THE SOLUM PROGRAM FOR PREDICTING TEMPERATURE PROFILES IN SOILS : MATHEMATICAL MODELS AND BOUNDARY CONDITIONS ANALYSES

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ABSTRACT

Building simulation programs normally do not take into account moisture effects in soils temperature determination. However, the presence of humidity can strongly affect the temperature distribution in soils due especially to the evaporation/condensation mechanisms and the strong variation of their thermophysical properties.

In order to calculate the temperature profiles in a more accurate way, we have developed the software Solum, which was conceived to model the coupled heat and moisture transfer in soils. The presented methodology is based on the theory of Philip and De Vries, using the thermophysical properties for three types of soil chemical composition. The governing equations were discretized using the finite volume method and a 3-D model was used for describing physical phenomena of heat and mass transfer in porous soils. The robust MultiTridiagonal-Matrix Algorithm (MTDMA) was used to solve the strongly-coupled problem.

In conclusion, we showed the effects of boundary conditions for the soil such as solar radiation, water table and adiabatic and impermeable surfaces on the temperature and moisture content profiles.

INTRODUCTION

Data of the Energy Secretariat of the São Paulo state, Brazil, show that the electric energy consumption in Brazilian buildings represents more than thirty percent of the total consumed (www.energia.sp.gov.br). However, the energy conservation potential in existing buildings can reach 30% and in new buildings this value can rise up to 50% (Lamberts *et al*., 1997).

In the 70´s many simulation programs such as BLAST (1977), DOE-1 (1978), NBSLD (1974), ESP (1974), TRNSYS (1975) and more recently ENERGY PLUS (1999) and DOMUS (2001) had been developed to simulate the building energy performance so that rational policies of energy conservation could be efficiently applied. However, most of those codes present some simplifications on their calculation routines of heat transfer through the ground.

Regarding the ground heat transfer some aspects should be clarified: the multidimensional phenomenon; the transient behavior and the great number of involved parameters, mainly when moisture is considered.

Some studies involving the pure conduction heat transfer through the ground can be found in the literature. Davies *et al.* (1995), using the finite-volume approach, compared multidimensional models and observed that the use of three-dimensional simulation provides better prediction of building temperature and heating loads than two-dimensional simulation, when these results are compared with experimental data. Zoras (2001) used a combination which incorporates structural response factors into a three-dimensional numerical solution of the conduction heat transfer equation.

In the works mentioned above, the conductivity and the thermal capacity are considered constant and the moisture effect is ignored.

The presence of moisture in the ground implies an additional mechanism of transport: in the pores of insaturated soil, liquid water evaporates at the warm side, absorbing latent heat of vaporization, while, due to the vapor-pressure gradient, vapor condenses on the coldest side of the pore, releasing latent heat of vaporization (Deru and Kirkpatrick, 2002).

Freitas and Prata (1996) elaborated a numerical methodology for thermal performance analysis of power cables on the presence of moisture migration in the surrounding soil. They utilized a two-dimension finite-volume approach to solve the governing equations.

Onmura et al. (2001) investigated the evaporative cooling effect of roofs lawn gardens and observed a reduction of up to 50 % in the heat flux through the ceiling.

Due to the numerical instabilities caused by the effect of latent heat at the boundaries, Wang and Hagentoft (2001) presented numerical method based on an algorithm that combines an explicit model with relaxation schemes. In this model, a criterion for time step determination is developed to ensure numerical stability.

A mathematical method to solve the equations of heat and mass transfer in porous media, was developed by Mendes et al. (2002) to ensure numerical stability as well. Their method avoids numerical oscillations by using a generic and robust algorithm (MTDMA – MultiTridiagonal-Matrix Algorithm) to solve simultaneously the governing equations.

In this work, we have developed the software Solum, which was conceived to model the coupled heat and moisture transfer in soils. The governing equations, based on the theory of Philip and De Vries (1957), were discretized using the finite-volume method. The MTDMA algorithm was used to solve a 3-D model to describe the physical phenomena of heat and mass transfer in porous soils.

In soil simulation, some parameters such as the boundary conditions, initial conditions, simulation time period (warm-up), simulation time step and grid refinement have to be carefully chosen and combined in order to reach accuracy without using excessive computational processing.

In this way, this paper deals with different types of boundary conditions for a sandy silt soil submitted to the weather data of the city of Curitiba for long-term simulations. In the results section of the present paper, we showed the effects of boundary conditions such as solar radiation, water table and adiabatic and impermeable surfaces on the temperature and moisture content profiles for simulation time periods of 1, 2, 5, 10, 20, 30, 40 and 50 years.

MATHEMATICAL MODEL

The governing equations, based on the theory of Philip and De Vries (1957), to model heat and mass transfer through porous media, are given by Eqs. (1) and (2). The energy conservation equation is written in the form

$$\rho_0 c_m(T,\theta)\frac{\partial T}{\partial t} = \nabla.(\lambda\ (T,\theta)\nabla T) - L(T)(\nabla.\mathbf{j}) \tag{1}$$

and the mass conservation equation as

$$\frac{\partial\theta}{\partial t} = -\nabla.\left(\frac{\mathbf{j}}{\rho_l}\right) \tag{2}$$

The total flow ($\mathbf{j}$) is given by summing the vapor flow ($\mathbf{j}_v$) and the liquid flow ($\mathbf{j}_l$). The vapor flow can be described as

$$\begin{aligned}\frac{\mathbf{j}}{\rho_l} = &-\left(D_T(T,\theta)\frac{\partial T}{\partial x} + D_\theta(T,\theta)\frac{\partial\theta}{\partial x}\right)\mathbf{i}\\ &-\left(D_T(T,\theta)\frac{\partial T}{\partial y} + D_\theta(T,\theta)\frac{\partial\theta}{\partial y}\right)\mathbf{j}\\ &-\left(D_T(T,\theta)\frac{\partial T}{\partial z} + D_\theta(T,\theta)\frac{\partial\theta}{\partial z} + \frac{\partial K_g}{\partial z}\right)\mathbf{k}\end{aligned} \tag{3}$$

where $D_T = D_{Tl} + D_{Tv}$ and $D_\theta = D_{\theta l} + D_{\theta v}$.

The upper surface of the physical domain (Fig.1) is exposed to short and long-wave radiations, convection heat transfer and phase change as boundary conditions.

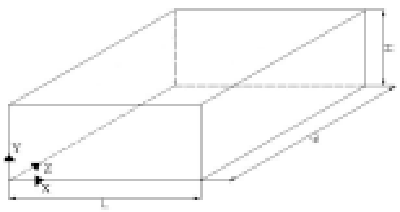

Figure 1. Physical domain of soil

This way, for *Y=H,* the energy balance becomes

$$-\left(\lambda(T,\theta)\frac{\partial T}{\partial y}\right)_{y=H} - \left(L(T)j_v\right)_{y=H} = h\left(T_\infty - T_{y=H}\right) + \alpha q_r + L(T)h_m\left(\rho_{v,\infty} - \rho_{v,y=H}\right) - \varepsilon R_{ol} \tag{4}$$

and the mass balance is written as

$$-\frac{\partial}{\partial y}\left(D_\theta(T,\theta)\frac{\partial\theta}{\partial y} + D_T(T,\theta)\frac{\partial T}{\partial y}\right)_{y=H} = \frac{h_m}{\rho_l}\left(\rho_{v,\infty} - \rho_{y=H}\right). \tag{5}$$

The others surfaces were all considered adiabatic and impermeable as well.

In the water table condition, the nodes of lower surface were saturated and it was not considered any resistance to the mass flow.

Equations 4 and 5 show a vapor concentration difference, $\Delta\rho_v$, on their right-hand side. This difference is between the porous surface and air and is normally determined by using the with values of previous iterations for temperature and moisture content, generating additional unstability. Due to the numerical instability created by this source term, the solution of the linear set of discretized equations normally requires the use of very small time steps, which can be exceedingly time consuming especially in long-term soil simulations; in some research cases, a time period of several decades is simulated, taking into account the tridimensional transfer of heat and moisture transfer through a very refined grid.

In order to rise that simulation time step, Mendes et al. (2002) presented a procedure to calculate the vapor flow, independently of previous values of temperature and moisture content. In this way, the term ($\Delta\rho_v$) was linearized as a linear combination of temperature and moisture content, viz.,

$$\left(\rho_{v,\infty} - \rho_v(s)\right) = M_1(T_\infty - T(s)) + M_2(\theta_\infty - \theta(s)) + M_3 \quad (6)$$

where

$$M_1 = A\frac{M}{\Re}\phi \ ,$$

$$M_2 = \frac{M}{\Re}\left(\frac{P_s(s)}{T(s)}\right)^{prev}\left(\frac{\partial\phi}{\partial\theta(s)}\right)^{prev},$$

$$M_3 = \frac{M}{\Re}\left[\left(\frac{P_s(s)}{T(s)}\right)^{prev} R(\theta^{prev}(s)) + \phi_\infty(R(T_\infty) - R(T^{prev}(s)))\right].$$

SIMULATION PROCEDURE

The governing equations were solved using the finite-volume methodology (Patankar, 1980). It was used the Cartesian coordinates for the geometry of the problem. The differential equations were integrated in each control volume and a fully-implicit scheme was adopted for the time derivatives. The MTDMA (MultiTridiagonal-Matrix Algorithm; Mendes *et al.*, 2002) was used to solve a 3-D model to robustly describe the physical phenomena of the strongly coupled heat and mass transfer in porous soils. In this algorithm, the dependent variables are obtained simultaneously, avoiding numerical divergence caused by the evaluation of coupled terms from previous iteration values.

In this study, the soil was considered sandy silt type (properties taken from Oliveira *et al.*, 1993).

The boundary conditions, the simulation time period (warm-up), the size of the physical domain, the simulation time step, the grid refinement, the convergence errors and the computer run time required are important simulation parameters, which have to be chosen very carefully in order to accurately predict temperature and moisture content profiles in soils under different sort of weather data. In this way, this paper deals with different types of boundary conditions in soils submitted to the weather data of the city of Curitiba for a simulation time period as long as 60 years.

In the present analysis, we have chose a physical domain of 5m x 5m and a depth of 20m and a uniform mesh constituted by 4961 nodes (11 x 41 x 11) was used.

The TRY (Test Reference Year) weather data for the city of Curitiba-Brazil (South latitude of 25.4°) was used, with a constant convection heat transfer coefficient of 10 W/m²K, an absorptivity of 0.5 and a constant long-wave radiation loss of 30 W/m². These conditions were applied to the ground upper surface. The other surfaces were considered adiabatic and impermeable. The water table condition was considered only as a boundary condition for the lower surface. Figs. 2 and 3 present the Curitiba temperature and relative humidity from December 25th to December 31st.

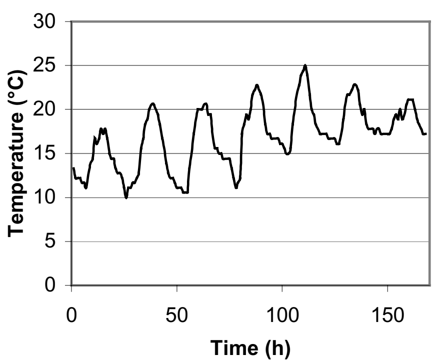

Figure 2. Dry bulb temperature in Curitiba (TRY weather file) from December 25th to December 31st.

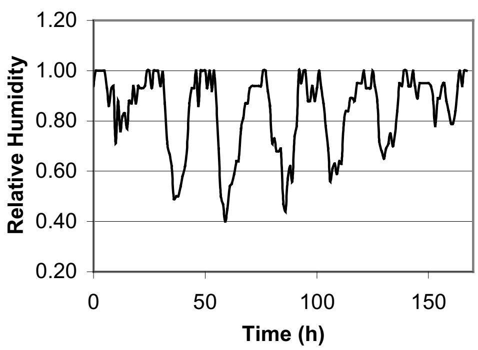

Figure 3. Relative humidity in Curitiba (TRY weather file) from December 25th to December 31st.

RESULTS AND DISCUSSIONS

In Figs. 4 and 5, we have repeated the weather data of the City of Curitiba for a period of 50 years, using a time step of 2 hours. As initial conditions for the soil, a temperature of 15 °C and a volumetric moisture content of 4% were utilized.

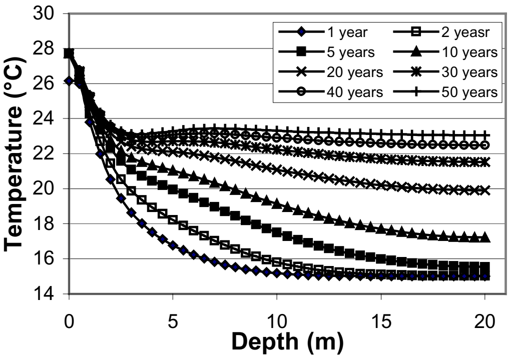

Figure 4. Temperature profiles of the sandy silt soil at 12 pm on December 31st .

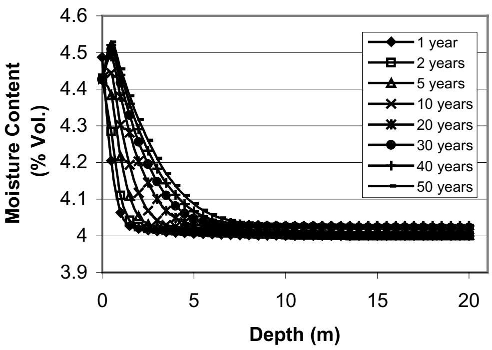

Figure 5. Moisture content profiles of the sandy silt soil at 12 pm on December 31st .

Figs. 4 and 5 present the temperature and moisture content spatial distributions at 12 pm on December 31st for 1, 2, 5, 10, 20, 30, 40 and 50 years. It is possible to notice in Figs. 4 and 5 that the temperature and moisture content values remain constant, in essence, at the interface between ground and air. We also remark a very slow evolution especially in the moisture content distribution.

The time evolution differences in the temperature and moisture content profiles are mainly attributed to the high thermal and hygric soil capacities. As we could expect, the temperature and moisture content average values at deeper soil sections are directly associated to the yearly average magnitudes for temperature, solar radiation and partial vapor pressure.

On the other hand, at shallow soil sections, we find average values of temperature and moisture content connected to daily averages of those weather parameters listed in the paragraph above. This temperature and moisture content distribution tendency are sharply related to the Biot number for both heat and moisture transfer. For example, for the moisture transfer (Bi_m), this dimensionless number can be written as:

$$Bi_m = \frac{h_m L}{D_\theta},$$

which means, for high Bi_m numbers, the soil hygric resistance is much higher than the corresponding surface resistance.

Fig. 5 shows also that, at a depth of 5m, the moisture content profile hardly changes with time.

In this way, we repeated the same simulation that produced Figs. 4 and 5 for a 5-m depth. In this case, we could observe the effect of choosing a shallower soil physical domain. Therefore, Figs. 6 and 7 demonstrate that the temperature and moisture content at the soil free surface remained practically unaltered when we compare them to those values shown in Figs. 4 and 5.

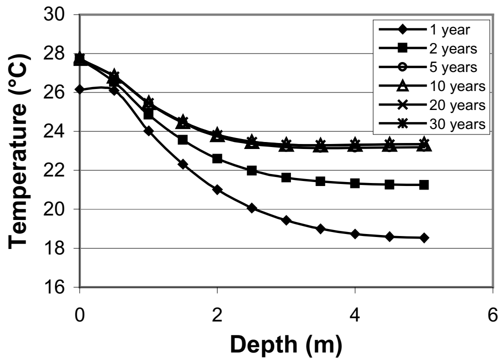

Figure 6. Temperature profiles in a soil physical domain with a 5-m depth.

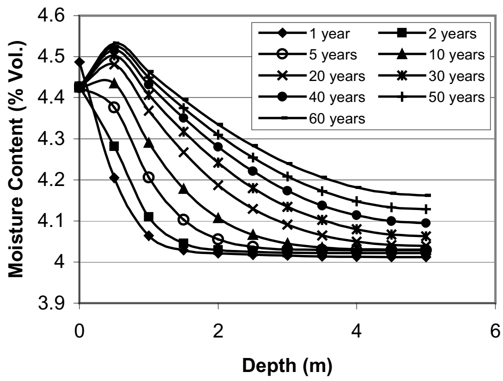

Figure 7. Moisture content profiles in a soil physical domain with a 5-m depth.

However, we can remark, by comparing Figs. 4 and 5 with Figs. 6 and 7, that the profiles considerably change when the considered simulated soil depth decreases from 20m to 5m.

On the other hand, as computer run time plays an important role in yearly building simulations, a depth of 5m could be suggested whereas the main temperature and moisture content gradients are close to the upper surface, which affects more significantly the building thermal behavior than the gradients on the vertical direction.

Figs. 8 and 9 show the temperature and moisture content profiles when a water table boundary condition is considered. In this case, it was neglected the presence of a resistance to the vapor flow between the water free surface and the lower ground surface, i.e., the water table was considered on direct contact with the soil bottom surface. Thereupon, a Dirichlet boundary condition was considered so that the volumetric moisture content assumed the soil porosity value. In that manner, this would represent an extreme situation for the analysis of the water table effects on the temperature and moisture content profiles.

It can be observed, in Figs. 8 and 9, the meaningful influence of moisture content gradients on the regulation of the temperature profile. The mathematical model takes this effect into account by the presence of the phase change term in the energy conservation equation (Eq. 1). In other words, the vapor flow divergent, at the imbibition front, decreases the temperature whereas the evaporation term becomes dominant when compared to the heat diffusion contribution.

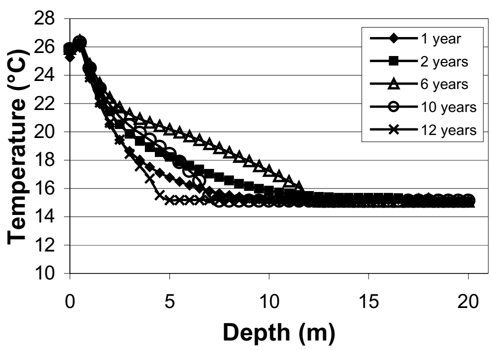

Figure 8. Temperature profiles of the sandy silt soil considering water table as the bottom surface boundary condition.

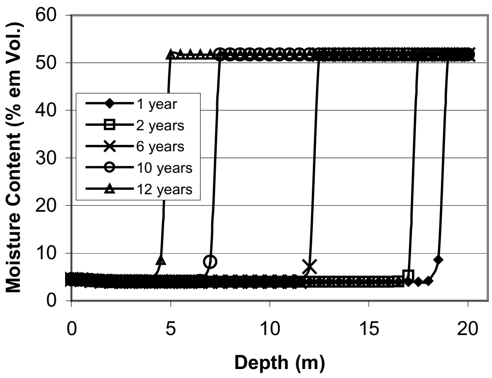

Figure 9. Moisture Content profiles of the sandy silt soil considering water table as the bottom surface boundary condition.

CONCLUSIONS

In this work, we presented a mathematical model to analyze the coupled heat and moisture transfer in soils. The MTDMA (MultiTridiagonal-Matrix Algorithm) was utilized to solve the heat and mass transfer governing equations in porous soils. This method avoids numerical unstabilities by solving simultaneously the governing equations, allowing the use of high time steps which are very important for long-term simulation of the tridimensional heat and mass transfer in soils, with transport coefficients highly dependent on the moisture content.

In the results section, we presented temperature and moisture content profiles of the sandy silt soil for 1, 2, 5, 10, 20, 30, 40 and 50 years.

We noticed the importance of simulation time period on the estimation of temperature and moisture content profiles. As computer run time plays an important role in yearly building simulations, a depth of 5 m was suggested whereas the main temperature and moisture content gradients are close to the upper surface, affecting somewhat the thermal behavior of low-rise buildings.

For further work, we will analyze the simultaneous tridimensional ground heat and moisture transfer integrated to a building simulation code.

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NOMENCLATURE

c_m - meam specific heat (J/kg K)

D_{Tl} - liquid fase transport coefficient associated to a temperature gradient (m²/s K)

D_{Tv} - vapor fase transport coefficient associated to a temperature gradient (m²/s K)

$D_{\theta l}$ - liquid fase transport coefficient associated to a moisture content gradient (m²/s)

$D_{\theta v}$ - vapor fase transport coefficient associated to a moisture content gradient (m²/s)

D_T - mass transport coefficient associated to a temperature gradient (m²/s K)

D_θ - mass transport coefficient associated to a moisture content gradient (m²/s)

h_m - mass convection coefficient (m/s)

h - convection coefficient (W/m² K)

j_v – vapor flow (kg/m² K)

j - total flow (kg/m² K)

L - latent heat of vaporization (J/kg)

M – molecular mass (kg/kmol)

P_s – satured pressure (Pa)

prev – previos iteration

q_r - solar radiation (W/m²)

$\Re$ - universal gas constant (J/kmol K)

R_{ol}- long-wave radiation (W/m²)

T - temperature (°C)

t – tempo (s)

α - solar absorptivity

ε - emissivity

ϕ - relative humidity

λ - thermal conductivity (W/m² K)

ρ_0 -solid matrix density (m³/kg)

ρ_l - water density (kg/m³)

θ - volume basis moisture content (m³/m³)