

## USING AN INVERSE METHOD TO EVALUATE ENVELOPE THERMAL PROPERTIES

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### ABSTRACT

The objective of this work is to present an inverse method solving the transient heat-transfer problem in walls aiming to estimate its thermal properties. The procedure uses a finite difference numerical scheme, simulated in the environment SIMSPARK that is non object oriented and allows solving highly non linear problems. A method aiming to estimate building envelope thermal characteristics is elaborated knowing experimental in situ measurements. The results present a good agreement between the forward and the inverse method respectively on a high-weight concrete and then on a Phase Change Material wall. Extrapolation of the method on a building model is realized to find the required air ventilation rate for a summer and a winter climate. Finally, the procedure is applied on a multilayered wall to estimate the U-value using real experimental data. The study shows clearly the possibility to identify a modification occurred on the wall composition and the moment at which this modification happens. The method can be used for different applications and provides a good accuracy on the results.

### INTRODUCTION

Since the mid eighties, building energy simulation tools have been developed very rapidly, in direct relation to the calculation performance of computers. At the early stages of design, they are more adapted and more accurate to perform estimations on energy saving and thermal comfort. Due to this progress, the simulation of building models by the inverse method is becoming easier.

One application of the inverse heat conduction problem is to assess the wall thermal properties from a measured temperature profile (Antonopoulos and Vrachopoulos, 1995; Chang and Chang, 2009).

The present study elaborates an inverse method aiming to estimate building envelope thermal characteristics, knowing experimental in situ measurements (given indoor and outdoor temperatures and heat fluxes on surfaces). Once the method is operative, it could be used for optimization purposes.

In this study we analyze the inversion model assessment for numerical simulation data and for experimental data. In both cases, there is a model of a transient wall with or without temperature-dependant thermal properties. We also try to extrapolate the method of application for another kind of problem such as a whole building ventilation rate calculation. Finally, an investigation on a multilayered wall subjected to modification of its thermal properties will show the possibilities for the method to be used for a real case of study.

### INVERSION MODEL FROM NUMERICAL SIMULATION DATA

This section describes an inversion method based on a heat conduction transfer through a single-layered wall. This wall is a separation between an interior ambiance maintained at a constant temperature and the outside environment. First we consider the case of the wall being made of high-weight concrete and then the case of a wall made of Phase Change Material wallboard.

In this part, no real experimentation was carried out: the results come from direct classic simulation or "forward model simulation". First, we treat the problem with the forward method in order to collect simulation numerical data. This output data will be introduced as an input to the inverse model where we can set system parameters as unknown.

The procedure developed is a numerical model programmed and simulated in the environment SIMSPARK (SIMSPARK platform is the tool used for these simulations). It is based on SPARK, an object-oriented equation-based environment (Lawrence Berkeley National Laboratory and Ayres Sowell Associates Inc, 2003). The interest of this environment in our case is that it is non-oriented. The inversion of a problem is made easier, it being unnecessary to reprogramme the entire model. Also, the use of graph-theory to decompose the problem into strong components and the robustness of the solver (Sowell et al., 2004) allows it to solve highly non-linear problems.

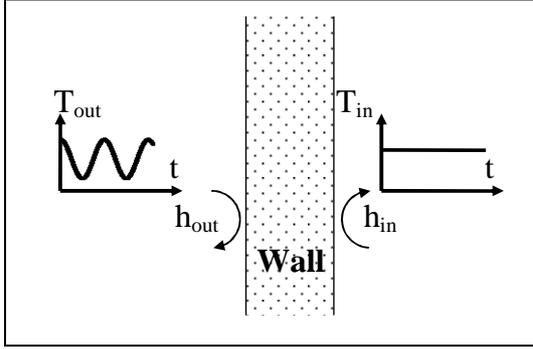


Figure 1: Interior and exterior conditions on the wall

## Methodology

The transient one-dimensional heat conduction in a single layered wall may be expressed as

$$\lambda \frac{\partial^2 T(x,t)}{\partial x^2} = \rho c \frac{\partial T(x,t)}{\partial t} \quad 0 \leq x \leq L, \quad t > 0, \quad (1)$$

Where  $x$  and  $t$  are the space and time coordinates respectively,  $T(x,t)$  is the temperature, and  $\lambda$ ,  $\rho$ ,  $c$  and  $L$  the thermal conductivity, the density, the specific heat and the thickness of the wall respectively. The boundary conditions at the outdoor and indoor wall surfaces and the initial conditions may be written as

$$q_o(t) = h_o [T_o(t) - T(0,t)] + aI(t) \quad t > 0, \quad (2)$$

$$q_i(t) = h_i [T(L,t) - T_i(t)] \quad t > 0, \quad (3)$$

$$T(x,0) = F(x) \quad (4)$$

where  $T_o(t)$  and  $T_i(t)$  are the outdoor and indoor air temperatures respectively,  $q_o(t)$  and  $h_o$  are the heat flow and the heat-transfer coefficient at the outdoor wall surface respectively,  $q_i(t)$  and  $h_i$  are the corresponding quantities at the indoor wall surface, and  $F(x)$  is the initial temperature field.  $I(t)$ , and  $a$  are the incident total solar radiation and the thermal absorptance of the outdoor wall surface respectively.

The conduction equation is solved following the finite difference method using a uniform computational mesh of  $m$  nodes.

According to equation (1), for an element  $i$  of the wall of width  $dx$  at the time step  $j$ :

$$\phi_{i-1,i}^j - \phi_{i,i+1}^j = \rho_i \cdot c_i \cdot h \cdot w \cdot \Delta x \cdot \frac{dT_i^j}{dt} \quad (5)$$

Where  $\phi_{i-1,i}^j$  and  $\phi_{i,i+1}^j$  are respectively the conductive fluxes entering and exiting the element  $i$  at the time step  $j$ ,  $\rho_i$ ,  $c_i$  is the thermophysical properties element, and  $h \cdot w$ , the wall section dimensions.

The numerical resolution is based on an implicit differencing scheme for the second order using a variable time step (Tittlein, 2008). The numerical procedure is written below for the inverse determination of the specific heat using measured values for the temperatures and heat flows.

$$\frac{dT_i^j}{dt} = \frac{T_i^{j-2} \Delta t_i^2 - (\Delta t_i + \Delta t_{i-1})^2 T_i^{j-1} + (\Delta t_{i-1}^2 + 2\Delta t_i \Delta t_{i-1}) T_i^j}{\Delta t_i \Delta t_{i-1}^2 + \Delta t_i^2 \Delta t_{i-1}} \quad (6)$$

$$c_i = \frac{(\phi_{i-1,i}^j - \phi_{i,i+1}^j) (\Delta t_i \Delta t_{i-1}^2 + \Delta t_i^2 \Delta t_{i-1})}{\rho_i h w \Delta x (T_i^{j-2} \Delta t_i^2 - (\Delta t_i + \Delta t_{i-1})^2 T_i^{j-1} + (\Delta t_{i-1}^2 + 2\Delta t_i \Delta t_{i-1}) T_i^j)} \quad (7)$$

Defined in this way, the inverse method avoids the iteration process encountered necessary when minimizing the difference between calculated and measured temperature at specific points on the wall.

## Application of the method for wall models

### Case of a structural concrete wall

We consider the case of a 10 cm thick high-weight structural concrete wall. The material specific heat is 980 J/°C.kg, the density, 2300 kg/m<sup>3</sup>, and the thermal conductivity, 1.8 W/m.K. The indoor temperature is a constant and equal to 20°C. The outdoor temperature has a periodic variation according to the relation:

$$T_o(t) = T_m + T_a \cos\left(\frac{2\pi}{T} t + \phi_0\right) \quad (8)$$

where  $T_m$  and  $T_a$  are respectively the mean value and the amplitude temperature fluctuation, through a period  $T$  equal to 24 hours. For the first example, temperature oscillations occur between 0 and 15°C, meaning that  $T_m = 7.5^\circ\text{C}$  and  $T_a = 7.5^\circ\text{C}$ .

We performed a simulation on the forward model to collect temperature and heat flux values through the wall over a period of 30 days.

Fig. 2(a) represents the temperature profile within the wall during the first 3 days. At  $t=0$ , the temperature field is initialized for indoor and outdoor temperatures respectively equal to 20°C and 15°C so the temperature field  $F(x)$  is linear.

As stated above, the wall is defined under transient heat transfer conditions.

From the simulation output are collected the inside and outside surface temperatures. Adding those

values to the outdoor and the indoor temperatures in input, the “inverse simulation” is performed setting the heat capacity as an output and keeping the rest of the system parameters as known values.

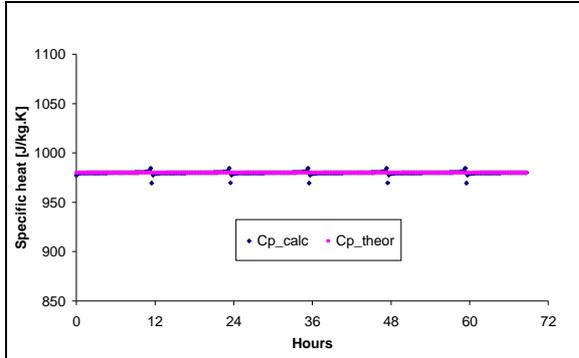


Figure 2: Calculated values of specific heat for the case of concrete wall.

As we can see in fig. 2, the estimation of the specific heat is very stable and near  $979.9 \text{ J.kg}^{-1}.\text{K}^{-1}$  (the theoretical value is  $980 \text{ J.kg}^{-1}.\text{K}^{-1}$ ) with a standard deviation of  $\sigma=1.33$ . The results sequence show periodic oscillations appearing each time the wall temperature derivative is equal to zero. Considering a single interval of 12h, for example  $t=24\text{h}$  to  $t=36\text{h}$ , the mean value for the specific heat becomes  $979.95$  and the standard deviation  $0.33$  which is acceptable. Therefore, the oscillations effects are negligible. Consequently, we can say that the inverse method show a good agreement with the theoretical value used in the forward model.

### Case of a PCM wall

We expect that the inverse method that we are developing will be a way of determining an appropriate wall material combination (for given environment conditions), the possibility to have a Phase Change Material should not be rejected. This is why in this section we are interested in applying the previous method for this kind of material. It also allows us to test the validity of the method for a highly non-linear problem.

The product studied here is commercialized by the company DuPont de Nemours. It consists of 60% microencapsulated PCM, which has a melting temperature of  $22^\circ\text{C}$ . The final form of the PCM material is a 5 mm-thick flexible board with a density of  $1019 \text{ kg/m}^3$ .

The law of PCM behavior used here is the one proposed by Kuznik et al (2008). It is based on a Gaussian curve of the specific heat according to temperature fitted on experimental measurement (cf. Fig. 3 and equation (9)).

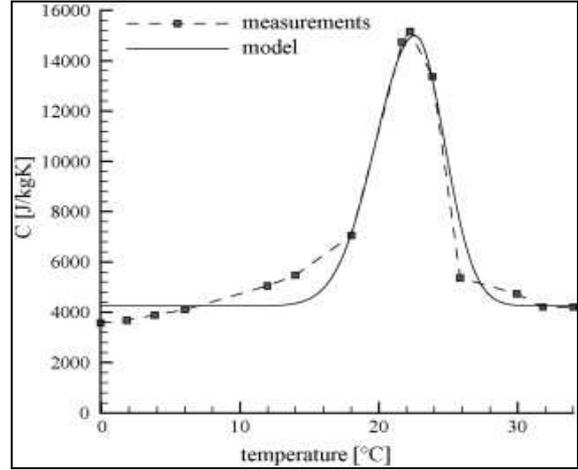


Figure 3: Experimental results and model used by Kuznik et al (2008)

$$C = \begin{cases} 4250 + 10750 \cdot e^{-\left(\frac{22.6-T}{4}\right)^2} & \text{if } T \leq 22.6^\circ\text{C} \\ 4250 + 10750 \cdot e^{-\left(\frac{22.6-T}{3}\right)^2} & \text{if } T > 22.6^\circ\text{C} \end{cases} \quad (9)$$

Thermal conductivity is considered as constant ( $0.2 \text{ W/(m.K)}$ ). Heat transfer in the board is modeled using the classical finite difference scheme presented above, but in each node, heat capacity is linked to the node temperature by equation (1).

As for the case of the structural wall of concrete, the wallboard is submitted to a periodic temperature oscillation on one side and to a constant temperature on the other (Fig. 1). In order to scan the entire phase-change temperature zone, the temperature varies from  $0$  to  $40^\circ\text{C}$  on one side of the wallboard and is maintained at  $24^\circ\text{C}$  on the other side. Then inside temperatures of the PCM are evaluated by a forward model simulation and considered as experimental results for the rest of the study.

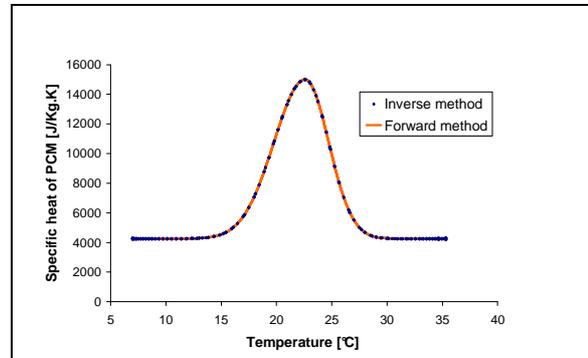


Figure 4: Comparison of the theoretical heat capacity and heat capacity evaluated by the inverse method

These “experimental” results are used as input to the inverse model simulation in order to find the heat capacity of the wallboard at each time step. The same numerical scheme as for the concrete wall has been followed. The PCM heat capacity over time for the forward and the inverse model can be seen on Fig. 4. The heat capacity evolution can be subdivided in three intervals, respectively for  $T < 16^\circ\text{C}$ , then  $16^\circ\text{C} < T < 26^\circ\text{C}$  and finally  $T > 26^\circ\text{C}$ .

For the first and the last intervals, it is visible that like for the previous case, the PCM model behaves quite similar to concrete and becomes less accurate.

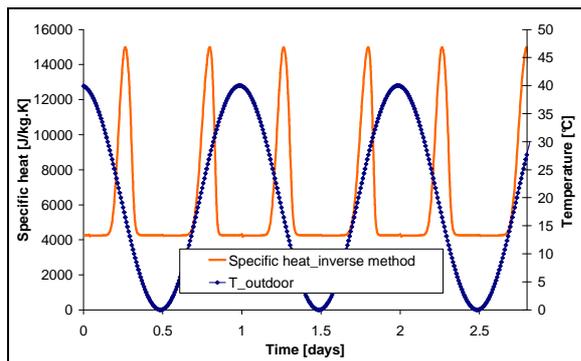


Figure 5: Heat capacity in a node of PCM

Fig. 5 shows that numerical problems can occur when the derivative of the material temperature becomes equal to zero. Equation (5) shows that for a temperature derivative value near to zero, heat capacity would be infinite. In order to avoid this numerical problem and continue the simulation after this point, an infinitely small number ( $10^{-23}$ ) is added to the derivative of temperature in the program. The deviation observed over this interval is  $25.69\text{J}\cdot\text{kg}^{-1}\cdot^\circ\text{C}$  which represent 0.6% of the mean value. As shown, the inverse method is providing results that are practically the same as the forward method. Furthermore, the model developed could be used to make a better characterization of the PCM wallboard and improve the analysis of its thermal behavior at a macroscopic scale.

### Application of the inverse method for a building model

The results presented before lead us to think that direct inversion of the problem as done earlier for the concrete and PCM walls could be applied to a larger problem such as a whole-building simulation.

The building studied here is a two-storey low-energy consumption house that is to be built in grounds of the National Institute for Solar Energy (Chambéry, France). This building is part of an experimental platform called INCAS. Its net floor area is about  $100\text{ m}^2$ . The envelope is made of 20 cm of heavy concrete and 20 cm of insulating material.

A standard forward model simulation is performed with the SIMSPARK platform. This model considers heat transfer across the walls with the method presented above. Indoor radiation is evaluated using fictitious surface method and considering sunspot evolution on the inside walls (Tittlein et al., 2008). Ventilation control is provided by a mechanical heat recovery ventilation system and is based on the evolution of which acts on the inside and outside temperatures. Over-ventilation can be forced by bypassing the heat recovery system when it is needed. Climatic conditions of Chambéry are considered.

The indoor temperature  $T_i(t)$  and the heating power  $P(t)$  are then considered as “experimental” results. The aim of the inverse model simulation is to find the air change rate  $Q(t)$ . In order to do that, the regulation system has been removed and the differentiator presented in equation (6) is used to calculate the indoor air temperature derivative.



Figure 6: 3D view of an experimental building of INCAS project

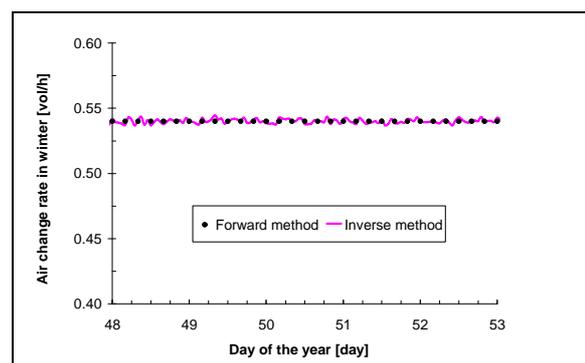


Figure 7: Air change rate with forward and inverse methods during winter

Fig. 7 shows that, in winter, when no over-ventilation is required, the air change rate found with the forward model simulation is very close to that of the inverse model. Inversion in this case is then very efficient. During the summer, when over-ventilation is required, Fig. 8 shows that the inverse method

allows following air change rate even if it changes very quickly. However, it will be noticed that the inverse method generates discontinuities and gives results that do not exactly correspond to the forward method. These differences are due to numerical difficulties in evaluating temperature derivatives when high change-rates occur.

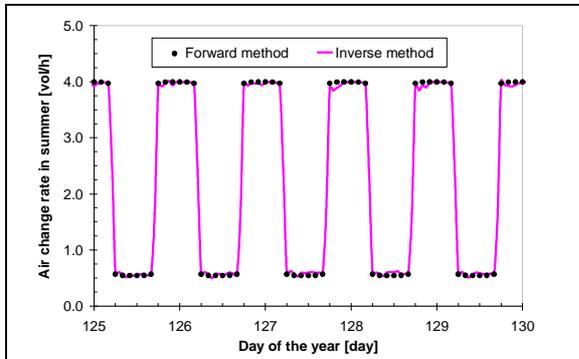


Figure 8: Air change rate with forward and inverse methods in summer (with over-ventilation)

## INVERSION FROM EXPERIMENTAL DATA

### Estimation of a multilayered wall U-value

In the context of the System Identification Competition III (SIC III) organized by the DYNASTEE (DYNAMIC Analysis, Simulation and Testing applied to the Energy and Environmental performance of buildings) network (Enriquez et al., 2008), a considered case study is presented and made available on internet<sup>1</sup>. It consists of a monitored wall in a residential building. The wall is multilayered, as shown in Fig. 10, is built with lightweight concrete block and a cavity providing the insulation. The objective is to improve the wall thermal performance by filling the cavity with insulating material. This should result in lower energy consumption and improved comfort for the occupants.

Experimental data corresponding to the wall case study (cf. Fig. 9) is also accessible. Monitoring was carried out during the winter. A heat flux meter was fixed to the internal surface of the wall.

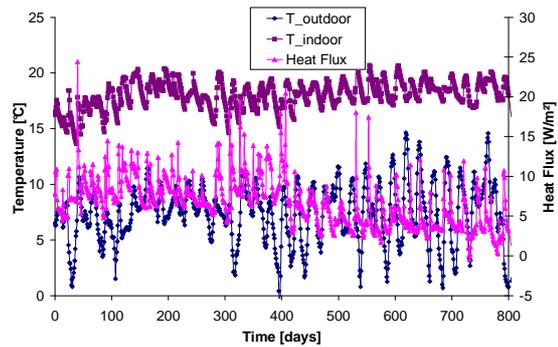


Figure 9: Heat flux, inside and outside temperatures

A temperature sensor was mounted in the room and an external air temperature sensor in a solar radiation shield fixed to the exterior of the north-facing wall. Measurements were logged at 1minute intervals and hourly averages recorded. 817 hours of data was collected.

The main goal of this part is to see how far the inverse method described above can determine the U-values before and after filling the wall cavity. We assume that neither layer thickness nor thermal conductivity is determined.

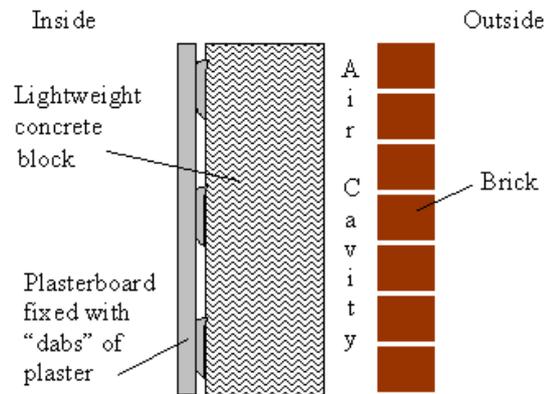


Figure 10: Section of the monitored wall

Here, the inverse problem is solved in a different way: according to the available experimental data, the boundary conditions are the heat flux entering the inside environment and the indoor and outdoor temperatures. We estimate the wall U-value at each time step.

<sup>1</sup> [www.dynastee.info](http://www.dynastee.info)

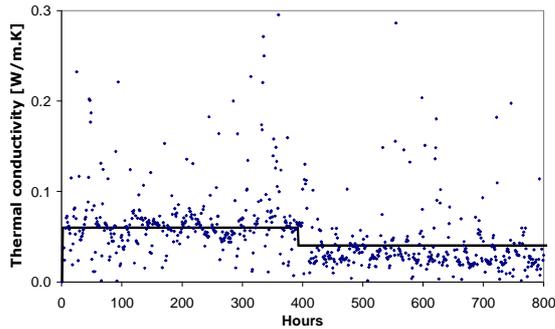


Figure 11: Estimated value of the multilayered wall thermal conductivity

The results of the simulation shown on fig.11 are mainly divided in two sequences: for the first 390 hours, the U-value was initially estimated to be equal to  $0.63 \text{ W/m}^2$  doing an average calculation. To exclude the values that are not representative (extreme and negative values) we made a selection keeping the sets of data between  $U_{av}-2\sigma$  and  $U_{av}+2\sigma$ . Then the average and the standard deviation were recalculated in order to have a more accurate estimation. The modified average for the U-value gives  $0.59 \text{ W/m}^2$  and the standard deviation 0.29. At  $t=390\text{h}$ , there is a significant break in the results succession. The wall properties have been modified by the introduction of the insulation material at this time stage. Doing the same procedure as before to estimate the U-value after  $t=390\text{h}$ , we find a mean value of  $0.26 \text{ W/m}^2\text{K}$  with a standard deviation of 0.5.

Nevertheless, the simulation result also shows a lack of accuracy: the scattered calculated values reveal the effect of the air circulating inside the air-cavity. The physical interaction of the naturally ventilated air is directly affecting the measured temperatures and heat flux values. The mathematical model developed at the beginning should be improved by considering the natural ventilation inside the wall.

As shown, the inverse method can reproduce the experimental behaviour of the wall. Despite the dispersion of the simulation results, it provides a good estimation of the thermal properties of the wall before and after the change of wall composition. However, it also identifies the day on which the insulation material layer was added. Nevertheless, the mathematical model of the wall must be improved by taking into consideration the natural ventilation effect to improve the accuracy of the inverse model. Also, the application of the inverse model can be further investigated to appreciate the effect of the added insulation on the thermal response of wall (capacitance, time constants etc.).

## CONCLUSION

Methods have been developed for the estimation of the thermal properties of structural elements under real, transient, non-periodic conditions using simulated or "taken on-site" temperature and heat flow measurements. The accuracy and robustness of the present method have been verified successfully in the case of the transient wall heat-transfer problem, regarding the possibility of a non-constant thermal properties material like the PCM. It has also been verified for the case of a whole building ventilation study. Finally, further investigations could be undertaken on the model considering the air circulating interaction in the wall cavity layer. Despite its lack of accuracy, the model gave a satisfactory estimation.

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