ESTIMATION OF HORIZONTAL ILLUMINANCE BY MEASURED SKY LUMINANCE DISTRIBUTION

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ABSTRACT
Horizontal diffuse illuminance and direct illuminance were estimated by measured sky luminance distribution to investigate the consistency among global, diffuse and direct illuminance for a long-term IDMP data in Kyoto, Japan. In order to estimate the diffuse and direct illuminance by sky luminance distribution, it is necessary to identify the direct circumsolar region of the sky. This study introduced a new method to identify the direct region by comparing the measured diffuse illuminance and the calculated illuminance came from outside the supposed region. As a result, angular distance of the direct region was identified as 10 degree from the sun. The range was verified by comparing two diffuse luminous efficacies: one was calculated by measured sky luminance and radiance, and another was calculated by illuminance and irradiance. They were modeled by clearness index and confirmed to vary similarly as clearness varied.

INTRODUCTION
The daylight measurements in Kyoto were tested by CIE data quality control test and it was found that the passing rate was lowest in the test of the consistency among global, diffuse and direct illuminance. The rate was as low as about 70%, while the rates exceeded 95% in most of the other tests. Improvement of the consistency is necessary to acquire the data of high quality.
It is essential to consider the measuring devices to improve the consistency. Direct illuminance is defined as illuminance comes from the circumsolar region of extremely high luminance within the opening angle of direct meter. In earlier studies or measurements, however, half opening angles of direct meters were between 2.5 and 5.5 degree and had not been unified. The optimum opening angle should be examined on the basis of the consistency of direct and diffuse components.
Analysis of sky luminance distribution is also necessary to improve the data quality. Horizontal illuminance can be calculated by sky luminance distribution and configuration factors in theory. But it is a problem that the higher illuminance measurements cannot be acquired because the sensor of sky scanner is shut to protect itself in the range of higher luminance, although they contribute highly to the total illuminance. This is avoided in the proposed method by using the luminance outside the region.

MEASUREMENT
Daylight measurement system
Measurement was carried according to the guide of CIE International Daylight Measurement Program at a research class IDMP station (35.3N, 135.8E, 90m above S.L) in Kyoto, Japan. The measuring system is on the rooftop of a four-storyed building. The surroundings of the station are shown in Figure1. Horizontal global, diffuse, and direct normal components of illuminance and irradiance, and zenith luminance were recorded every minute. The radius and width of the shadow bands for diffuse measurements was 25cm and 5cm respectively. Cylinders of sensors for direct components rotate along diurnal course of the sun. The shadowing bands and the sighting devices of direct component were adjusted every several days. The half opening angle of direct illuminance or irradiance meter, in Figure2, was 2.5 degree.

Figure1 Surroundings of the site

Figure2 Half opening angle $\theta$ of direct meter
Sky scanner

Sky luminance were scanned every fifteen minutes during the day. Figure 3 shows the sky scanner on site. Sensor of sky radiance distribution was added in September 2000. The sky scanner measured the luminance of 145 points on the sky hemisphere as shown in Figure 4. It starts at the point of 6 degree in altitude in south, rotates and raises the altitude at intervals of 12 degree up to the zenith. Azimuth of the measuring points are shown in Figure 4. The figure also shows the order of the measurement. It took about three minutes to scan entire sky hemisphere. Effect of sudden change of the weather was negligible. Half opening angle of the luminance meter was 5.5 degree.

For the purpose of protection of the sensor, upper limit of measurement was assigned to 106 kcd/m². The sky scanner used in this study was an older type, but even the most advanced scanner available in 2003 in Japan has a limit of 50 kcd/m². The values exceed the limit were recorded as overflowed data.

DATA

Data between May 1993 and April 1994 throughout a year were analyzed in this paper. Illuminance and irradiance measurements for every 15 minutes at the same time of the sky scanner measurements were used. Data of solar altitude under 5 degree was removed to reduce the effects of assumed obstacles around the measurement system shown in Figure 1. Number of the measured sky luminance distribution was 5,904. 30.7% of the measured luminance distribution contained one or more overflowed records of luminance.

Correction of the diffuse components

Diffuse components were corrected by the following equation by Drummond (Drummond, 1956). Figure 5 shows the annual variation of the correction factor for the shadow band. The factor changes between 1.04 and 1.14.

\[
\gamma = \frac{1}{1 - \frac{2b}{\pi r}} \cos^2 \Delta (\sin \phi \sin \delta t_{00} + \cos \phi \cos \delta \sin t_{00})
\]

where,

\( \gamma \): Correction factor
\( \delta \): Latitude of the site
\( \phi \): Sun’s declination
\( b \): Width of the shadow band
\( r \): Radius of the shadow band
\( t_{00} \): Hour angle of the sun at sunset (in radians)

Data quality control test

After diffuse components were corrected, 14,537 daylight measurements were tested by CIE data quality control test and 10,237 of them passed all the tests. The passing rate was lowest in Test-2 about the consistency among global, diffuse and direct components. The equations of Test-2 are shown below. There are two levels of acceptance. The passing rate of Test-2.2 about illuminance was 70.42% and lower than that of test-2.1 (about irradiance), 89.58%, when acceptance ranges were set 15%. The rate exceeded 95% in most of the other tests. As for other yearly data between December 1994 and November 1995, and between July 1998 and June 1999, the passing rates were also lower in Test-2.2, 71.32% and 73.83% respectively. In most of the failed cases, the left sides of the equation were larger than the right sides.

The inclination angle of the shadow bands and the optical sight of direct meters were readjusted every several days, because of the inadequacies of measurement personnel or weather conditions, although daily adjustment is desirable. Therefore data in the next days of readjustment was picked out and analysed in order to inspect the effects of adjustment errors. For these data of fifty-nine days passing rate improved when diffuse components were higher, on the other hand the rate changed little as direct components changed. The effects of the ill-adjustments of the rate of direct components were not recognized.

Test-2.1: \( E_{vg} = (E_v, \cos \theta_{v}, + E_{vd}) \pm 15(25)\% \)

Test-2.2: \( E_{vg} = (E_v, \cos \theta_{v}, + E_{vd}) \pm 15(25)\% \)

\( \Delta \): 45°
DISCUSSION AND RESULT ANALYSIS

Estimation of the global horizontal illuminance

The sky scanner covers only about two thirds of the entire sky hemisphere because half open angle of the sensor is 5.5 degree (Nakayama et al., 2002). But here we suppose that the hemisphere is partitioned into 145 segments and the luminance of each measuring point represents that of the quadrangle segment around the point (Figure6). Global horizontal illuminance \( E \) can be estimated as follows.

\[
c_i = \frac{S_n''}{\pi \cdot n_k}
\]

\( c_i \) is a configuration factor of \( S_n'' \), the projected area of an objective segment \( S_n' \) on the horizontal plane. \( n_k \) is a number of the elements in the band of objective altitude. If \( a \) and \( b \) are the altitude angles across the segment, \( S_n'' \) can be written as,

\[
S_n'' = \pi (\cos^2 a - \cos^2 b) / n_k
\]

Global horizontal illuminance \( E \) is a total of the products of each configuration factor \( c_i \) by each measured luminance \( L_i \).

\[
E = \sum_{i=1}^{145} \pi \cdot L_i \cdot c_i
\]

Data of days without overflowed records was chosen (n=1493), where horizontal illuminance \( E \) can be obtained by Equation (3). Effects of solar altitude and angular distance from the sun on the coefficients of correlation between the calculated illuminance \( E \) and the measured illuminance \( E_{vg} \) were examined. The examination was necessary because 1) The luminance in one third of the hemisphere area could not be measured, 2) A segment was included in direct region if the center of it was in a supposed angular distance, 3) Measured luminance in lower altitude were not so reliable.

Figure7 shows the coefficients between the calculated illuminance \( E \) by Equation (3) and the measured illuminance \( E_{vg} \) in different groups of solar altitude. The coefficient became lower when the solar altitude exceeded 45 degree and it was lowest at the groups of solar altitude between 65 and 70 degree. The coefficients between the measured luminance \( L_z \) by zenith luminance meter and the measured zenith luminance \( rr145 \) by sky scanner are also shown in the figure and both coefficients varied similarly. It can be said that the effects of solar altitude on the correlation were recognized.

Figure8 shows the coefficients of different angular distance groups. The coefficients were almost constant around 0.9 under 100 degree of angular distance, and then increased as the angular distance increased. But the effects of angular distance were not so large as those of solar altitude.
In order to eliminate the effects of excessively large luminance, it is not sufficient to delete the overflowed data. Data was limited to the overcast sky conditions when the diffuse illuminance $E_{vd}$ was greater than ninety-five percent of the global illuminance $E_{vg}$. The sample size was 3970. The regression equation of the global illuminance by calculated illuminance $E$ by Equation (3) and solar altitude $h_s$ was as follows.

$$E_{vg} = 1.0903 \cdot E + 80.071 \cdot h_s + 608.50 \quad (4)$$

$E_{vg}$ is the estimated global illuminance. Figure 9 shows the relationship between $E_{vg}$ estimated by Equation (4) and the measured global illuminance $E_{vg}$. Coefficient of correlation was 0.935 and Root Mean Square Error was 0.3575.

This equation was applied for the estimation of diffuse illuminance which came from the entire hemisphere in all sky conditions. Equation (5) is the estimation equation of diffuse illuminance when diffuse illuminance is replaced to global illuminance in Equation (4).

$$\tilde{E}_{vd} = 1.0903 \cdot E + 80.071 \cdot h_s + 608.50 \quad (5)$$

**Procedure to identify the direct region**

If diffuse components of illuminance are estimated by measured luminance distribution, direct region around the sun can be identified as follows.

1) Luminance of the measured segments within a supposed angular distance $\theta$ from the sun were eliminated.
2) Mean luminance of the remained segments were substituted to segments in the direct region.
3) $E$ was calculated by the luminance of the segments in both direct and diffuse regions.
4) $\tilde{E}_{vd}$ was estimated by Equation (5).
5) RMSE between $E_{vd}$ and $\tilde{E}_{vd}$ was calculated.
6) The angular distance of the direct region was identified by comparing the values of RMSE among different angular distances.

Here the angular distance of the segment was supposed to be equal to that of the center point of the segment where the luminance was measured. Table 1 shows the number of the points within an angular distance from the sun and the number of overflowed luminance records within the distance, for the case of 10 degree for example. About 16% of the segments within 10 degree contained only one element ($n=919$). These cases were eliminated, because the luminance of the neighboring segments could not be considered, which might drop outside the region by chance.

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0.7
0.75
0.8
0.85
0.9
1

0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160

Angular distance from the sun [°]

0.20000 0.40000 0.60000 0.80000 1.00000

Estimated illuminance by Eq. (4) [lx]

Measured global illuminance $E_{vg}$ [lx]

$\tilde{E}_{vg}$ is estimated global illuminance. Figure 9 shows the relationship between $\tilde{E}_{vg}$ estimated by Equation (4) and the measured global illuminance $E_{vg}$. Coefficient of correlation was 0.935 and Root Mean Square Error was 0.3575.
Figure 10 Direct and diffuse component around the sun

Figure 11 Procedure to identify the circumsolar angular distance

Figure 12 Angular distance and the RMSE between the measured and estimated diffuse illuminance

Direct region around the sun

Figure 12 shows the RMSE values between the measured illuminance $E_{vd}$ and the estimated diffuse illuminance $\tilde{E}_{vd}$ for different angular distances from the sun. The angular distance of the direct region was identified to be 10 degree where the RMSE was the lowest. The coefficient of correlation was 0.939 for 10 degree.

Direct region of irradiance

Sensor of sky radiance distribution was added to the sky scanner in October 2000 and data for eight months was also available to compare the range of the direct region of illuminance and that of irradiance. Figure 13 and Figure 14 show the values of RMSE for illuminance and irradiance. Region for direct illuminance was identified as 8 degree. The RMSE values for irradiance were similarly low between 7 and 10 degree and the region was identified as 8.5 degree for irradiance.

The difference of the range of illuminance between the data in 1993 and in 2000 might be caused by the different measuring seasons in a year. The data in 2000 showed, however, the angular distance of illuminance and irradiance were almost the same for the same periods.

Luminous efficacy estimation by measured illuminance and irradiance

Luminous efficacy is defined as a ratio of luminous flux to radiant flux. In previous studies, various explanatory variables were used in the luminous efficacy models. Diffuse luminous efficacy can be obtained by luminance and radiance distribution, once direct region was identified by the method proposed above. That is, alternative diffuse efficacy model was formed by measured illuminance and irradiance. Both were regressed by clearness index and compared. Clearness index, ratio of global horizontal irradiance to extraterrestrial horizontal irradiance, was often used as an explanatory variable.
There are several formulae of global luminous efficacy for Japanese data. Global luminous efficacy by clearness \( K \) by Muneer for data in Chofu, Toyota and Fukuoka in Japan is as follows (Muneer, 1995).

\[
\eta_{G,M} = 136.6 - 74.541K + 57.3421K^2
\]

Inanuma used \( E_{eg} \) and \( \sin h_s \) as explanatory variables as follows (Inanuma et al., 2002).

\[
\eta_{G,is} = (0.1394 + 0.2075\sin h_s - 0.3042\sin^2 h_s \\
+ 0.1471\sin^3 h_s \cdot E_{eg}^{-0.06}
\]

Igawa proposed a formula by zenith angle \( \theta_s \) (Igawa et al. 1999).

\[
\eta_{G,ig} = (210.12 + 8.86\theta_s)E_{eg}^{-0.1} + (-29.24 + 90.21\theta_s \\
- 102.21\theta_s^2 + 54.16\theta_s^3 - 10.98\theta_s^4)E_{eg}^{0.1}
\]

RMSE of these equations was 0.117, 0.075 and 0.094 respectively for Kyoto data in 1993-1994.

The regression equation of global luminous efficacy \( \eta_G \) by measured global irradiance \( E_{eg} \) and clearness index in this study was as follows,

\[
\eta_G = (-183.7 - 73.1K + 123.4K^2 \\
- 72.3K^3) \cdot E_{eg}^{0.0904}
\] (6)

The equation by clearness index was,

\[
\eta_G = 160.1 - 173.3K + 303.5K^2 - 195.8K^3
\] (7)

Or in the fourth order,

\[
\eta_G = 166.6 - 304.8K + 1016.6K^2 \\
- 1545.1K^3 + 822.4K^4
\] (8)

RMSE of Equation (6), (7) and (8) for the Kyoto data was 0.0583, 0.0640 and 0.0586 respectively. Equation (8) of single variable is most convenient for use.

\[
\eta_D = 176.6 - 563.6K + 1760.6K^2 \\
- 2021.9K^3 + 765.1K^4
\] (9)

Equation of diffuse luminous efficacy by Muneer is as follows (Muneer, 1995).

\[
\eta_{DM} = 130.2 - 39.828K + 49.979K^2
\] (10)

Luminous efficacy estimation by sky luminance and radiance distribution

As mentioned above the direct regions around the sun were identified as 10 degree both for the illuminance and the irradiance. \( \bar{\eta}_D \), a ratio of the estimated diffuse illuminance \( \bar{E}_{ed} \) to the estimated diffuse irradiance \( \bar{E}_{ed} \), is an alternative expression of diffuse luminous efficacy. In the estimation of \( \bar{E}_{ed} \), sum of the products of configuration factor by measured radiance was replaced in the same form of Equation (5). The equation of diffuse luminous efficacy by sky luminance and radiance distribution was as follows,

\[
\bar{\eta} = 177.3 - 369.6K + 1132.6K^2 \\
- 1238.3K^3 + 413.3K^4
\] (11)

Figure 14 compares Equation (9), (10) and (11) for diffuse efficacy by clearness index. It is notable that luminous efficacy was remarkably high for low clearness in equation (9) and (11) than that in the model of Muneer. The mean \( \sqrt{\text{standard deviation}} \) of the efficacy calculated by measured illuminance and irradiance was 168.0 ± 25.6, 153.9 ± 8.1 and 146.6 ± 7.6 lm/W for the class of the clearness of 0-0.05, 0.05-0.10 and 0.10-0.15 respectively. It is certain that the efficacy was high and widely ranged for low clearness in the Kyoto data, but the regression
equations in the fourth order seemed to overestimate the efficacy for lower clearness under the effect of the clearness in the lowest class. But the effect was not so much in total because the relative frequency of the lowest clearness class was 5.0%, while 12.4% and 12.0% for the second and the third classes.

As for the comparison of Equation (9) and (11), the diffuse efficacy by measured illuminance and irradiance was about 10 percent smaller than that by luminance and radiance distribution. But both moved similarly as clearness index moved. It can be said that this fact shows the validity of the method of identification in this study to a certain extent.

![Figure 15: Comparison of diffuse luminous efficacy](image)

**CONCLUSIONS**

The range of the direct region around the sun was identified on the basis of diffuse illuminance and sky luminance distribution measured in Kyoto, Japan. The range was verified in relation to diffuse luminous efficacy, although further studies are needed for other sites. Conclusions are as follows.

1. The passing rate of CIE data control test of consistency among global, diffuse and direct components (Test-2) was 70.42% for illuminance and 89.58% for irradiance respectively.
2. Adjustment of the sensor direction improved the passing rate when diffuse components were higher, while the rate changed little as direct components changed.
3. The range of the direct region from the sun was identified to be 10 degree in angular distance.
4. A model to estimate the diffuse luminous efficacy by clearness index was introduced.
5. Diffuse luminous efficacy calculated by sky luminance distribution and radiance distribution varied similarly to the model by illuminance and irradiance as clearness varied.

**NOMENCLATURE**

- $E_{vg}$: Measured global illuminance
- $E_{vd}$: Measured diffuse illuminance
- $E_{ag}$: Measured global irradiance
- $E_{ad}$: Measured diffuse irradiance
- $L_i$: Measured luminance
- $c_i$: Form factor of a sky segment #i
- $E$: Calculated illuminance by sky luminance distribution and configuration factors
- $h_s$: Solar altitude
- $K$: Clearness index
- $\theta_z$: Zenith angle
- $\tilde{E}_{vd}$: Estimated global illuminance
- $\tilde{E}_{vd}$: Estimated diffuse illuminance
- $\eta_D$: Diffuse luminous efficacy
- $\eta_G$: Global luminous efficacy
- $\tilde{\eta}_D$: Diffuse luminous efficacy estimated by luminance and radiance distribution
- $\square$: Angular distance from the sun

**REFERENCES**


