a second stage, while basically still considering a two-dimensional situation, a rudimentary thermal

probe was introduced that represents a heater mounted axially in a stainless steel shell, which is filled with glass, as encountered in an actual experimental thermal probe apparatus. While modelling the probe, the resolution of the spatial grid has been refined with a factor 100. Simulations were carried out for a probe inserted into stabilized water (agar), glycerine with fibre, toothpaste, and PTFE. These simulations were then validated using data obtained through recent laboratory studies, confirming that percentage errors for conductivity and diffusivity are lowest for the model of the glass filled probe (de Wilde et al., 2009).

OBJECTIVE

The underlying main aim of this work is to improve the understanding of the probe to sample conductance H [$\text{Wm}^{-2}\text{K}^{-1}$], and how this may be determined with confidence, so that the analysis of thermal probe data can be achieved confidently. At present Solver routines are used to determine the values of thermal conductivity, diffusivity and probe conductance, as described in de Wilde et al. (2009). If the practical probes could be calibrated so that the probe conductance was known, or could be repeatedly determined, then the probe technique would give conductivity and diffusivity values confidently. A particular difficulty arises when the thermal probe is used to study low density low thermal conductivity materials.

METHODOLOGY

The research described in this paper sets out with further use of the program Voltra (Physibel 2005), which allows calculation of transient heat transfer using the energy balance technique, and which was used in the earlier work

The paper demonstrates issues arising when using Voltra to study probe to sample conductance H in detail, which necessitates the move to a different modelling approach.

A solution has been found by shifting towards general engineering finite element analysis (FEA) software. An overview of the motivation for moving the modelling methodology is given, followed by a comparison of Voltra and FEA results to demonstrate continuity of the research.

Based on the simulation results, possible values for the probe to sample conductance H will be tentatively suggested.

VOLTRA MODELLING

The first simulation work reported in this paper is based on the use of the program Voltra (Physibel 2005), which allows calculation of transient heat transfer using the energy balance technique. Use of Voltra ensures continuity of the probe model and applicability of earlier validation work.

The initial two-dimensional line source model in Voltra consisted of a very large slice of sample with outer dimensions of 2400 by 2400 mm, but with a thickness of only 1 mm. The boundary conditions on both faces of this slice of material have been defined as adiabatic, rendering the material infinite in the direction perpendicular to these faces. The line heat source is modelled at the middle of this slice, again perpendicular to the faces, positioned at 1200 mm from the boundaries of the sample. The default spatial grid has a mesh size of 10 microns. The Voltra program always uses a rectangular grid. See figure 2.

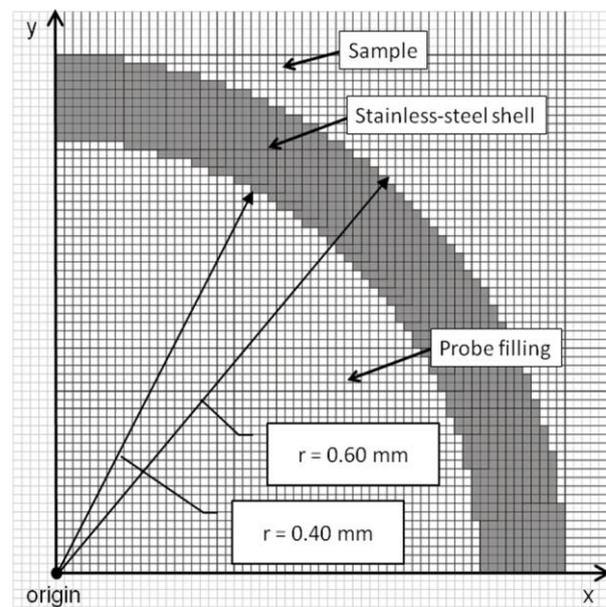


Figure 2: Representation of the Voltra model of the elementary thermal probe.

The time dependent heat transfer through a material will have both thermal resistive and thermal capacitive components. In order to select appropriate values for the three surface heat transfer coefficients present in the probe for input into the Voltra model a steady state overall heat conductance H_s [$\text{Wm}^{-2}\text{K}^{-1}$] has been calculated. The probe is initially assumed to consist of an axial heater, radius r_1 mounted in a probe filling contained in a stainless steel cylindrical shell. This shell has internal and external radii r_2 , (0.4 mm) and r_3 , (0.6 mm) respectively. In the practical application of the thermal probe great care is taken to

ensure the best possible thermal contact between the external surface of the probe and the sample.

For the heat transfer from the heater to the sample five heat transfer components are identified:

1. Heat transfer from the heater surface to the probe filling, H_1 at r_1 , (assumed to be 10 microns)
2. Thermal conduction through the probe filling of conductivity λ_f , from r_1 to r_2 .
3. Heat transfer from the probe filling to the internal surface of the probe steel shell, H_2 at r_2 .
4. Heat conduction through the steel shell of conductivity λ_s , from r_2 to r_3 .
5. Heat transfer from the external surface of the steel shell to the sample, H_3 at r_3 .

The Voltra model was constructed to study the thermal performance of the probe when immersed in agar, and used material properties selected from Table 1.

Following Hagan (1999) an expression for the total resistance between the probe heater and an assumed position just beyond the probe exterior and in the sample was established. It was assumed that the probe was in a steady state, and the total resistance was determined relative to the external probe surface area. The equation below gives an expression for the steady state overall probe conductance H_s .

$$H_s = 1 / [r_3 / (r_1 H_1) + (r_3 / \lambda_f) \ln(r_2 / r_1) + r_3 / (r_2 H_2) + (r_3 / \lambda_s) \ln(r_3 / r_2) + 1 / H_3]$$

Using this expression in a spread sheet provided a vehicle for quickly assessing the possible orders of magnitude of H_1 , H_2 and H_3 that are required for the Voltra model, and gave a value of the steady state overall heat conductance H_s . Practical studies have demonstrated that the observed value of the thermal probe conductance lies in the range 700 +/- 30% $\text{Wm}^{-2}\text{K}^{-1}$, (de Wilde 2009) However, these practical values are obtained from the time dependent behaviour of the probe; the probe and sample are

clearly not in a steady state. How are these two parameters, the time dependent and steady state values of the probe conductance, related? In the case of the built environment, building components show steady state thermal transmittances, U-values, which are usually less than the time dependent admittance values, U-values some 25% of the corresponding admittances. In order to proceed with this preliminary modeling of the probe conductance, the steady state conductance H_s was assumed to have a value in the range 150 to 250 $\text{Wm}^{-2}\text{K}^{-1}$.

From above it is clear that not all the input data for this spread sheet are confidently known, and three problem areas in this calculation of H_s can be identified.

- The heater in a practical probe consists of a hairpin and results in an asymmetric heat input. As a result the value of r_1 is uncertain, although for the modelling it has been assumed to be 10 microns.
- The values of the surface heat transfer coefficients H_1 , H_2 and H_3 are unknown and must be guessed.
- The practical probe is filled with quartz spheres in air, with a packing fraction 0.68. This probe filling may have three heat transfer routes, (i) conduction via the touching quartz spheres, (ii) convection in the air spaces between the quartz spheres, and (iii) radiation transfer again in the air. The spread sheet calculation for the quartz air filling assumed that the convection and radiation transfers were negligible and that the conductivity of the filling was 0.68 times the conductivity of quartz, or $0.68 * 8$, that is 5.4 $\text{Wm}^{-1}\text{K}^{-1}$. Various values of $H_1 = H_2$ and H_3 were entered until a suitable range of H_s values was achieved.

*Table 1
Material properties input data for the thermal model.*

MATERIAL	DENSITY	CONDUCTIVITY	SPECIFIC HEAT CAPACITY	DIFFUSIVITY
	kg/m^3	W/mK	J/kgK	$10^7 \text{ m}^2/\text{s}$
Air	1.29	0.024	993	187
Quartz glass	2600	8	756	40.7
Mixture of quartz + air	1768	5.45	832	37.1
Steel, heater and shell	7900	16	460	44
Agar sample	1000	0.61	4180	1.46

Another aspect of this work was to compare the Voltra modelling data for two thermal probe designs. Firstly, there is the practical probe with axially mounted heater and quartz air filling contained in a relatively heavy steel shell. The rise in probe temperature is determined by a thermojunction mounted close to the centre of the probe, not necessarily on the axis. Secondly, there is the Blackwell probe (1954) consisting of a thin walled shell, where $(r_3 - r_2) \ll r_3$. In this second Voltra model r_2 was 0.58 mm making the assumption that (0.6 – 0.58) mm is very much smaller than 0.60 mm. In this second version of the probe the heater is wound on the exterior of the probe. Here the rise in probe temperature is measured by a thermometer in contact with the inside surface of the thin steel shell.

The interest in this second probe design arises because Blackwell gives a recipe for measuring, albeit approximately, the probe conductance at short times. However, the Voltra model was unable to provide sufficiently small time intervals to fully explore this probe model.

For both the practical probe, with its axial heater, and the Blackwell probe, and to enable the surface heat transfer coefficients, H_1 , H_2 and H_3 to be deployed uniformly at the various surfaces, a 10 micron grid was established in the Voltra platform, as shown in figure 2.

VOLTRA RESULTS

Table 2 shows the results for the two modelling experiments. In each case the numerical data from the Voltra simulations were analysed using a Solver 4.3 routine (de Wilde 2009) to determine the minimum time beyond which the linear section of the rise in probe temperature versus natural logarithm of the elapsed heating time could be assumed. The Solver 2.3 routine was then used to determine the thermal conductivity, the diffusivity and the probe conductance H_r . All the Voltra modelling runs were undertaken with the probe assumed to be placed in agar. The results of the Solver 2.3 analysis of the data always gave the conductivity ($0.61 \text{ Wm}^{-1}\text{K}^{-1}$) to within 2% and the diffusivity ($1.46 \cdot 10^{-7} \text{ m}^2/\text{s}$) to within 5%. This reflects the usual results obtained with the thermal probe and the analysis routines used here. The values of the probe conductance H_r are shown in table 2. Runs 1 to 3 in table 2 give the results for the modelled practical probe, where the axial heater is surrounded by a quartz spheres in air filling. In all these first three runs the Solver 2.3 value of the probe conductance is less than the steady state value. This is not what was expected. Earlier it was suggested that the ratio H_r/H_s might be of the order of four. Here the average value of this ratio is 0.42. The most acceptable agreement occurs with run

3, where the values of H_r and H_s are of the same order of magnitude as the experimentally observed values of H ; that is in the range 400 to 900 $\text{Wm}^{-2}\text{K}^{-1}$. Runs 4 to 9 give the results for the modelling of the Blackwell probe with the exterior mounted heating element. The values of the surface conductances H_1 , H_2 , and in runs 4 and 5, the value of H_3 are marked as “infinite”, that is not modelled in Voltra. This implies that the surface conductances are perfect, that there is no thermal resistance. In the spread sheet used to determine the steady state probe conductance perfect thermal conductance was denoted by a value of $10^9 \text{ Wm}^{-2}\text{K}^{-1}$. Table 2 runs 4 to 9 demonstrate that it is extremely difficult to draw any real conclusions. The results for the practical probe, run 3, and the Blackwell air filled probe, run 6, suggest a measure of agreement between the Solver 2.3 produced probe conductance and the steady state value. Further application of the steady state probe conductance spread sheet with $H_1 = H_2 = 1160 \text{ Wm}^{-2}\text{K}^{-1}$ and $H_3 = 580 \text{ Wm}^{-2}\text{K}^{-1}$ gave a steady state probe conductance of $695 \text{ Wm}^{-2}\text{K}^{-1}$. Further modeling studies will be undertaken with input conductances of this order.

The results do not allow any firm conclusions on the probe to sample conductance H to be drawn. The thermal probe simulation research, being positioned in the field of building physics, started out with using appropriate dedicated building simulation software (Voltra). However, by the increased focus on fundamental details (probe to sample conductance H , interaction between constituent parts of the probe like heater wire, thermojunction and shell) the boundaries for application of the original Voltra model apparently have been reached. The distribution of the thermal resistance at the three interfaces, as the surface area increases with radius, is difficult to model in the rectangular grid used by Voltra. Further complications arise as the heat transfer may not be by conduction in air, but a mixture of all three heat transfer routes, conduction, convection and radiation. Apart from the reliability of computational results, the run time of the simulation is also pushing practical limits.

Table 2
Values of the probe conductance.

	HEATER POSITION	PROBE FILLING	SURFACE CONDUCTANCE [WM ² K ⁻¹]			FROM SOLVER	FROM STEADY STATE SUM
			H ₁	H ₂	H ₃	H _r	H _s
1	axis	quartz air	7500	7500	4000	2480	4180
2	axis	quartz air	7500	7500	400	260	2520
3	axis	quartz air	750	750	400	261	454
4	exterior	air	infinity	infinity	infinity	2510	221
5	exterior	agar	infinity	infinity	infinity	4900	5574
6	exterior	air	infinity	infinity	400	236	212
7	exterior	agar	infinity	infinity	400	246	2822
8	exterior	air	infinity	infinity	4000	1520	220
9	exterior	agar	infinity	infinity	4000	1780	5078

(1) Infinity means not modelled in Voltra, and set to equal 10⁹ in the steady state sum.

(2) Note : All numerical sets gave excellent values for the conductivity and diffusivity of sample, agar.

GENERAL ENGINEERING FEA APPROACH

In order to move forward a different modelling approach has been introduced, using general engineering finite element analysis (FEA) software. FEA is widely used by the engineering community for the investigation of an extensive range of problems, including static and dynamic structural behaviour, fluid flow and heat transfer (Zienkiewicz et al., 2005).

FEA has a number of specific advantages over application-specific tools such as Voltra:

1. FEA packages have a library of different elements for the representation of different physical scenarios. Computational efficiency is achieved through the use of 1D, 2D or 3D elements where appropriate.
2. Individual elements can be described by a relatively large number of nodes. This results in increased computational accuracy (since the variable of interest can have a complex variation across each element), and the ability to represent complex geometries more realistically. For example, a quadrilateral element may be defined by 8 nodes (4 at each corner, 4 at the middle of each edge), enabling element edges to be curved. This will permit accurate representation of the probe geometry.
3. The FEA method can couple different physical scenarios in a given simulation – for example, a heat transfer problem could incorporate thermal expansion / thermal stresses and the effects of

moisture content. Properties and boundary conditions may be nonlinear if desired.

4. Most FEA software can represent heterogeneous, anisotropic materials, and is commonly used in the analysis of composite laminates (Barbero, 2007). Although data is notoriously difficult to obtain, this capability could be applicable to anisotropic construction materials, especially natural materials such as cob and straw-bale.
5. There is no practical limit to the spatial or time-domain resolution. This means that the detailed internal structure of the thermal probe can be modelled if desired, and its behaviour examined at very early times. As in all thermal simulations, numerical accuracy requires consistency between the time step (δt), the spatial resolution (δx) and the thermal diffusivity of the material (α) according to:

$$\delta t = \frac{(\delta x)^2}{\alpha}$$

This means that a very fine structure will require a small computational time step. Similarly, information about the temperature at small times will require a refined FEA mesh.

The advantages of using general engineering FEA software need to be balanced against a relatively high initial cost, and the need for sufficient expertise in running specific programs. The specific FEA package adapted for the thermal probe simulation work is Strand7 (2009).

FEA THERMAL PROBE MODELLING

In order to replicate earlier Voltra results the probe first has been modelled as a long thin heater wire embedded in a uniform medium of much larger dimensions, and front and back faces have been modelled as adiabatic. This renders the heat transfer problem essentially axisymmetric, since the temperature distribution is a function only of radial distance. A simple FEA representation uses the y-axis as the axis of revolution, and employs quadrilateral plate elements (see figure 3). In the FEA model the mesh can be refined (i.e. element size reduced) as required to give acceptable accuracy, taking into account the fact that temperature gradients are highest near the probe. Note the relatively few elements required to represent the problem. The element dimension in the y-direction is arbitrary, since there is no temperature gradient along this axis. This model is not yet aiming at a detailed representation of the probe itself, so the left-hand edge represents a point at the outer surface of the probe (diameter $D = 2$ mm). The heat input to the test material is simply

calculated by expressing the output from the line source as a flux, given by $Q = 3 \text{ (W/m)} / (\pi D) = 477 \text{ W/m}^2$.

For reasons of comparison, the model is run with the hypothetical thermal properties used in the original calculations ($\rho = 1000 \text{ kg m}^{-3}$, $\lambda = 0.01 \text{ W m}^{-1} \text{ K}^{-1}$, $C_p = 100 \text{ J kg}^{-1} \text{ K}^{-1}$, giving a diffusivity of $\alpha = 10^{-7} \text{ m}^2 \text{ s}^{-1}$), see de Wilde et al. (2008). In figure 3, the smallest element size is 1 mm, so we use a time step of 0.1 s.

Note that the FEA software also allows, in 2D, to move from a circular geometry (as appropriate for the thermal probe) to a rectangular geometry (as appropriate for most building details, and the standard grid configuration for Voltra). See figure 4. This will be used in future work; it is not applied in the 1D axisymmetric simulations discussed above.



Fig. 3: Axisymmetric representation of material surrounding thermal probe. The model comprises 24 quadrilateral elements and 123 nodes.

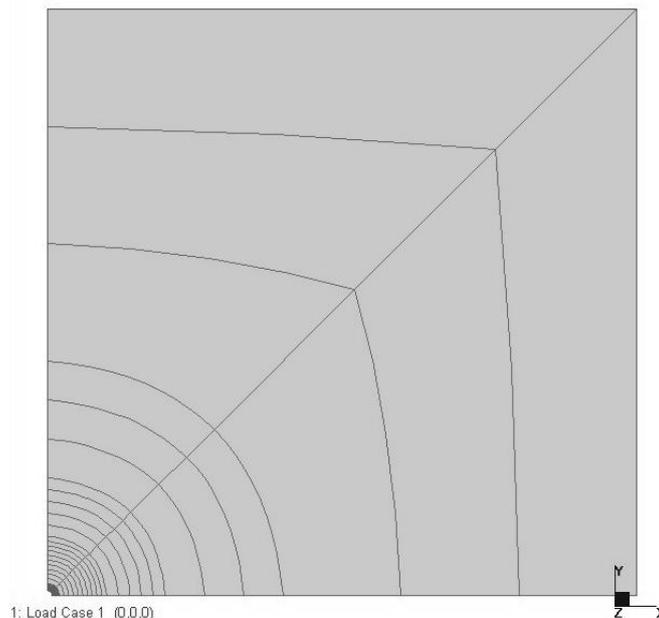


Figure 4: combination of circular and rectangular grids in 2D, for future work.

FEA (STRAND7) RESULTS

Figure 5 shows the calculated probe temperature for the theoretical material using all three numerical methods: FEA, Voltra and numerical solution of the analytical solution, provided by the Blackwell long time expression with the four constants, equation (2) of de Wilde (2009). Good agreement can be observed between both the methods and the Blackwell theory at long times, with all three returning the input thermal properties as derived from slope and intercept of the curve. The deviations at small times are more complex. The ‘theory’ curve is derived from an approximate expression in which some terms have

been neglected, and hence does not converge properly as $t \rightarrow 0$. The disagreement between the two numerical models at $t < 100$ s is likely to be caused by the different representation of the heat source boundary condition. It is of interest that the FEA solution appears to converge towards the analytical solution much earlier than the Voltra model.

It should also be noted that the FEA calculation required a CPU runtime of only 12.6 s on a medium-specification laptop. Voltra simulation of similar cases, in contrast, can take 23 hours.

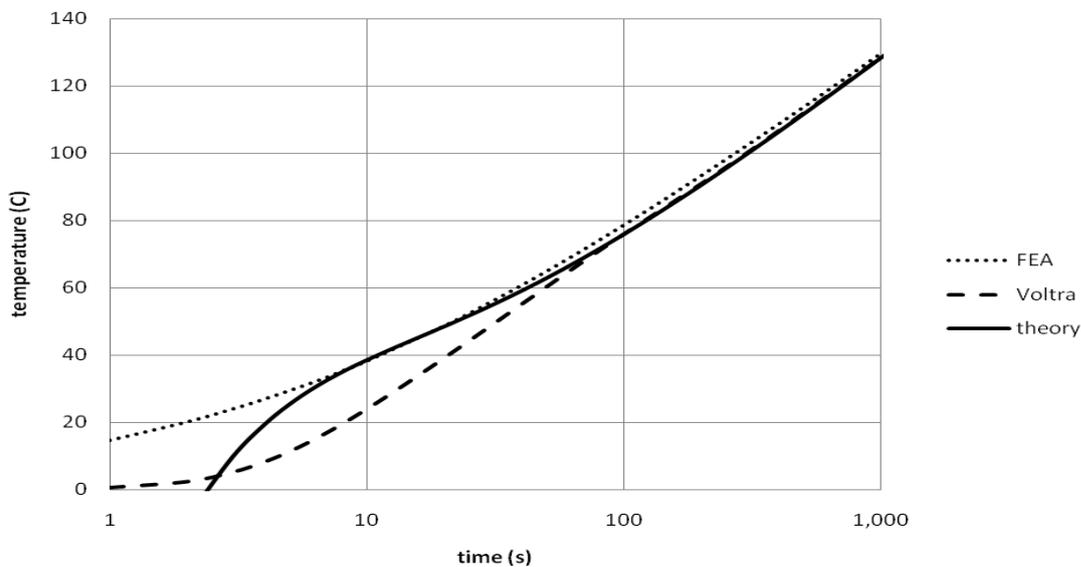


Figure 5: Comparison between probe temperature vs. time for FEA, Voltra and analytical results.

CONCLUSIONS AND REMARKS

1. The probe to sample conduction value H remains an object of investigation. Initial work is reported in this paper, but remains inconclusive.
2. In order to move the research forward, a modelling paradigm shift has taken place from dedicated building physics transient heat transfer software to general engineering multipurpose finite element analysis software. For a 1D model of a thermal probe, results obtained with either method have been compared to each other, and to an analytical solution.
3. Results obtained with FEA give results that are much closer to the Blackwell theory at short times, and hopefully will lead to better understanding of the probe behaviour at these short times.
4. The work reported in this paper, and the transition from one modelling approach to another, serves as a reminder for the need to constantly check that an appropriate simulation methodology is applied in any research project. The need to provide continuity with earlier research efforts needs to be balanced with accuracy of the results, and practical considerations.
5. Future work will investigate potential consequences of using material properties as measured by a thermal probe in-situ, rather than by the hot box method, on the overall thermal performance of buildings.

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