NUMERICAL ANALYSIS ON THE RELATIONSHIP BETWEEN RATE OF OUTDOOR AIR SUPPLY AND TOTAL AIR AGE

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ABSTRACT
Air age is an important parameter to assess indoor air quality in ventilated rooms. In order to consider the contribution of ductwork and recirculation, the concept of total air age was proposed for ventilation systems with ductwork and recirculation in previous study. In this paper, about 16 typical cases are simulated with computational fluid dynamics after introducing the method to calculate total air age for ventilation systems with only one AHU and one room in which several inlets and outlets exist. The total air age in the breathing zone is compared for 4 kinds of flow pattern and 4 kinds of mixing ratio between fresh air and return air with the same amount of supply air. It is shown that the total air age in breathing zone is very different even if with the same amount of outdoor air supply.

KEYWORDS
Indoor air quality (IAQ), Rate of outdoor air supply, Total air age, Computational fluid dynamics (CFD)

INTRODUCTION
It is reported that people spend almost 90% of time indoors. Indoor air quality (IAQ) is therefore important for the health of people. In order to maintain an acceptable indoor air quality, minimum rate of outdoor air supply is usually required in different kinds of ventilation standards. However, the indoor air quality is affected by airflow pattern, mixing ratio between fresh air and return air and so on even if the amount of outdoor air is the same. Therefore, air age rather than the amount of outdoor air is usually used as an index to assess the indoor air quality. Since the air age of supply air inlets is usually taken as 0 (100% fresh) in traditional experimental and numerical methods, the concept of total air age was proposed by Li et al. (2003a) to include the influence of recirculation and ductworks. Total air age was defined as the time that has elapsed since fresh air element enters the ventilation system from the fresh air intake. Compared to the traditional air age, total air age can be used as an absolute value to evaluate the indoor air quality of each ventilated room even if it is not ventilated by the same air handling unit.

The total air age can be obtained by combining the room air age with the air age in ducts. The room air age can be solved using computational fluid dynamics (CFD) tools and the air age in ducts solved using the one-dimensional flow characteristic, which includes the mixing process between flows with different ages. Li et al. (2003a) have reported that a direct solution without iteration is available when there is only one air handling unit (AHU) in the system. The approach is developed based on the fact that the air age at all air inlets in the room is the same. However, this does not apply when the length and velocity to the inlets is different. In this circumstance each inlet may have totally different air age. An algorithm to calculate total air age in a room ventilated by multiple AHUs was developed by Li et al. (2003b), which still needs multiple CFD runs to reach a final solution though it is more computationally effective than the iteration method.

In this paper, we developed an algorithm to calculate total air age in a room ventilated with several inlets and outlets by one AHU, or even the direct fresh air supply. The approach needs neither multiple CFD runs nor iterations. Based on the approach, about 16 typical cases are simulated with CFD. The total air age and room air age in the breathing zone is compared for 4 kinds of flow pattern and 4 kinds of mixing ratio between fresh air and return air with the same amount of supply air, in order to numerically analyze the relationship between rate of outdoor air supply and total air age.

CALCULATION OF TOTAL AIR AGE
According to previous study by Li et al. (2003b), the air flow in ductwork can be treated as one dimensional flow, and the air age change in non-branching duct and mixing process can be calculated by equations (1) and (2), respectively:

$$\Delta \tau = \frac{L}{u_{duct}} \quad (1)$$

$$\tau_{mix} = \frac{\sum_{i=1}^{n} Q_i \tau_i}{\sum_{i=1}^{n} Q_i} \quad (2)$$
where $\Delta \tau$ is air age increase in duct, $L$ is length of duct, $u_{duct}$ is air velocity in duct, $\tau_{mix}$ is air age after mixing with several branches, $\tau_i$ is air age of the $i$th branch. $Q$ is air flow rate of the $i$th branch.

Since all the lengths of ductwork and air flowrate are known, the air age change in all ductwork and AHU can be calculated with Eqs. (1) and (2). Then we take a ventilation system with one AHU supplying air to one room as example to analyze the air age relationship in ductwork and AHU.

The relation of air age of the $k$th inlet in the room with the supply air age of AHU can be written as:

$$\tau_{ki} = \tau_{SA} + \Delta \tau_{ki}$$  \hspace{1cm} (3)

where $\tau_{ki}$ is air age of the $k$th inlet, $\tau_{SA}$ is air age of supply air just after mixing with fresh air in the AHU, $\Delta \tau_{ki}$ is air age increase from the supply air of the AHU to the $k$th inlet.

The relation of air age of the $k$th outlet in the room with the return air age of AHU can be written as:

$$\tau_{RA} = \sum_{k=1}^{N_R} \left[ \alpha_{ko} (\tau_{ko} + \Delta \tau_{ko}) \right]$$ \hspace{1cm} (4)

where $\tau_{RA}$ is air age of return air before mixing with fresh air in the AHU, $N_R$ is number of air outlets in the room from the AHU, $\alpha_{ko}$ is ratio of the $k$th outlet flow rate to air flow rate of the AHU, $\Delta \tau_{ko}$ is air age increase from the $k$th outlet to the AHU before mixing.

The relation of fresh air age, return air age and supply air age in AHU is:

$$\tau_{SA} = f\tau_{FA} + (1-f)\tau_{RA}$$ \hspace{1cm} (5)

where $f$ is ratio of fresh air flow rate to the supply air flow rate in the AHU, $\tau_{FA}$ is air age of fresh air before mixing with return air in the AHU.

Based on the previous work (Sandberg, 1981; Kato et al., 1992; Li et al., 1997), the transport equation of air age in ventilated room can be written as:

$$\frac{\partial \tau_p}{\partial t} + \frac{\partial (U_j \tau_p)}{\partial x_j} = \frac{\partial}{\partial x_j} (\Gamma \frac{\partial \tau_p}{\partial x_j}) + 1$$ \hspace{1cm} (6)

Boundary conditions are:

$$\begin{align*}
\tau_p &= \tau^i_{in} & \text{at } i\text{th inlet} \\
\frac{\partial \tau_p}{\partial n} &= 0 & \text{at outlets}
\end{align*}$$ \hspace{1cm} (7)

where $\tau_p$ is air age at any point $p$ in the room, $t$ is time, $U_j$ is velocity at $x, y, z$ directions ($j = 1, 2, 3$), $x_j$ is $x, y, z$ axis ($j = 1, 2, 3$), $\tau^i_{in}$ is air age of the $i$th inlet.

When the air age at all inlets are 0, the air age at any point, $\tau^R_p$, called as room air age (Li et al., 2003a), can be obtained by Eq. (6) with the following boundary conditions:

$$\begin{align*}
\tau_p &= 0 & \text{at inlets} \\
\frac{\partial \tau_p}{\partial n} &= 0 & \text{at outlets}
\end{align*}$$ \hspace{1cm} (8)

In order to consider the contribution of supply air to contaminant distribution, Li and Zhao (2004) proposed the concept of accessibility of supply air (ASA). Yang et al. (2004) defined the ASA to an arbitrary point $p$ from the $i$th inlet as:

$$A_{si,p}(\tau) = \frac{\int (C_p(t)dt)}{C_{s,i} \tau}$$ \hspace{1cm} (9)

where $C_{s,i}$ is the tracer gas concentration of $i$th inlet, $C_p(t)$ is the tracer gas concentration of point $p$ at moment $t$ when the initial concentration is 0 and all the inlets concentrations are 0 except that the $i$th one is $C_{s,i}$.

The accessibility of the $i$th inlet to point $p$ at steady state is simply written as $A_{si,p}$, which is $\frac{C_p(\infty)}{C_{s,i}}$ and equals to $SVE4$ proposed by Kato et al. (1992).

Once the flow field is steady, the transport equation of air age is a linear one and Superposition Theorem can be applied. Therefore, the air age at any point $p$ in the room can be written as:

$$\tau_p = \sum_{k=i}^{N_s} (\tau_{ki}A_{ki,p}) + \sum_{k=1}^{N_{DF}} (\tau^D_{k,p} A_{k,p}^{DF}) + \tau^R_p$$ \hspace{1cm} (10)

where $N_S$ is number of supply air inlets in the room from the AHU, $N_{DF}$ is number of direct fresh air inlets in the room, $A_{ki,p}$ is steady state accessibility of the $k$th inlet to point $p$ in the room, $A_{k,p}^{DF}$ is steady state accessibility of the $k$th direct fresh air inlet to point $p$, $\tau^D_{k,p}$ is air age of the $k$th inlet of direct fresh air in the room.

After inserting Eq. (3), the air age can be written as:
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\[ \tau_p = \tau_{si} + \sum_{k=1}^{N_k} A_{k,ip} + \sum_{i=1}^{N_i} (\Delta \tau_{ki} A_{ki,ip}) + \sum_{k=1}^{N_k} (\tau_{k,DF} A_{k,ip}^{DF}) + \tau_{R}^{p} \]

(11)

Then air age of outlets in the room can be written as:

\[ \tau_{ko} = \tau_{si} + \sum_{i=1}^{N_i} (\Delta \tau_{ki} A_{ki,ko}) + \sum_{k=1}^{N_k} (\tau_{k,DF} A_{k,ko}^{DF}) + \tau_{R}^{ko} \]

(12)

where \( A_{k,ko} \) is steady state accessibility of the \( k \)th inlet to the \( k \)th outlet, \( A_{k,ko}^{DF} \) is steady state accessibility of the \( k \)th direct fresh air inlet to the \( k \)th outlet, \( \tau_{R}^{ko} \) is room air age at the \( k \)th outlet when air age of all inlets is 0.

From Eqs. (4), (5) and (12), we can get the supply air age of AHU:

\[ \tau_{SA} = \frac{f \tau_{FA} + (1 - f) \sum_{ko=1}^{N_o} \left( \alpha_{ko} \sum_{ki=1}^{N_i} (\Delta \tau_{ki} A_{ki,ko}) + \sum_{k=1}^{N_k} (\tau_{k,DF} A_{k,ko}^{DF}) + \tau_{R}^{ko} + \Delta \tau_{ko} \right)}{1 - (1 - f) \sum_{ko=1}^{N_o} \alpha_{ko}} \]

(13)

The air age distribution in the ventilated room can then be calculated with Eq. (11).

**CASE STUDY**

**Case description**

A three-dimensional room with table, computer, occupant and lamps inside, which is ventilated by four different ventilation types, is studied under non-isothermal condition. For the first one, the mixing type (TYPE1), there are two inlets (No.1 and 2) at the east wall below the ceiling and two outlets (No.3 and 4) at the bottom of the east wall, as showed in Figure 1. In the second type (TYPE2), two inlets (No.1 and 2) and two outlets (No.7 and 8) are located at the upper east wall and lower west wall respectively. In the

\[ \sum_{i=1}^{N_i} (\Delta \tau_{ki} A_{ki,ko}) + \sum_{k=1}^{N_k} (\tau_{k,DF} A_{k,ko}^{DF}) + \tau_{R}^{ko} \]

\[ \frac{f \tau_{FA} + (1 - f) \sum_{ko=1}^{N_o} \left( \alpha_{ko} \sum_{ki=1}^{N_i} (\Delta \tau_{ki} A_{ki,ko}) + \sum_{k=1}^{N_k} (\tau_{k,DF} A_{k,ko}^{DF}) + \tau_{R}^{ko} + \Delta \tau_{ko} \right)}{1 - (1 - f) \sum_{ko=1}^{N_o} \alpha_{ko}} \]

\[ \sum_{i=1}^{N_i} (\Delta \tau_{ki} A_{ki,ko}) + \sum_{k=1}^{N_k} (\tau_{k,DF} A_{k,ko}^{DF}) + \tau_{R}^{ko} \]

\[ \frac{f \tau_{FA} + (1 - f) \sum_{ko=1}^{N_o} \left( \alpha_{ko} \sum_{ki=1}^{N_i} (\Delta \tau_{ki} A_{ki,ko}) + \sum_{k=1}^{N_k} (\tau_{k,DF} A_{k,ko}^{DF}) + \tau_{R}^{ko} + \Delta \tau_{ko} \right)}{1 - (1 - f) \sum_{ko=1}^{N_o} \alpha_{ko}} \]
third type (TYPE3), two inlets (No.1 and 2) and two outlets (No.5 and 6) are located at the upper east wall and upper west wall respectively. In the last one, the displacement type (TYPE4), there are four inlets (No.3, 4, 7 and 8) at the lower east and west wall and two outlets (No.9 and 10) at the ceiling. All four types ventilation have an exhaust (No.11) located at the ceiling. Each ventilation type can change 4 kinds of mixing ratio $f$ between fresh air and return air with the same amount of supply air. $f$ can be equal to 25%, 50%, 75% and 100%. Thus, there are total 16 ventilation methods. Figure 1 shows the schematic of the case and corresponding boundary conditions of the 16 cases are shown in Table 1 and 2.

Table 1 Configuration

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Table 2 Flow boundary conditions and total heat transfer for the cases

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<th>QUANTITIES</th>
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<td>10m</td>
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<tr>
<td>Computer heat transfer</td>
<td>140W</td>
<td>Return air duct length</td>
<td>10m</td>
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<tr>
<td>Lamp heat transfer</td>
<td>40W×2</td>
<td>Fresh air duct length</td>
<td>2m</td>
</tr>
<tr>
<td>Air flowrate</td>
<td>0.128m$^3$/s</td>
<td>Supply air temperature</td>
<td>21℃ (case1-12)</td>
</tr>
<tr>
<td></td>
<td>(7.2ACH)</td>
<td></td>
<td>23℃ (case13-16)</td>
</tr>
</tbody>
</table>

Simulation tool

CFD simulations were carried out for the purpose of flow visualization as well as the calculation of air velocity, temperature, ASA and room air age of distribution. STACH-3 (Zhao et al, 2003), a well-validated CFD program, was adopted to study the problems numerically. To account the turbulent flow in a room, a zero-equation turbulence model (Chen and Xu, 1998) was used in these cases.

RESULTS AND DISCUSSION

Figure 2 shows the total air age distributions for TYPE1 ventilation method with different fresh air ratios at the same amount of supply air, calculated using the algorithm describes above. The total air age in the breathing zone ($y=1m$) can be very different with the different mixing ratio between fresh air and return air with the same amount of supply air, though the distribution patterns are similar with each other. Nevertheless, the value of room air age with different fresh air ratio has little discrepancy. It is obvious that the total air age can be influenced by the fresh air ratio strongly. Thus, we can not ignore the influence of the fresh air ratio when indicating the air quality conditions in the ventilation room.

The distributions of total air age for TYPE1--4 when the fresh air ratio is equal to 25% have been calculated and the results in the breathing zone ($y=1m$) are presents in Figure 3. We can see from the figure that the total air age is about 2100--2400s for all 4 types ventilation method, however, the distribution patterns are different from each other. The results indicate that with a fresh air ratio of 25%, the total air age can be 4 times more than the room air age, when the room air age is about 400--600s, as the difference is due to the mixing of return air and fresh air in the AHU and air movement in ducts.

It is also shown in Figure 3 that the total air age in breathing zone is very different in 4 types of ventilation even if with the same amount of outdoor air supply and same amount of supply air. If we use the rate of outdoor air supply as an index to assess
the IAQ, the results would be the same in these 4 types of ventilation. Here we can use total air age as an index to assess the IAQ better than the rate of outdoor air supply.

The Figure 4(a) and (b) give the comparison of the average of room air age and total air age for each ventilation method respectively. The average of the total air age with the fresh air ratio 25% is about 2 times, 3 times and 4 times of the average of the total air age with the fresh air ratio 50%, 75% and 100% respectively. It also shows that the averages of room air age and total air age with displacement ventilation (TYPE4) have been affected mostly by the fresh air ratio.

![Figure 2](image-url)

*Figure 2 Total air age distributions for TYPE1 with different fresh air ratios in section y=1m.*
In principle, the greater the fresh air ratio is, the fresher the air in the ventilated rooms is. It is clearly showed in Figure 4 that the greater the fresh air ratio is, the smaller the averages of total air age for all ventilations are, but the averages of room air age may increase or decrease. The comparison proves that the total air age would be more accurate than the room air age to indicate the air quality conditions in the room by considering recirculation and air residence time in the ducts (where air may get polluted).

In these 16 cases, the air age increased in the ducts is 55 seconds in the **TYPE1~3**, and is 110 seconds in the **TYPE4**. When the fresh air ratio is small, compared with the extraordinary high value of the total air age, we can ignore the increase of air age in the ducts for simplifying the calculation of the total air age. On the contrary, when the fresh air ratio is larger than 50%, the increase of the air age in the ducts can probably reach 5~25% of the total air age in the ventilated room, which therefore can not be ignored.

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**Figure 3 Total air age distribution for TYPE1~4 when the fresh air ratio is equal to 25% in section y=1m.**
CONCLUSION

In order to analyze the relationship between rate of outdoor air supply and total air age in a ventilated room, an algorithm to calculate total air age in a room ventilated by one AHU with several inlets and outlets is developed. 16 cases are studied to compare the total air age for different kinds flow pattern and mixing ratio between fresh air and return air with the same amount of supply air.

It is shown that the total air age in breathing zone can be influenced by the fresh air ratio strongly and is very different even if with the same amount of outdoor air supply. The comparison proves that the total air age would be more accurate than the room air age and the amount of outdoor air supply to indicate the air quality conditions in the room by considering recirculation and air residence time in the ducts. When the fresh air ratio is less than 50%, the increase of air age in the ducts can be ignored to simplify the calculation.

For the further research, we may put forward the reasonable range of total air age and the required rate of outdoor air supply for different kinds of flow pattern and mixing ratio between fresh air and return air. The results will be helpful on determining the amount of fresh air when designing the ventilation systems.

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REFERENCES


