DEVELOPMENT OF A SIMULATION TOOL FOR CALCULATING MAXIMUM ALLOWABLE COOLING TIME IN CLOSED TYPE COOLING TOWER

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ABSTRACT

In winter, when the external air temperature is below zero, there is a risk of damage to coils in a closed type cooling tower due to freezing of cooling water. In warm climates, a method of draining cooling water from the coil can not be adopted to prevent freezing because a cooling load exists even in winter. It is necessary to make warm water circulate through the coil in closed type cooling tower. In this case, it isn’t clear when to operate an electric heater and a circulating warm water pump. The authors developed a simulation tool to calculate the maximum allowable cooling time in a closed type cooling tower taking into account the pressure increase due to freezing of cooling water and the allowable elastic deformation. Based on the calculated maximum allowable cooling time, an optimal operational method for frost prevention can be determined.

INTRODUCTION

Recently, the internal heating load is increasing due to the office automation of buildings. Therefore, even in warm regions, there are air conditioning needs in winter, and intermittent operation of air conditioners is required. In this case, because it is not possible to remove the water from the coil of a closed type cooling tower or AHU, the following three methods are adopted to prevent accidental freezing of the coil.

a. Adding antifreeze liquid to cooling water.
b. Installing an electric heater to the cooling water system.
c. Making warm water circulate through the coil in closed type cooling tower.

Method a is evaluated as not desirable due to the increase in initial costs, decrease in heat exchange performance of the coil due to the use of antifreeze and increase in environmental load caused by the disposal of the main antifreeze component which is toxic ethylene glycol. The idea behind methods b and c is simple and both are easy to execute. However there are problems associated with the formulation of an optimal control method for operating the electric heater and pump for circulating warm water based on external conditions such as the external air temperature or wind velocity.

Especially in warm regions where the external air temperature rarely falls below zero throughout the year, servicing of freeze prevention systems for coils in a closed type cooling towers or AHU is currently almost never performed. In addition, assuming that freezing prevention measures (b or c) for air conditioning equipment were adopted in the design stage, the designer or equipment manufacturer do not indicate a clear method of operating the equipment and leave the operation control of the freezing prevention system to an operator, accidents causing freezing damage to the coils of a closed type cooling tower or AHU as a result of improper operation control are often reported.

In order to formulate the optimal operation control policy at varied outdoor air temperature and wind conditions for a warm water circulating type freezing prevention system for a closed type cooling tower, the authors have developed a simple simulation tool to calculate the maximum allowable cooling time for coils in a closed type cooling tower. In this paper, a summary of this tool and the calculation results for maximum allowable cooling time of an actual closed type cooling tower are reported.

REVIEW OF PAST RESEARCH

London (1943) firstly researched freezing problems of water in tubes as a Stefan problem and sought a relation between the amount of freezing and the cooling time. Goodman (1958) assumed a temperature distribution in an ice layer and proposed an approximate mathematical technique utilizing a “heat-balance integral” for determining the location of the melt line in heat-conduction problems involving a change of phase. Muehrbauer (1965) improved this method and applied it freezing problems, but precise solutions could be obtained only in limited situations.

Murray (1959) proposed a method of dividing a frozen section and a not-yet frozen section into a fixed number of separate temperature points, and determining the position of the freezing aspect using a method of interpolation with reference to a finite difference between movable temperature points and fixed temperature points.

Kodo (1960) proposed a differential equation for one dimensional heat conduction describing the change of the ice inner diameter associated with freezing of the inside surface of a tube and calculated an approximate solution of the cooling time required to freeze all the water in the tube.

Tien (1967) and Cho (1969) applied Goodman's method to a problem regarding the distribution of sensible heat in a solid phase, solid-liquid coexisting phase and a liquid phase. They calculated the temperature distribution of each phase and obtained a solution that closely resembled an exact solution of the semi-unlimited temperature field.

The solution to the “heat-balance integral” method can be obtained simply. However, it is possible to apply only when the shape of the temperature field and the boundary conditions are simple and has the disadvantage that a departure error exists when an initial temperature field is specified.

Katayama (1974) proposed an enthalpy method which did not consider a solid/liquid boundary by using a concept of an apparent calorific capacity including latent heat and in which the physical values depend on temperature. Voller’s (1981) improvement resulted in a useful analysis method which can be applied to complicated temperature fields.

However, methods of analyzing water freezing problems in a tube which as discussed above sought a relationship between the freezing quantity and freezing time, but problems associated with pressure increase or decrease in freezing point which accompany freezing have not been examined. Sugawara (1983) deduced a relation between displacement of the freezing interface and the amount of pressure increase inside a tube when water freezes in a sealed tube from the relational expression of Clausius-Clapeyron, and proposed a basic analytical method to prevent tube fracture as a result of freezing.

Inaba (1990) reported the following results for freezing experiment on city-water bending tubes. 1) Since the local heat transfer coefficient of the bending section is large, promotion of freezing is retarded. 2) Since the straight tube in the vicinity of the bending section freezes first, the bending section is blocked. 3) The water in the bent section finally is in a closed state, and the tube is destroyed by freezing.

Chiba (2003) reported the following results for freezing damage experiments on coils in an AHU. 1) Freezing almost always begins from bent sections and the connection sections. 2) Because the material strength of the tube deformation section is reduced by repeated prior freezing to a level which does not reach a level which destroys the coil with freezing, and the tube is destroyed by freezing although the external air conditions do not reach the conditions for freezing destruction of the tube. Then, Chiba prepared a detailed simulation program which develops Sugawara's analytical in three dimensional finite element method concerning the water freezing problem in a coil under steady flow in order to simulate freeze position and the growth direction of the water in the coil of the AHU, the change in internal pressure, and the distortion of the tube wall. Experimental results were compared in order to verify the precision of the simulation program. These results were used to examine the change of the inside pressure of a tube.

However, although the precision of many methods is good regards the freezing damage of circular tubes and the coils, but practical application is hindered by the complicated nature of the calculation. Because of that, the authors have developed a simple analytical tool based on the energy balance at process of the water in coil from initial temperature, cooling to 0 C, until it freezes.

DERIVATION OF FORMULA FOR ALLOWABLE COOLING TIME OF COIL

The problem to be handled in this paper is to simulate the cooling and freezing process of stationary water in a copper tube coil of a closed type cooling tower, when the external air temperature takes a minus value. In addition, as mentioned earlier, water in the bending section is closed airtight by freezing of the straight tube even with when there is a steady flow in the coil. Eventually the bending section is damaged due to the freezing of the stationary water. This simulation tool applies this situation in the same way.

The simulation model of freeze process of the stationary water in the tube is shown in Figure 1. First of all, the water temperature in the tube from an initial temperature changes to 0 C. Then, freezing is commenced from the inner wall surface of the tube. When the water the in tube undergoes a phase change to ice, the volume increases and pressure on the inside wall of the tube occurs which causes distortion of the tube wall. Fracturing of the copper tube will not occur if the increase in of tube diameter is within the elastic extension range of the copper. Therefore the maximum allowable freezing quantity in the copper tube can be calculated from the allowance elastic extension of the copper tube.

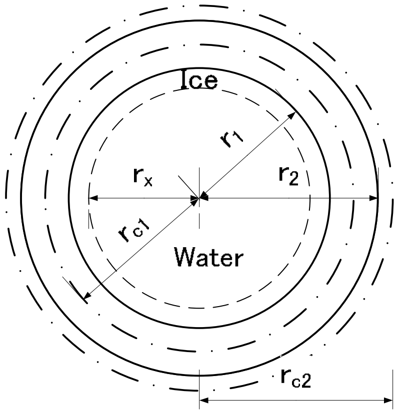

Figure 1 Computation model allowable freezing quantity in tube

Assumed Conditions of the Simulation Model

The following assumptions have been made in order to avoid various convergence calculations since the purpose of this research to develop a simple simulation tool for coil freezing destruction problem which can be calculated in an Excel environment of Excel.

1) Generally, since there is an expansion tank and a pressure safety valve in a chilled water pipeline, the pressure increase of a system in which freezing occurs inside the coil can not exceed the allowable pressure of the safety valve. But, because the straight section in tube freezes first and the water in the bent section becomes closed, the simulation model assumes that entrance and outlet of the coil is closed and that the water in the coil is in a stationary state.

2) The increase in volume due to freeze of the water in the tube is applied to the increase in all tube diameters.

3) In order to avoid convergence calculations regarding internal pressure of tube and the fall in the freezing point temperature, the fall in the freezing temperature which accompanies the pressure rise inside of tube is ignored.

Cooling time for Sensible heat exchange

The variation in sensible heat in the water in the tube Q

$$Q = c_w M\,(t_{w0} - t_{wh}) \tag{1}$$

The heat quantity Qc, which is discharged by the heat transmission of the tube wall

$$Q_c = 2\pi LKh\ \Delta t_m \tag{2}$$

Here, Δt_m is (time) logarithmic mean temperature difference between the water temperature in the tube and the air temperature near the outside surface; it is calculated with formula (3).

$$\Delta t_m = \frac{t_{wh} - t_{w0}}{\ln[(t_a - t_{w0})/(t_a - t_{wh})]} \tag{3}$$

From heat quantity balance of Q and Qc, we can coalescing formula (1) and formula (2) to obtain the cooling time for sensible heat exchange h which is time taken for the water temperature in tube to fall from an initial temperature fall to t_{wh}, and is expressed by formula (4).

$$h = \frac{c_w r_1^{\,2} \rho_w \ln[(t_a - t_{w0})/(t_a - t_{wh})]}{2K} \tag{4}$$

The heat transmission coefficient K of the tube wall is calculated by expression (5).

$$\frac{1}{K} = \frac{1}{\alpha_1 r_1} + \frac{1}{\lambda_1}\ln\frac{r_2}{r_1} + \frac{1}{\alpha_2 r_2}\ \text{(In cooling)} \tag{5}$$

The heat transfer coefficient α1 on the surface of the tube wall is obtained from expression (6) and expression (7), considering natural convection between the water in tube and the inside surface of tube wall (SHASEJ, 1995).

$$Nu1 = 3.65 + \frac{0.0668(d/L)\,\mathrm{Re}\,\mathrm{Pr}}{1 + 0.04[(d/L)\,\mathrm{Re}\,\mathrm{Pr}]^{2/3}} \tag{6}$$

$$Nu\,1 = \frac{\alpha_1 2 r_1}{\lambda_w} \tag{7}$$

Because forced convection is caused on the outside surface of tube wall by the external wind velocity, the heat transfer coefficient on the external surface of tube wall is obtained from expression (8) and expression (9) (SHASEJ, 1995).

$$Nu2 = 1.11C\,\mathrm{Re}^m\,\mathrm{Pr}^{0.31} \tag{8}$$

$$Nu\ 2 = \frac{\alpha_2 2 r_2}{\lambda_a} \tag{9}$$

Here, C and m are coefficients which depend on the Reynolds number.

Maximum allowable freezing time

Calories absorbed by freezing in Δh time interval

$$Q_s = 2\pi L\varphi\rho_w (r_{x,t-1}^2 - r_{x,t}^2) \tag{10}$$

On the other hand, because the formula to calculate the heat quantity discharged by the heat transmission of the tube wall at Δh time zone is the same formula (2), it is omitted here.

However, when ice freezes on the inside surface of tube wall, the convective heat transfer ratio of the boundary aspect between the ice and the water is infinite and because resistance of heat conduction of the ice exists, the coefficient of heat transmission from the inside surface of the ice to the air near the tube external surface by formula (11) is calculated.

$$\frac{1}{K} = \frac{1}{\lambda_i}\ln\frac{r_{c1}}{r_x} + \frac{1}{\lambda_1}\ln\frac{r_{c2}}{r_{c1}} + \frac{1}{\alpha_2 r_{c2}}\quad \text{(In freeze)} \tag{11}$$

Moreover, the arithmetic expression of heat transfer coefficient α_2 of the outside surface of the tube is similar to formula (8) and formula (9).

From the heat quantity balance between the heat transmission quantity in the tube wall and the heat quantity absorbed by freezing, coalescing formula (2) and formula (10), it becomes deformed and the inside surface radius of the ice after the Δh time is calculated by formula (12).

$$r_x = \sqrt{r_{x,t-1}^2 - \frac{K\,\Delta h\,\Delta t}{\varphi\rho_w}} \tag{12}$$

Since the total mass of remaining water and the mass of the ice which freezes inside the tube is equal to the mass of the water before the freezing, the outside radius of the ice, r_{c1}, is calculated by formula (13).

$$\rho_w \pi r_x^{\,2} + \rho_i \pi[(r_{c1})^2 - r_x^2] = \rho_w \pi r_1^2 \tag{13}$$

Since the sectional area of the tube wall is always fixed, the relational expression between the warp of the tube inside radius ε and the warp ε_o of the tube outside radius shown expression (14) is established, and as a result, the tube outside radius r_{c2} can be calculated.

$$\frac{1}{(1+\varepsilon)^2} = 1 - \left(\frac{r_{c2}}{r_{c1}}\right)^2 \left\{1 - \frac{1}{(1+\varepsilon_o)^2}\right\} \quad (14)$$

Therefore, the tube outside radius r_{c2} after time of Δh is calculated from expression (15).

$$r_{c2} = r_2 (1 + \varepsilon_o) \quad (15)$$

The thickness of ice after time of Δh can be calculated by expression (16).

$$D_i = r_{c1} - r_x \quad (16)$$

On the other hand, the maximum allowable elasticity expansion of the nonferrous metals material is 0.2% in general (National Observatory, 2005) as shown in Figure 2. In other words, for a copper tube, the inside radius of the tube $r_{1,\max}$ of the maximum allowable elastic extension is $1.002r_1$.

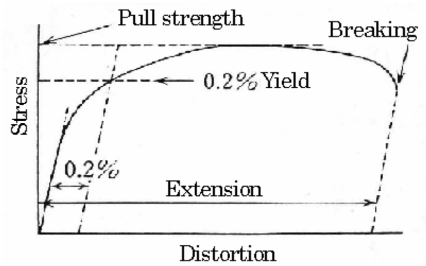

Figure 2 Stress-strain diagram of nonferrous metals material

When the physical values of ice and water and $r_{1,max}$ are substituted into formula (12), the inside surface radius of the ice, $r_{x,max}$, is calculated by formula (17).

$$r_{x,\max} = \sqrt{\frac{\rho_w - 1.002^2 * \rho_i}{\rho_w - \rho_i}} r_1 = \sqrt{\frac{999.84 - 1.002^2 * 920}{999.84 - 920}} r_1 = 0.977 r_1 \quad (17)$$

Therefore, the maximum allowable ice thickness $D_{i,max}$ is calculated by the difference between the maximum allowable tube inside radius $r_{i,max}$ and maximum allowable ice inside surface radius $r_{x,max}$, and it becomes $0.025r_1$.

$$D_{i,\max} = 1.002\, r_1 - 0.977\, r_1 = 0.025\, r_1 \quad (18)$$

By increasing calculation time H for each step Δh, formula (2), and formulae (10) - (16) are calculated repeatedly until a ice thickness D_i equal to $D_{i,max}$. The total calculation time is the maximum allowable freezing time.

Limiting pressure for copper tube coil freezing damage

The experimental results (Sato,1960), concerning the relationship between the average shearing stress (τ) of a cross section of a brass tube resulting internal pressure and the distortion (ε) of the optional point in the tube wall is shown in Figure 3, and the experiment conditions of each case are shown in Table 1. The distribution of the shearing stress τ of the brass tube in the section is calculated by Figure 3. The secondary recursive between τ and ε is calculated from Figure 3, formula (19) and becomes.

$$\tau = 8.2714 + 0.9644\,\varepsilon - 0.0162\,\varepsilon^2 \quad (19)$$

Moreover, the internal pressure of the tube is obtained by formula (20).

$$p = 2\int_{r_{c1}}^{r_{c2}} \tau \frac{dr}{r} \quad (20)$$

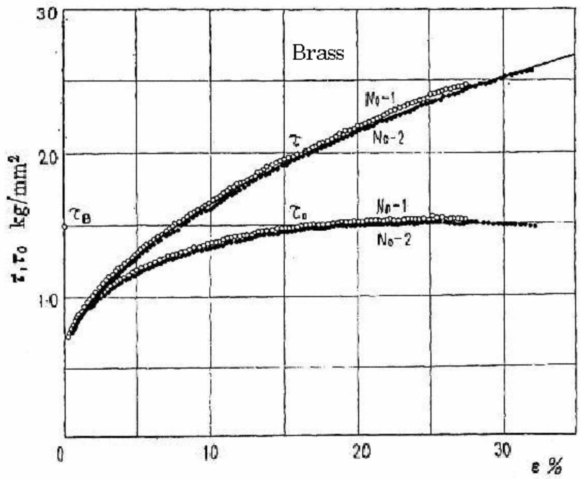

Figure 3 The relation between the distortion and the straightening shearing stress on inside radius of the brass tube

Extreme pressure of freezing damage of the brass tube is calculated by formula (21).

$$P_B = 2\tau_B \log\frac{r_2}{r_1} \quad (21)$$

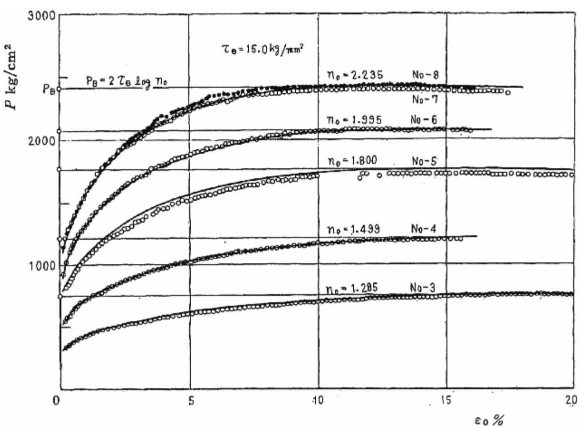

Figure 4 The relation between the distortion on outside radius of brass tube and limit pressure inside brass tube

It is understood that limiting allowable shearing stress τ_B value of the brass tube is 14.71 [kPa] from the experimental result in Figure 3. From formula (21) the calculation result related to distortion ε_o of the internal pressure of the brass tube and the outside radius which were calculated and the comparison of the measured value are shown in Figure 4. The relationship between the internal pressure of the brass tube and the distortion in the outside radius of the brass tube at various wall thicknesses can be estimated by formulas (19) - (21).

Table 1 Experiment condition by Sato

	d_o [mm]	d [mm]	d_o/d
No.1	17.99	16.02	1.123
No.2	18.00	16.02	1.123
No.3	18.00	14.01	1.285
No.4	17.99	12.00	1.499
No.5	18.00	10.00	1.800
No.6	17.99	9.02	1.995
No.7	17.99	8.05	2.235
No.8	18.00	8.04	2.235

EXAMPLE OF CALCULATING THE CLOSED TYPE COOLING TOWER

In order to formulate the optimal operation control policy for a warm water circulating type freezing prevention system for a closed type cooling tower, it is necessary to confirm the maximum allowable cooling time at varied outdoor air condition. A calculation example of maximum allowable cooling time for an actual closed type cooling tower is reported in the following.

Outline of object closed type cooling tower

A figure showing the overall operation of the heat source which possesses object closed type cooling tower is shown in Figure 5 and a summary of the object cooling tower is shown in Table 2.

Table 2 Outline of object closed type cooling tower

Cooling capacity [kW]	965	d_o [mm]	19.05
Holding water quantity [m^3]	1.5	d [mm]	18.25

Table 3 Computational condition

Initial water temperature [C]	10		
External wind velocity [m/s]	0.5	10	
External air temperature [C]	-1 (Osaka)	-5 (Sandai)	-10 (Hakotate)

Calculation conditions

The external wind velocity was assumed to have a maximum value of 10 m/s and a weak value of less is 0.5 m/s. An initial water temperature in the coil of the object closed type cooling tower is set to 10.0 C, and the external air temperature assumed a heating load design temperature in three regions of -1.0 C (Osaka), -5.0 C (Sendai), and -10.0 C (Hakodate) (SHASEJ, 1995). The condition for calculation shown in Table 3, main physical properties value is shown in Table 4.

Calculation result and consideration of the maximum allowable cooling time

The calculation condition of each case and the allowable cooling time of the coil are shown in Table 5. Matters related to the external air temperature and the wind velocity relationship between the allowable cooling time of a closed type cooling tower are shown in Figure 6. The following findings are obtained.

1) The allowable cooling time of the coil is shortened by lowering of the air temperature or increasing the wind velocity passing on the coil external side. However, the influence of external air temperature on the allowable cooling time of the coil is larger than the passing wind velocity on the coil external surface.

2) The required time for the maximum allowable freezing time in the coil is longer when external air temperature at -1.0 C and shorter when the external air temperature at -5.0 C and -10.0 C in comparison to the required time for the water temperature in the coil to fall from 10 C to 0 C.

3) When an initial water temperature in the coil is 10.0 C, the allowable cooling time of the coil is 0.63 hours or less in the cold region where the external air temperature in winter is -5.0C or less (Sendai or Hakodate). It is thought that a warm water circulation type freeze prevention system should always be operated. On the other hand, it is necessary to operate the warm water circulating pump of the warm water circulation type freeze prevention system with an electric heater for one hour after closed type cooling terminates operation because there is only one allowable cooling time of the coil in warm region (Osaka ,the external air temperature is -1.0 C), when the wind velocity passing on the external side of the coil is 10 m/s.

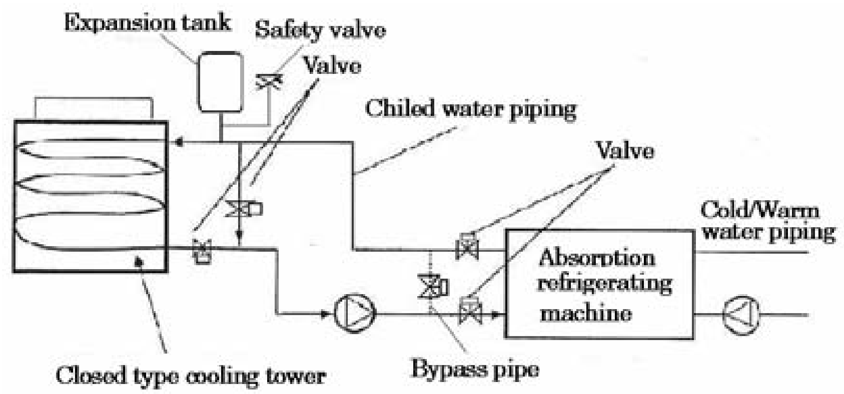

Figure 5 The heat source system which possesses object closed type cooling tower

Table 5 Calculation condition of each case and allowable cooling time of coil

External wind velocity	0.5 [m/s]		10 [m/s]	
External air temperature	Cooling time [minute]	Freezing time [minute]	Cooling time [minute]	Freezing time [minute]
-1.0 [C]	50	67.5	16	48.6
-5.0 [C]	23	14.6	7	4.7
-10.0 [C]	15	7.6	5	2.3

Table 4 Physical properties value

C_w [J/(kg・K)]	4,198	λ_w [W/(m・K)]	576×10^{-3}
υ_a [m^2/s]	15×10^{-6}	λ_a [W/(m・K)]	25.7×10^{-3}
φ [kJ/kg]	335	λ_i [W/(m・K)]	2.2
λ_1 [W/(m・K)]	386		

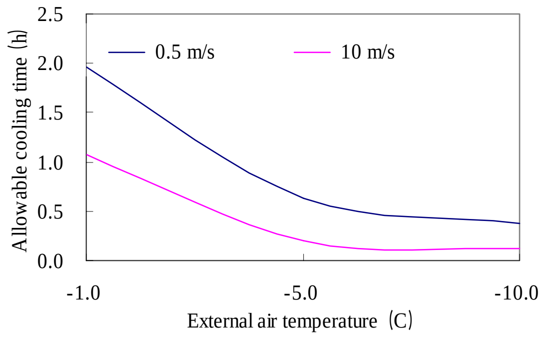

Figure 6 Relation between external air temperature/ wind velocity and allowable cooling time

Verification of inside pressure of tube at maximum allowable freeze quantitative time

The size and material of the copper tube in object closed type cooling tower differs from the size and material of the experimental material of Sato (1960), but as mentioned earlier, it can apply (19) - (21). If amount of the radius in the copper tube (=0.2%), the maximum allowable elasticity expansion is substituted for expression (19) and the shearing stress τ in the section of the copper tube becomes 8.273kPa. The internal pressure at the maximum allowable freezing time becomes 16.563kPa since this τ value, r_{c1}, and r_{c2} are substituted for formula (20) and is integrated.

Moreover, when τ_B value of the inside radius, outside radius of the copper tube coil in the actual closed type cooling tower is substituted into formula (21), the extreme pressure P_B value of the object copper tube coil becomes 54.816 [kPa], it is 3.3 times greater than the inside pressure of copper tube at the maximum allowable freezing time. This shows that the freezing destruction does not occur in the copper tube coil at the amount of the maximum allowable freezing time.

SUMMARY

In this paper, the simple simulation tool to calculate the allowable cooling time of the coil in a closed type cooling tower which considers the allowance elastic extension of the copper tube coil was developed.

This simple simulation tool is very convenient because it can be used for calculations in an Excel environment. If heat transfer rate on inside of tube α_1 or circulation water volume can be obtained, this tool can be applied to the freezing calculation when water flows in the tube.

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SIGN

C_w ：Specific heat of water [J/(kg・K)]
d ：Diameter inside tube [m]
d_o ：Outside diameter of tube [m]
D_i ：Thickness of ice [m]
$D_{i,max}$ ：Largest allowable thickness of ice [m]
h ：Cooling time for Sensible heat exchange [s]
H : Calculation time for freezing [s]
K ：Heat transmission coefficient of tube wall [W/(m・K)]
L ：Length of tube [m]
M ：Mass of water [kg]
Nu_1 ：Free convection Nusselt in inside surface of tube
Nu_2 ：Free convection Nusselt in outside surface of tube
Pr ：Prandtl number
P_B ：Limit pressure of frozen damage of brass tube [kPa]
Q ：Cooling heat loss of water in tube [J]
Qc ：Heat transmission quantity of tube wall [J]
Qs ：Calorie absorbed to freezing [J]
Re ：Reynolds number
t_{w0} ：Initial water temperature in tube [C]
t_{wh} ：Water temperature after h time [C]
t_a ：External air temperature [C]
Δh ：Computing time step to frozen [s]
Δt_m ：Logarithm mean temperature difference [C]
r_1 ：Inside radius of tube [m]
$r_{1,max}$ ：The allowance inside radius of tube [m]
r_2 ：Outside radius of tube [m]
r_{c1} ：Inside radius of tube (=outside radius of ice) after elastic deformation [m]
r_{c2} ：Outside radius of tube after elastic deformation [m]
r_x ：Inside radius of ice [m]
$r_{x,max}$ ：Maximum inside radius of ice [m]
$r_{x,t-1}$ ：Inside radius of ice in computing t-1 time step [m]
α_1 ：Heat transfer rate on inside of tube [W/(m^2・K)]
α_2 ：Heat transfer rate on outside of tube [W/(m^2・K)]
ε ：Distortion of radius inside of tube [%]
ε_o ：Distortion of radius outside of tube [%]
λ_1 ：Thermal conductivity of tube [W/(m・K)]
λ_a ：Thermal conductivity of air [W/(m・K)]
λ_i ：Thermal conductivity of ice [W/(m・K)]
λ_w ：Thermal conductivity of water [W/(m・K)]
ρ_w ：Density of water [kg/m^3]
τ ：Section shearing stress of tube [kPa]
τ_B ：Limit section shearing stress of tube [kPa]
ν_a ：Kinematic viscosity of air [m^2/s]
ψ ：Coagulation heat of ice [kJ/kg]