

TWO-PHASE APPLICATION OF MULTI-OBJECTIVE GENETIC ALGORITHMS IN GREEN BUILDING DESIGN

Weimin Wang¹, Radu Zmeureanu¹, and Hugues Rivard²

¹Department of Building, Civil, and Environmental Engineering,
Concordia University, Montreal, Canada

²Department of Construction Engineering, ETS, Montreal, Canada

ABSTRACT

This paper presents the application of multi-objective genetic algorithms for green building design to minimize two conflicting criteria: the life-cycle cost and the life-cycle environmental impact. Environmental impact categories considered in this study include energy and non-energy natural resources, global warming, and acidification. Variables focus on building envelope-related parameters. The application of multi-objective genetic algorithms is divided into two phases. The first phase intends to help designers in understanding the trade-off relationship between the two conflicting criteria. The second phase intends to refine the performance region that is of the designer's interest.

INTRODUCTION

Since buildings are major energy consumers and have a large impact on the global climate change and other energy-related environmental issues, it is important to incorporate environmental performance as a criterion in building design. Green building is a recent design philosophy that requires the consideration of resources depletion and waste emissions during its whole life cycle (Woolley et al. 1997). Optimization can help in obtaining the most promising solution for green building design. Many studies have been made in this respect. Operating energy consumption or life-cycle cost is usually used as the single performance criterion in many optimization models (e.g., Coley and Schukat 2002; Nielsen 2002; Wetter and Wright 2004).

Most previous studies deal with either economical or environmental performance. In a few studies that have considered both economical and environmental performance criteria, two approaches were adopted: (1) one criterion is handled as a constraint (Nielsen 2002); and (2) the weighted sum technique is used (Hauglustaine and Azar 2001). Both approaches require designers to provide a priori information such as boundary value for the constraint and weights for the performance criteria. With little knowledge about the performance space of the problem in advance, designers may find it difficult to set appropriate values for those required inputs. Furthermore, only

one optimal solution is obtained for each run if the two performance criteria are treated separately or coupled together into one meta-criterion. The designer cannot learn the impact of the marginal change of one criterion on another just from a single optimal solution. Therefore, it is difficult to make cost-effective decisions without knowing the trade-off relationship between economical and environmental performance. To address these limitations, Wright et al. (2002) and Nassif et al. (2003) applied multi-objective genetic algorithms to optimize HVAC system design and control. However, they focused only on the performance space with no special efforts taken to acquire high-quality solutions distributed as wide as possible in the design space. As a result, designers cannot consider other important criteria not implemented in the optimization model when selecting a design alternative.

In this paper, we apply the multi-objective genetic algorithm in two phases to better aid designers in the decision-making process. The remainder of this paper is organized as follows. The optimization problem is described in the next section. It is followed by the presentation of the optimization algorithm. Then, the results for the optimization problem after two-phase application of the multi-objective genetic algorithm are presented and discussed.

OPTIMIZATION PROBLEM

A single-story office building located in Montreal, Canada, is to be optimized for both economical and environmental performance. The building has a total above-basement floor area of 500 m² with a 40-year life expectancy. Heating season is from November to March, and cooling season from June to August. The optimization problem is described with its variables and objective functions.

Variables

The list of variables considered in this study is given in Table 1. Orientation is a continuous variable indicating the angle in degrees between the true north and the building north. The shape of the building plan has two possible values: rectangular shape or L-

shape (Figure 1). The two shapes need to be defined with different parameters. For the rectangular shape, the variable r_0 ($=AF/EF$) is required. For the L-shape, the three variables, r_0 ($=AF/EF$), r_1 ($=DE/AF$), and r_2 ($=AB/EF$), are required. The wall tilt of each side is a continuous variable representing the angle in degrees between the outward normal of the ground and that of the wall surface. The variable *winRatio* is defined as the ratio between the window area and the total area for a given façade. The window type is allowed to vary with different facades. Seven available window types for this building design are listed in Table 2.

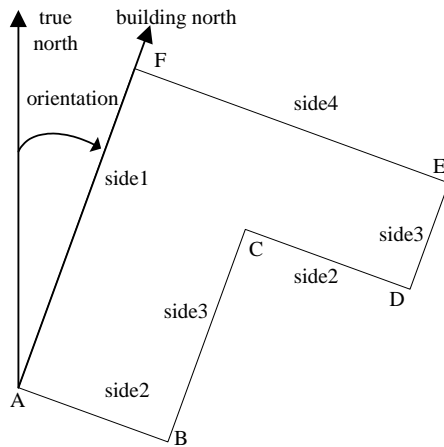


Figure 1 Illustration of the building shape and related variables

Steel frame and concrete frame are the two alternatives for the structural system. Both steel frame and concrete frame have the same two possible wall types: concrete block wall and steel-stud wall. However, they have different roof types and floor types. With the steel frame structural system, a steel deck on open web steel joist is used for both roof and floor, while with the concrete frame, a cast-in-place concrete flat plate system is used. Although there is no limitation on the number of wall layers and roof layers to be used as optimization variables, some of them (e.g., vapor barrier) are defined as constants because of their minor impacts on the two considered performance criteria. Thus, only the layers of cladding, wall insulation (including rigid insulation for both wall types and the cavity insulation for the steel-stud wall), and roof insulation are variables. Their discrete values are listed in Table 2, where the numbers leading the alternative materials indicate the indices in the corresponding data files. The *overhangType* is a discrete variable that defines whether overhangs exist above windows and the materials used. Overhangs may be favorable for some facades but not for others due to the close relationship between the orientation and the sun-shading effect. Therefore, each façade has an *overhangType* variable, which can take one of two

possible values: no overhang and aluminum overhang. The *overhangDepth* refers to the distance between the wall and the outer edge of the overhang and it is allowed to vary between 0.1 and 1.2 m. The distance from the top of the windows to the overhang is fixed to be 0.2 m.

It can be noted that not all variables in Table 1 are used to determine a building design. Some variables such as orientation and *winRatio* on each side are always active because their values are necessary to define each building design alternative. However, some variables may be inactive in certain situations. For example, if the variable *structuralSystem* takes the value of concrete frame, all the variables affiliated to the steel frame are inactive. Variables at higher levels (e.g., *structuralSystem*) are called structured variables. A structured variable may be controlled by another one, which leads to the existence of multi-level structured variables. For example, the structured variable *wallType* is a sub-level variable of *structuralSystem*.

Objective Functions

Both life cycle cost (LCC) and life cycle environmental impact (LCEI) are selected as the two objective functions to be minimized. Let X denotes a variable vector, the general expressions to calculate LCC and LCEI are:

$$LCC(X) = IC(X) + OC(X) \quad (1)$$

$$LCEI(X) = EE(X) + OE(X) \quad (2)$$

where, IC is the initial construction cost (\$) of a building including exterior walls, windows, the roof, the floor, and overhangs; OC is the present worth of life-cycle operating costs (\$) that comprise energy consumption cost and peak demand cost; EE is the embodied environmental impact (MJ) due to natural resource extraction, building material production, on-site construction, and transportation associated with the above phases; OE is the environmental impact (MJ) due to the energy consumed by a building for heating, cooling, and lighting over its operation phase. Both EE and OE are evaluated using expanded cumulative exergy consumption, which is a unifying indicator of all considered impact categories, that is, the energy and non-energy natural resources, the global warming, and the acidification (Wang 2005).

A simulation program based on the ASHRAE toolkit for building load calculations (Pedersen et al. 2000) was developed to calculate both LCC and LCEI. Using the heat balance method, the ASHRAE toolkit calculates hourly heating or cooling loads for a given day per month that may correspond to (1) the average weather condition; or (2) the extreme weather condition. The hourly loads corresponding to the

average weather condition are used to estimate the operating energy consumption accounting for the efficiency of the heating and cooling system. The operating energy consumption is then used to calculate energy consumption cost and the operating environmental impacts. The hourly loads corresponding to the extreme weather condition are used to calculate peak demand cost. The simulation program calculates *IC* and *EE* based on: the building formulation as defined by the selected variables; the unit construction cost of building elements; and the unit embodied impact data of building elements.

MULTI-OBJECTIVE GENETIC ALGORITHM

Because the optimization model presented in the previous section has two objective functions and a set of discrete variables, the multi-objective genetic algorithm (GA) is an appropriate method to be used in this study. A major advantage of multi-objective GA lies in its ability to locate multiple Pareto optimal solutions in a single run. A solution is said to be Pareto optimal if and only if it is not dominated by any other solution in the performance space. If solution X_1 dominates another solution X_2 , it implies that X_1 is non-inferior to X_2 for all the considered performance criteria but it is better than X_2 for at least one criterion.

The multi-objective GA implemented in this study has the following steps. An initial population of individuals is usually randomly generated. Then, the objective functions of each individual representing a potential solution are evaluated by the simulation program, and they are used to calculate individual's fitness according to the rank-based fitness assignment strategy (Fonseca and Fleming 1998). Based on the fitness values, selection is used to give robust individuals higher opportunity to reproduce. The crossover and mutation operations are then employed on the selected individuals to form a new generation of population. Finally, the elitism strategy is used to copy the non-dominated individuals of the old population to an external population and removes those dominated ones in the external population. The above steps from individual evaluation to elitism are repeated on the new population and until reaching a predefined maximum number of generations.

In this study, the application of multi-objective GA is divided into two phases with different purposes. The first phase aims to obtain the diversity of Pareto solutions in terms of objective function values so that designers can understand the trade-off relationship between the two conflicting criteria through the Pareto front (e.g., the dashed line in Figure 2). The second phase aims to refine the interesting region as shown in Figure 2. The interesting region is the area between the curves *AB* and *CD*. *AB* is the part of the

Pareto front around the chosen goal point, while *CD* is formed after the function values of all the points on *F1* are increased by a small value ϵ to be discussed later. The second phase intends to obtain the diversity of Pareto solutions in terms of variable values so that designers can select a design alternative while accounting for other performance criteria not considered in the optimization model.

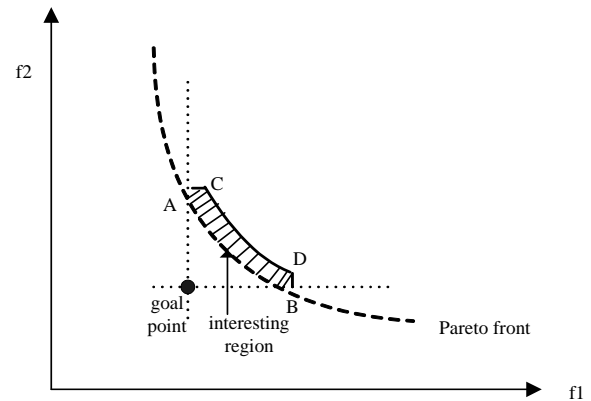


Figure 2 Illustration of the transition from phase 1 to phase 2

The different purposes in the two phases are achieved through the following measures:

Different functions are used to evaluate individuals. In the first phase, the original objective functions LCC and LCEI are used. After a goal point is chosen by the designer, the second phase is actually a goal programming problem, so the following functions are used:

Minimize:

$$\text{DiffLCC}(X) = \langle \text{LCC}(X) - \text{costGoal} \rangle \quad (3)$$

$$\text{DiffLCEI}(X) = \langle \text{LCEI}(X) - \text{impactGoal} \rangle \quad (4)$$

where, *costGoal* and *impactGoal* are the values of LCC and LCEI corresponding to the chosen point; the bracket operator $\langle \rangle$ returns the value of the operand if that value is positive; otherwise, it returns zero.

Different niche sharing strategies are employed in the fitness assignment. As an important technique to maintain diversity in Pareto solutions, the niche sharing requires that close individuals in the population be penalized by a reduction in fitness. In the first phase, niche sharing is carried out in the performance space, so the distance between two individuals is evaluated in terms of their functions values with Equation 5 (Deb 2001). In the second phase, niche sharing is carried out in the variable space, so the distance between two individuals is evaluated in terms of their variable values with Equation 6 (Everitt 1993).

$$d_{ij} = \sqrt{\sum_m \left(\frac{f_i^m - f_j^m}{f_{max}^m - f_{min}^m} \right)^2} \quad (5)$$

$$d_{ij} = \frac{\sum_k (w_k \cdot s_{ij}^k)}{\sum_k w_k} \quad (6)$$

where, d_{ij} is the distance between the two individuals i and j ; m represents the index of objective function f ; max and min respectively represent the maximum and minimum value for the considered objective function in a population; k is the index of variables; w is the weight assigned to the k -th variable; s is the dissimilarity between two individuals in terms of the k -th variable. The calculation of the dissimilarity s between two individuals depends on the variable type. For a discrete variable X_k , s is equal to 1 if X_k takes different values for the two individuals; otherwise, it is 0. For a continuous variable, s is calculated as (Everitt 1993):

$$s_{ij}^k = \left| \frac{X_i^k - X_j^k}{X_u^k - X_l^k} \right| \quad (7)$$

where, u and l represent the upper bound and the lower bound for the k -th variable X_k .

Different scopes of non-dominated individuals are used in elitism. To illustrate this point, the concept of ϵ -dominance (Laumanns et al. 2002) needs to be introduced. For the ϵ -dominance, solution X_1 dominates X_2 if the following two conditions are satisfied: (1) $\forall i, f_i(X_1) \leq (1 + \epsilon)f_i(X_2)$; and (2) $\exists j, f_j(X_1) < (1 + \epsilon)f_j(X_2)$, where, ϵ is a small number; i and j indicate the indices of the objective functions for minimization. When ϵ is equal to zero, ϵ -dominance turns to its special case: the conventional dominance. The concept of non-dominated individuals is involved in the step of elitism at two places: (i) for each generation, the non-dominated individuals are copied to the external population; and (ii) when new non-dominated individuals are added to the external population at a generation, those that become dominated are removed. In other words, only non-dominated individuals are kept in the external population. In the first phase, the concept of conventional dominance is employed to find the non-dominated individual set. In the second phase, the concept of ϵ -dominance is employed to find non-dominated individuals. Let F denotes the set of individuals in the external population from the first phase that are located in the interesting region. In the second phase, the non-dominated individuals are enlarged to include all those that ϵ -dominate at least one member in F . The value of ϵ can be regarded as the extent to which that the designer has no preference with all the solutions around a given one.

Different distance metrics are used in the clustering technique. Because the external population has a predefined capacity, it may not accommodate all the elites found in the evolution process. In this case, the clustering technique (Everitt 1993) is employed to remove some individuals located in crowded regions. Equations 5 and 6 are used respectively for the two phases to evaluate the distance between individuals in the external population. Thus, the individuals close in the performance space are removed in the first phase, while those individuals close in the variable space are removed in the second phase.

RESULTS AND DISCUSSION

Phase 1

For the MOGA implementation, the following parameters are specified: crossover probability=0.9, mutation probability=0.01, maximum number of generations=300, population size=40, external population capacity=30, the selection operator is the binary tournament selection.

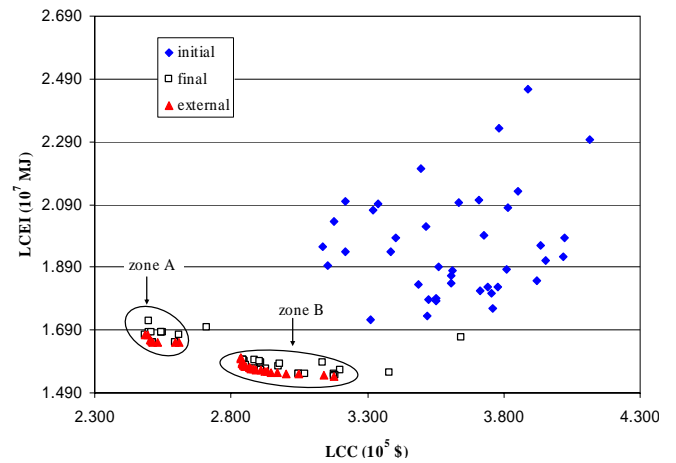


Figure 3 Distribution of initial, final, and external population

Individuals in the initial, final, and external population are distributed in the performance space as illustrated in Figure 3. It can be seen from this figure that:

- The initial randomly produced individuals are widely distributed, while the final population is clustered to the lower left corner. The final population is close to the external population. These two observations demonstrate a good convergence.
- The role of optimization is noticeable. The average values of the LCC and LCEI are $3.616 \cdot 10^5$ \$ and $1.960 \cdot 10^7$ MJ in the initial population, while they reached $2.814 \cdot 10^5$ \$ and $1.591 \cdot 10^7$ MJ in the external population after the optimization.

- The Pareto front, consisting of all the points in the performance space for Pareto solutions, is composed of two isolated zones *A* and *B*. Solutions in Pareto zone *A* have lower cost, and solutions in zone *B* have lower environmental impact.

Because of the space limitation, only a representative set of non-dominated solutions in the external population are listed in Table 3 with the corresponding values for variables and objective functions. The individuals are arranged in increasing order of the life-cycle cost. Some abbreviations are used to save space. The numbers in the column heads indicate the side indices as shown in Figure 1. The actual values corresponding to the indices of the discrete variables are defined in Tables 1 and 2.

Some major findings from Table 3 are as follows:

- The two allowed structural configurations have caused the two isolated Pareto zones. The steel frame system (S.F. in Table 1) is used in the Pareto zone *A* while the concrete frame system (C.F.) is used in the Pareto zone *B*.
- The orientation angle has mostly converged to zero because buildings elongated on the east-west axis are advantageous for energy performance without any extra cost.
- The shape of the building has converged on a rectangular shape over an L-shape. For the rectangular shape, the aspect ratio can vary with different Pareto solutions because the optimal values of aspect ratio are different for LCC and LCEI.
- The wall tilt of each façade has converged to the lower bound (75°) because it decreases the roof area, thereby reducing the initial cost and the heat exchange with the outdoor environment.
- The window type on side2 (south orientation) has converged to the low-e double glazing with coating (emissivity=0.2) on the exterior of the inside pane. However, window types on the other three orientations do not converge. This is due to the flat surface of the functional relationship between performance and the window types on the north, east, and west facades.
- The window ratio on the north, east, and west facades have converged to the lower bound while it varies for the south facade. Accordingly, there should be no overhang on the north, east, and west facades; however, overhang is a suitable passive solar technique for south-facing windows. The overhang depth is related to the window ratio: larger window ratio requires longer overhang projected out of the wall.
- The steel-stud wall (stud) is the optimal wall type for both steel-frame and concrete-frame structural systems. For the steel-stud wall, the cavity

insulation together with the steel stud converged to the minimum thickness (102 mm) of available alternatives while the rigid insulation converged to the maximum thickness (76 mm). Most solutions converged to the vinyl cladding as the optimal selection because it has the lowest initial cost.

- The roof insulation has converged toward the thickest alternatives for the two considered roof insulation materials because the level of roof insulation has a large impact on the building performance.

Assuming that the designer has interests with the solutions closing to the goal point at $LCC=2.850 \times 10^5$ \$ and $LCEI=1.556 \times 10^7$ MJ. The solutions located around the goal point are shaded. The second phase of genetic algorithm is then applied.

Phase 2

In the second phase, the initial population comprises all thirty individuals from the external population of the first phase and ten individuals generated randomly. Since elites are used as the initial population, the maximum generation is reduced to 150 in the second phase. The value of ϵ is 2%.

In this phase, the weights used in Equation 6 are indicated in the rightmost column of Table 1. These weights are thus set and implemented according to the following guidelines: (1) the weight is 0 if a variable is inactive; (2) more weights are given to a high-level structured variable; (3) more weights are given to those variables that are closely related to architecture design. Because of the last guideline, the variables of window type and insulation are given zero weights.

All the solutions in the external population at the end of phase 2 are around the chosen goal point. Because of space limitation, only a representative set of them are listed in Table 4, where the variables with zero weights are not shown since no efforts is taken to diversify their values. By comparing the solutions in Table 4 and those with grey-background color in Table 3 (ID from 3 to 7), one can find that:

- Most variables take more diversified values at the end of phase 2. For example, two solutions with L-shape are found while the rectangular shape is the only choice in phase 1; orientation can vary from 0 to 52 degrees while it almost converges to zero in phase 1.
- Three variables, window ratio on side1 (i.e., north orientation), structural system configuration, and wall type, converge and take the same values in both two phases. This indicates that these variables have significant impact on the function values. Any deviation from their optimal values

will move the solutions outside the interesting region.

CONCLUSIONS

The genetic algorithm is a powerful technique to solve building simulation based optimization problems. The two-phase application of the multi-objective genetic algorithm complies with the ever-refining decision process in building design. The Pareto front obtained in the first phase helps designers to grasp the general trade-off relationship between the conflicting performance criteria. The diversified solutions of the second phase offer designers more freedom to determine an alternative when considering other criteria such as aesthetics.

ACKNOWLEDGEMENT

The authors would like to acknowledge the financial support provided by the EJLB Foundation and Natural Sciences and Engineering Research of Canada.

REFERENCES

Coley, D.A., Schukat, S. 2002. Low-energy design: combining computer-based optimization and human judgment. *Building and Environment*, 37, 1241-1247.

Deb K. 2001. Multi-objective Optimization Using Evolutionary Algorithms, John Wiley & Sons.

Everitt B.S. 1993. Cluster Analysis (3rd edition), Edward Arnold, London.

Fonseca C.M., Flemming P.J. 1998. Multiobjective optimization and multiple constraint handling with evolutionary algorithms-part 1: a unified formulation, *IEEE Transactions on Systems Man and Cybernetics: Part A*, 28, 26-37.

Hauglustaine J.M., Azar S. 2001. Interactive tool aiding to optimise the building envelope during the sketch design, Proceedings of the Seventh International IBPSA Conference, 387-394.

Nassif, N., Kajl, S., Sabourin, R. 2003. Two-objective online optimization of supervisory control strategy. Proceedings of the eighth international IBPSA conference, 927-934.

Nielsen T.R. 2002. Optimization of Buildings with Respect to Energy and Indoor Environment. Ph.D. Thesis. Department of Civil Engineering, Technical University of Denmark.

Pedersen C.O., Liesen R.J., Strand R.K., Fisher D.E. Dong L. Ellis P.G. 2000. A Toolkit for Building Load Calculations, ASHRAE.

Wang, W. A simulation-based optimization system for green building design. Ph.D. Thesis, Department of Building, Civil and Environmental Engineering, Concordia University, Canada.

Wetter, M., Wright, J.A. 2004. A comparison of deterministic and probabilistic optimization algorithms for nonsmooth simulation-based optimization. *Building and Environment*, 39, 989-999.

Woolley, T., Kimmins, S., Harrison, P., Harrison, R. 1997. Green building handbook. E & FN Spon, London.

Wright, J.A., Loosemore, H.A., Farmani, R. 2002. Optimization of building thermal design and control by multi-criterion genetic algorithm. *Energy and Buildings*, 34, 959-972.

Laumanns, M., Thiele, L., Deb, K., Zitzler, E. 2002. Combining convergence and diversity in evolutionary multiobjective optimization. *Journal of Evolutionary Computation*, 10, 263-282.

Table 2 Alternatives for the window type, cladding, and insulation

Variable	Alternatives
Window type	1. double clear 2. double reflective 3. double, e=0.2, on surface 3 4. double, e=0.1, on surface 3 5. double, e=0.2, on surface 2 6. double, e=0.1, on surface 2 7. triple, e=0.2, on surface 5
Cladding	1. clay brick 2. concrete block 3. wood siding 4. vinyl siding 5. steel siding 6. stucco
Rigid insulation for concrete block wall	3. 76 mm expanded polystyrene(EPS) 4. 102 mm EPS 5. 127 mm EPS 6. 152 mm EPS 10. 76 mm XPS 11. 102 mm extruded polystyrene(XPS) 12. 127 mm XPS 13. 152 mm XPS
Cavity insulation for steel stud wall	1. 102 mm fiberglass 2. 152 mm fiberglass 3. 102 mm rockwool 4. 152 mm rockwool
Rigid insulation for steel stud wall	1. 25 mm EPS 2. 51 mm EPS 3. 76 mm EPS 8. 25 mm XPS 9. 51 mm XPS 10. 76 mm XPS
Roof insulation	17. 76 mm EPS 18. 102 mm EPS 19. 127 mm EPS 20. 152 mm EPS 34. 51 mm Polyisocyanurate (PI) 35. 76 mm PI 36. 102 mm PI 37. 127 mm PI

Table 1 List of variables and their weights used in phase 2

Variable Name		Type*	Range	weight		
orientation		C	[0, 90]	2		
shape		S	Rectangular (R); L-shape (L)	3		
Rectangular	r ₀	C	[0.1, 1.0]	1		
'L' shape	r ₀	C	[0.3, 1.0]	1		
	ratioA	C	[0.2, 0.7]	1		
	ratioB	C	[0.2, 0.7]	1		
wallTilt _i (i=1,2,3,4)		C	[75, 105]	1		
winType _i (i=1,2,3,4)		D	1,2,3,4,5,6,7	0		
winRatio _i (i=1,2,3,4)		C	[0.2, 0.8]	1		
structuralSystem		S	Concrete frame (CF) Steel frame (SF)	8		
Concrete Frame	wallType		S	Concrete block wall (C) Steel-stud wall (S)	3	
	Wall Type1	Cladding	D	1,2,3,4,5,6	1	
		Insulation	D	3,4,5,6,10,11,12,13	0	
		Vapor barrier	T	Modified bitumen	-	
		Structure	T	200 mm concrete block	-	
		Finish	T	12.7 mm gypsum board	-	
	Wall Type2	Cladding	D	1,2,3,4,5,6	1	
		Insulation	D	1,2,3,8,9,10	0	
		Air barrier	T	Sheathing paper	-	
		Sheathing	T	12.7 mm gypsum sheathing	-	
		studInsulation	D	1,2,3,4	0	
		Vapor barrier	T	Polyethylene	-	
	Roof type		S	CIP concrete roof	-	
	CIP roof	Roofing	T	2-ply modified bitumen	-	
		Coverboard	T	12.7 mm mineral fiberboard	-	
		Insulation	D	17,18,19,20,34,35,36,37	0	
		Vapor barrier	T	polyethylene	-	
		Roof deck	T	175 mm CIP roof	-	
		Finish	T	16 mm acoustical tile	-	
	Floor type		T	CIP concrete floor	-	
	overhangType _i (i=1,2,3,4)		D	No overhang (No) Aluminum type (Yes)	1	
	Steel Frame	Wall types and wall layers			The same as concrete frame	
		Roof type		T	OWSJ roof	-
OWSJ roof		Roofing	T	2-ply modified bitumen	-	
		Coverboard	T	12.7 mm mineral fiberboard	-	
		Insulation	D	17,18,19,20,34,35,36,37	0	
		Vapor barrier	T	polyethylene	-	
		Roof deck	T	Steel deck above OWSJ	-	
Finish		T	16 mm acoustical tile	-		
Floor type		T	OWSJ floor with concrete finish	-		
overhangType _i (i=1,2,3,4)		D	The same as concrete frame	1		
overhangDepth _i (i=1,2,3,4)		C	[0.1, 1.2]	-		

* C: continuous; D: discrete; S: structured; T: constant

Table 3 Representative solutions of the external population at the end of phase 1

zone	ID	orien.	shape	r0	wall tilt		window type				winRatio		stru.	wall			roof	overhang type		overhang depth (m)	LCC (10 ⁵ \$)	LCEI (10 ⁷ MJ)
					1-4	1	2	3	4	1,3,4	2	type		clad.	Insu.	stud-Insu.		1,3,4	2			
A	1	0	R	0.93	75	6	5	4	6	0.2	0.20	SF	S	4	3	1	20	N	Y	0.10	2.483	1.675
	2	0	R	0.70	75	3	5	4	6	0.2	0.20	SF	S	4	3	1	37	N	Y	0.25	2.594	1.651
B	3	0.7	R	0.73	75	5	5	4	6	0.2	0.20	CF	S	4	10	1	37	N	Y	0.10	2.869	1.570
	4	0	R	0.62	75	5	5	4	6	0.2	0.22	CF	S	4	10	1	37	N	Y	0.17	2.881	1.566
	5	0	R	0.50	75	5	5	4	6	0.2	0.22	CF	S	4	10	1	37	N	Y	0.17	2.891	1.564
	6	0.7	R	0.55	75	5	5	3	6	0.2	0.28	CF	S	4	10	1	37	N	Y	0.32	2.910	1.561
	7	0	R	0.50	75	5	5	4	6	0.2	0.30	CF	S	4	10	1	37	N	Y	0.32	2.922	1.558
	8	0	R	0.50	75	5	5	4	5	0.2	0.33	CF	S	4	10	1	37	N	Y	0.32	2.933	1.556
	9	0	R	0.50	75	5	5	4	5	0.2	0.46	CF	S	4	10	1	37	N	Y	0.32	2.971	1.555
	10	0.7	R	0.50	75	5	5	4	6	0.2	0.66	CF	S	4	10	1	37	N	Y	0.61	3.048	1.549
	11	0	R	0.40	75	5	5	4	6	0.2	0.66	CF	S	3	10	1	37	N	Y	0.61	3.178	1.544

Table 4 Representative solutions of the external population at the end of phase 2

ID	orien.	shape	Rec. r0	L-shape		wall tilt				winRatio			stru.	wall type	clad.	overhang type				overhang depth (m)	LCC (10 ⁵ \$)	LCEI (10 ⁷ MJ)
				r1	r2	1	2	3	4	1	2	3				4	1	2	3			
1	22.7	R	0.95			75	75	75	75	0.20	0.20	0.21	0.28	S	4	N	N	N	N		2.867	1.605
2	51.7	R	0.95			76	75	75	75	0.20	0.28	0.20	0.24	S	4	N	N	N	N		2.903	1.602
3	7.8	R	0.98			76	75	75	79	0.20	0.28	0.20	0.24	S	4	N	N	Y	0.17	2.917	1.593	
4	6.4	L	0.81	0.53	0.68	76	77	75	75	0.20	0.28	0.20	0.20	S	4	N	N	N	N		2.922	1.588
5	0.7	L	0.79	0.53	0.56	75	75	75	75	0.20	0.20	0.20	0.20	S	4	N	Y	N	N	0.32	2.925	1.596
6	7.8	R	0.98			78	75	75	75	0.21	0.30	0.20	0.20	S	4	N	Y	N	N	0.69	2.937	1.592
7	7.8	R	0.4			79	79	77	76	0.20	0.30	0.22	0.20	S	4	N	N	Y	N	0.17	2.957	1.584
8	0	R	0.5			76	75	75	75	0.20	0.37	0.20	0.20	S	4	Y	N	N	N	0.91	2.963	1.570
9	0.7	R	0.62			90	77	84	76	0.20	0.28	0.20	0.22	S	4	N	Y	N	N	0.32	2.976	1.576
10	6.4	R	0.72			75	75	75	75	0.20	0.22	0.20	0.20	S	3	N	N	N	N		2.980	1.573