

TWO-OBJECTIVE ONLINE OPTIMIZATION OF SUPERVISORY CONTROL STRATEGY

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ABSTRACT

The set points of supervisory control strategy are optimized with respect to energy use and thermal comfort for existing HVAC systems. The set point values of zone temperatures, supply duct static pressure, and supply air temperature are the problem variables, while energy use and thermal comfort are the objective functions. The HVAC system model includes all the individual component models developed and validated against the monitored data of an existing VAV system. It serves to calculate energy use during the optimization process, whereas the actual energy use is determined by using monitoring data and the appropriate validated component models. A comparison, done for one summer week, of actual and optimal energy use shows that the on-line implementation of a genetic algorithm optimization program to determine the optimal set points of supervisory control strategy could save energy by 19.5%, while satisfying the minimum zone airflow rates and the thermal comfort. The results also indicate that the application of the two-objective optimization problem can help control daily energy use or daily building thermal comfort, thus saving more energy than the application of the one-objective optimization problem.

INTRODUCTION

Since the energy crisis of the 1970s, great efforts have been invested in minimizing the energy costs associated with the operation of the HVAC system. Computer-aided energy management techniques as well as rigorous applications of the variable air volume (VAV) concept were accepted as means of achieving an energy-efficient and comfortable building environment. Not all VAV systems are successful and efficient (Cappellin, 1997; Linder, 1997). Multiple factors contribute to this unfortunate situation, including the system control strategy. This lack of efficiency is due to the fact that they are not wide-interaction optimized. The performance of HVAC systems can be improved through optimized supervisory control strategies (ASHRAE, 1999). Set points can be adjusted by the optimized supervisor to improve the operating efficiency. Several studies have investigated the optimum set points of one or

two local-loop controllers. For example, Ke and Mumma (Ke, 1997 b) investigated the interactions between the supply air temperature and the ventilation requirement in order to determine the optimal supply air temperature; Englander and Norford (Englander, 1992) minimized the supply duct static pressure set point without sacrificing occupant comfort or the adequate ventilation, and Braun (Braun et al., 1989 b) determined the chilled water set point by optimizing chilled water systems. To date, the interaction between all primary and secondary system set points have not been investigated by using two objective genetic algorithms. This research takes into account this interaction between all these set points. Our objective is to optimize on-line HVAC supervisory control strategies by determining the optimal controller set points. This work investigates the multi-objective genetic algorithm optimization method. The genetic algorithm is applied to a wide range of scientific, engineering, and economic search problems (Goldberg, 1989). The genetic algorithm potential is studied for different applications: (i) the control of air-conditioning systems (Huang, 1997; Lam, 1995; Nordvik, 1991), and (ii) HVAC system design using the one-objective method (Asiedu, 2000; Wright and Farmani, 2001; Wright, 1996) and the two-objective method (Wright and Loosemore, 2001).

SYSTEM DESCRIPTION

The real time, one-line optimization of the HVAC supervisory control strategy is investigated through the optimization of existing HVAC system set points. Two VAV systems (AHU-4 and AHU-6) investigated in this research are installed at the *École de technologie supérieure (ETS)* campus. These systems were monitored starting in August 2001. The AHU-4 meets the load for 68 west perimeter zones, while the AHU-6 meets the load for 70 interior zones on the second floor. In this paper, the AHU-6 is only presented and consequently the zone reheat is excluded from the problem formulation. Secondary system control loops including (i) the 70-zone temperature control loops, (ii) the cooling and heating control loop, and (iii) the fan control loop, are investigated by determining their set points, which are the zone temperatures, the supply air temperature, and the duct static pressure,

respectively. The primary system control loops and their set points, as the chilled water supply temperature, the chilled water loop differential pressure, and the condenser water supply temperature, are not investigated in this study. The values of these set points were obtained directly through monitoring. It should be noted that presently, the supervisory control strategy of the ETS system adjusts the controller set points as predetermined fixed values, with the exception of the supply air temperature set point, which varies with the outdoor temperature and fan airflow rate.

PROBLEM FORMULATION

The optimization seeks to determine the set point values of the supervisory control strategy of the ETS system. These set points should be optimized for the operating consumption energy and the building thermal comfort. The optimization problem is formed through the determination of the problem variables, the constraints, and the objective functions.

Problem variables

The following are the problem variables for AHU-6:

- Zone temperature set points (70 variables)
- Supply duct static pressure set point
- Supply air temperature set point

The resulting problem variables consist of 72 variables for AHU-6, but the number rises to 138 variables for AHU-4 (not presented in this paper) through the addition of 68 zone supply air temperature variables in order to take into account the zone reheat.

Constraints

The constraints result from restrictions on the operation of the HVAC system. They cover the lower and upper limits of variables such as supply air temperature, zone temperatures, etc. The constraints also cover the design capacity of components. The fan and zone airflow rates, for instance, are restricted within the maximum and minimum limits. The following are the minimum limits:

- The minimum fan airflow rate is equal to 40% of design one
- The minimum zone airflow rates are determined by the ETS system operator, and are generally equal to 30% of the design zone airflow rate (ASHREA recommendation)

Through monitoring, it is observed that the minimum zone airflow rates determined above are not respected in some zones. Consequently, three minimum zone flow rate constraints are applied:

- *100% constraint*—the minimum zone airflow rates are exactly the same as the minimum zone airflow rates determined by the ETS system operator
- *90% constraint*—the minimum zone airflow rates are equal to 90% of the minimum zone airflow rates recommended (as mentioned above)
- *Without constraint*—the minimum zone airflow rates could be zero, and thus the zone VAV damper could shut off

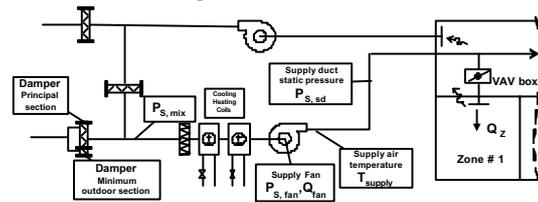


Figure 1 Schematic of VAV system

Regarding the maximum zone airflow rate limits, they vary with the supply duct static pressure set point ($P_{S, sd}$) which is the optimization variable. For each simulation (i^{th}), the maximum zone airflow rate $Q_{Z, max, i}$ corresponding to this variable ($P_{S, sd, i}$) is calculated as follows:

$$\dot{Q}_{z, max, i} = \dot{Q}_{z, max, design} \cdot \sqrt{\frac{P_{S, sd, i} - \Delta P_i}{P_{S, sd, design} - \Delta P_{design}}}$$

The minimum limit of the supply duct static pressure, considered as the constraint, is adjusted in order to ensure that the corresponding $Q_{Z, max, i}$ is greater than the current (i^{th}) zone airflow rate required to satisfy the zone load. The terms ΔP_{design} and ΔP_i represent the design and current (i^{th}) pressure drop between the static pressure sensor point and the zone VAV box inlet, respectively. The equation above can be simplified as:

$$\left(\dot{Q}_{z, max, i} \right)_{simplified} = \dot{Q}_{z, max, design} \cdot \sqrt{\frac{P_{S, sd, i}}{P_{S, sd, design}}}$$

Given that, $\Delta P_{design} > \Delta P_i$, and consequently $(Q_{Z, max, i})_{simplified} < Q_{Z, max, i}$, this simplification further ensures, for given (i^{th}) supply duct static pressure set point, that no zone box is starved for supply air. The zone PPD is also limited within the [5-10] range.

Objective functions

The set points of the supervisory control strategy are optimized in order: (i) to reduce energy use, and (ii) to improve thermal comfort, which are two objective functions.

Energy use includes the Chiller and fan power demand. The models of HVAC system and their

components used in calculating energy use are presented in the sections that follow.

The zone comfort is represented as the “Predicted Percentage of Dissatisfied” (PPD), and calculated using the following equation:

$$PPD=100-95 \cdot EXP[-(0.03353 PMV^4 + 0.2179 PMV^2)]$$

The predicted mean vote (PMV) is an index devised to predict the mean response of a large group of people according to the ASHRAE thermal sensation scale (ASHRAE, 1997). In a practical situation, the tabulated PMV values can be used to predict the performance of a VAV system for a combination of variables (Chen and Demster, 1995). For example, for a combination of 1 clo (I_{cl} clothing insulation), an ambient temperature of 24°C and a relative velocity of 0.3 m/s, the PMV value from Table 1 is -0.09.

Table 1
Predicted mean vote (PMV)

I_{cl} clo	Temp. °C	Relative Velocity (m/s)			
		<0.1	0.2	0.3	0.4
0.75	21	-1.11	-1.44	-1.66	-1.82
	23	-0.47	-0.78	-0.96	-1.09
	24	-0.15	-0.44	-0.61	-0.73
	25	0.17	-0.11	-0.26	-0.37
1.00	21	-0.57	-0.84	-0.99	-1.11
	22	-0.30	-0.55	-0.69	-0.80
	23	-0.02	-0.27	-0.39	-0.49
	24	0.26	0.02	-0.09	-0.18
	25	0.53	0.31	0.21	0.13

For the interior zones, operative zone temperatures are assumed to be equal to zone temperatures. In this paper, the zone air velocity is only assumed to be fixed at less than 0.1 m/s. However, the diffuser model used in determining the zone air velocity and the operative zone temperatures could be the subject of further research.

HVAC SYSTEM MODEL

In simulation and optimization calculations, the mathematical model of the HVAC system must include all the individual component models that influence the objective functions. For this research project, these component models are developed and validated against the recorded data of an existing VAV system, and are presented in the next sections. The outdoor airflow rate and fan power and airflow rate were determined using the damper and fan models, respectively. The cooling coil demand was calculated through the detailed cooling coil model developed from the ASHRAE HVAC 2 Toolkit (Brandemuel, 1993). The Chiller energy use is modeled using a simple equation developed by running regression analyses on the manufacturer’s design data (Chiller model). The valve and duct works are also modeled for this research, but they are not presented in this paper (Nassif et al., 2003).

Fan model

The fan model was introduced by Clark (Clark, 1985) to estimate airflow rates as a component of fluid flow networks. It uses fourth-order polynomial fits to the dimensionless head and efficiency to predict the fan pressure rise and power. This model is validated by comparing the calculated airflow rate with the recorded rate. Figure 2 shows the comparison of the airflow rate recorded and obtained using the fan model for July 25-31. The results indicate that the fan model accuracy varies between 2 and 3%.

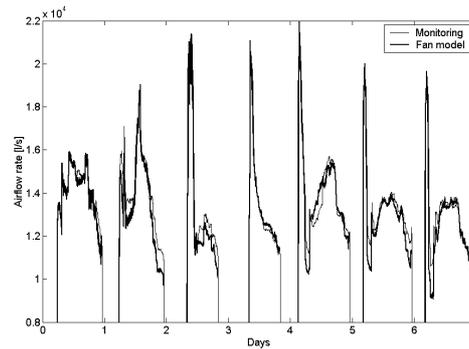


Figure 2 Comparison of airflow rate recorded and obtained by fan model for July 25 to 31

Damper model

The damper model is used to determine the outdoor airflow rate. It is based on the formula $Q = C * \Delta P^c$, where the coefficients C and c are determined from the manufacturer’s data.

This model is validated for three operation modes:

- When the damper is fully opened, the outdoor airflow rate calculated by the model is compared with the recorded fan airflow rate
- When the damper modulates, the outdoor airflow rate calculated using the model is compared with the outdoor airflow rate calculated through the temperature balance method, taking into account only the data when this method is applicable (difference between the return and outdoor air temperature is sufficiently large) (Schroeder et al., 2000)
- When the damper is at a known minimum outdoor air position, the outdoor airflow rates calculated using the model are compared with the manufacturer’s data and with the 10% fan airflow rate, which is considered as a minimum outdoor air intake rate in the monitored VAV.

Figure 3 shows a comparison of the airflow rate recorded and obtained by the damper model when the

damper is wide open for May 3 to 5. Figure 4 shows the outdoor airflow rate calculated using the temperature balance method (TBM), respecting the difference between the return and outside air temperatures, and using the damper model (DM) for December 1. The validation results of these three operation modes are 4%, 5%, and 1.8%, respectively.

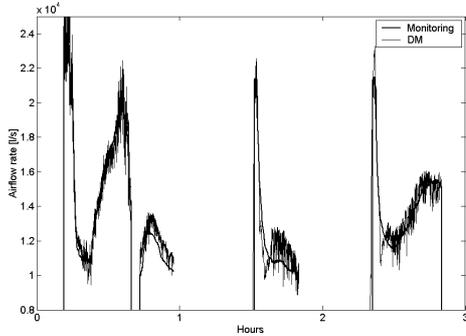


Figure 3 Comparison of airflow rate recorded and obtained by damper model (damper wide open)

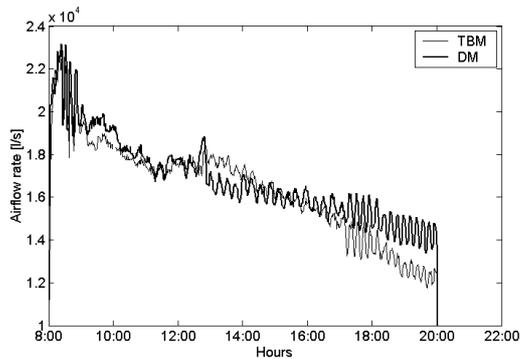


Figure 4 Comparison of outdoor airflow rate obtained by damper model (DM) and by temperature balance method (TBM)

Cooling coil model

The simplified (CCSIM) and detailed (CCDET) cooling coil models are developed based on the ASHRAE HVAC 2 Toolkit. The cooling coil models are validated by comparing the leaving cooling coil air temperature calculated using the model with that recorded by the system control of an existing VAV air handling unit. Since the opening of the cooling coil valve is recorded instead of the water flow rate, the valve model is also combined with the cooling coil model. The fan and duct air heat up are added to the simulated leaving cooling coil air temperature to calculate the supply air temperature. Figure 5 shows the supply air temperature recorded and obtained by cooling coil models for July 29. As was expected, the accuracy of the detailed model is better than that of the simplified one. They are 1.8 and 23% for the CCDET model and CCSIM model, respectively. Obviously, the CCDET model is only used in this research project.

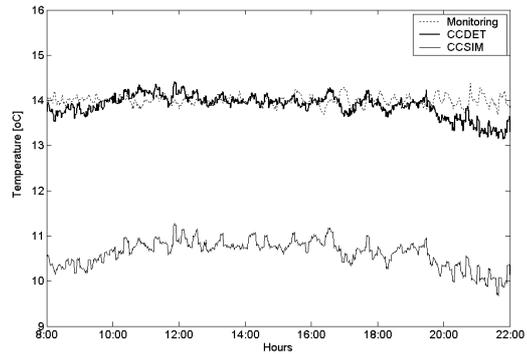


Figure 5 Comparison of air supply temperature recorded and obtained by cooling coil models for July 29

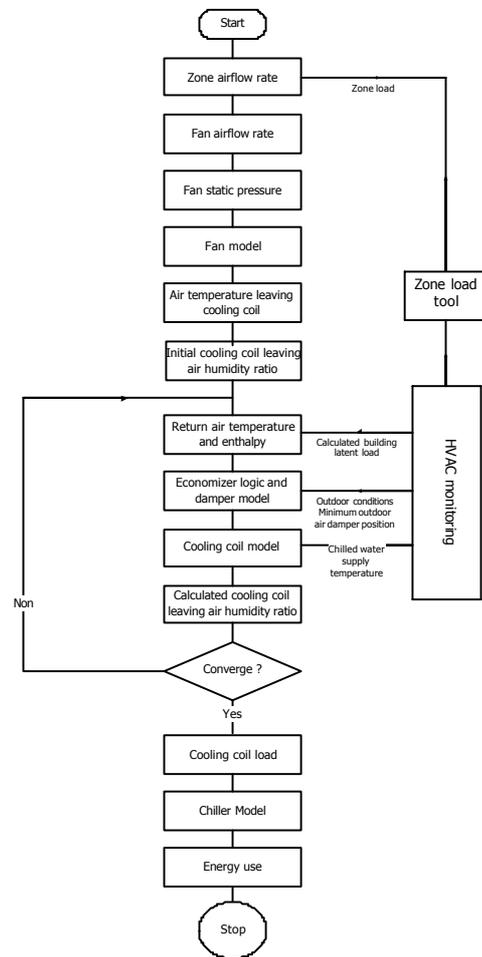


Figure 6 Flow diagram for simulation of VAV system required by optimization process

Integration of system model

Energy calculations, required by the optimization process, involve the construction of the complete VAV system model. This model comprises the HVAC component models presented above and some

calculations between the linked models and the data recorded by monitoring. Figure 6 shows the flow diagram of this model. The recorded outdoor conditions and the chilled water supply temperature are the inputs to the VAV system model as well as the zone loads and building latent load, allowing the calculation of energy use. The zone load is calculated using a simplified zone load tool, assuming that it is exactly the same as the current air-conditioning load. This assumption is required to compare the actual and optimal energy use. To determine the zone load, the recorded zone temperature, the supply air temperature, and the zone airflow rate are used. The building latent load is calculated using the recorded supply and return air temperature and relative humidity and the fan airflow rate. The load of each zone and the building latent load thus calculated are constant during the optimization process. When problem variables are generated by Genetic Algorithms during the optimization process, first, the required zone airflow rates, in order to satisfy zone loads, are calculated taking into account the differences between zone temperature and supply air temperature (problem variables). In the next step, the fan airflow rate $\dot{Q}_{fan,i}$ is calculated as the sum of zone airflow rates. The fan static pressure ($P_{S,fan,i}$), using another problem variable (supply duct static pressure), is then calculated using the following formula:

$$P_{S,fan,i} = \left(\frac{\dot{Q}_{fan,i}}{\dot{Q}_{fan,design}} \right)^2 * (P_{S,fan,design} - P_{S,sd,i}) + P_{S,sd,i}$$

The fan static pressure and the fan airflow rate are inputs to the fan model, allowing the calculation of the fan power consumption. Assuming an initial value for the cooling coil leaving air humidity ratio, the return humidity ratio and enthalpy can be calculated using a building latent load. The mixed air temperature and humidity ratio are only determined through the damper model when the damper is in the minimum outdoor position. In other cases, the economizer logic is used. The mixing plenum box static pressure ($P_{S,mix,i}$) required by the damper model is calculated as follows:

$$P_{S,mix,i} = \left(\frac{\dot{Q}_{fan,i}}{\dot{Q}_{fan,design}} \right)^2 * P_{S,mix,design}$$

A new value of the cooling coil leaving air humidity ratio to verify the assumed value as well as the cooling coil load are calculated in the cooling coil model. Finally, the Chiller power is calculated by using the Chiller model. On the other hand, the thermal comfort model determines the values of the zone and building PPD.

Actual energy use is calculated using monitoring data and the component models. Figure 7 illustrates the

flow diagram for this calculation. In this case, the recorded data are the inputs to the fan, damper, and valve models. Their outputs as well as the recorded chilled water supply temperature are the inputs to the cooling coil model used to determine the cooling coil load. The Chiller model uses the cooling coil load to calculate the Chiller power, while the recorded zone temperatures are the inputs to the thermal comfort model, used to calculate the value of the PPD.

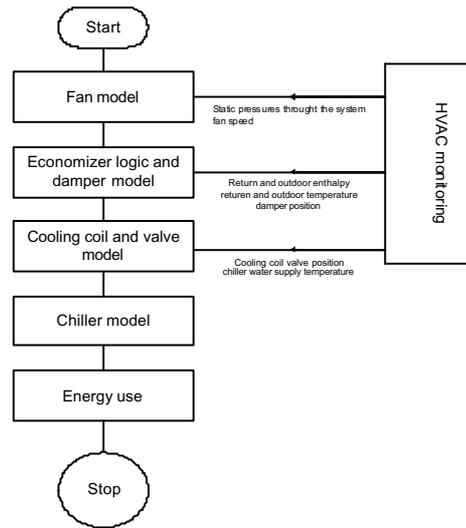


Figure 7 Flow diagram for simulation of existing and monitored VAV system

In real-time, on-line optimization, three following modifications will be required: (i) on-line measured data instead monitoring data will be used, (ii) zone load tool will be developed to predict the zone load taking into account the gradient of the temperature zone variation and (iii) minimum outdoor air damper position and chilled water supply temperature will be the problem variables.

OPTIMIZATION ALGORITHM

In this study, a genetic algorithm search method based on the mechanics of Darwin's natural selection theory was developed in order to solve the optimization problem. Since energy use and thermal comfort are the objective functions, the multi-objective genetic algorithm must be investigated. The principles of multi-objective genetic algorithm optimization are different from those of a single-objective genetic algorithm optimization. The main goal in a single-objective optimization is to find the global optimal solution, resulting in the optimal value for the single-objective function. However, in a multi-objective optimization, there is more than one objective function, each of which may have a different individual optimal solution. If there are sufficient differences in the optimal solutions corresponding to the different objectives, the objective functions are often recognized as

conflicting with one another. Multi-objective genetic algorithms with such conflicting objective functions give rise to a set of optimal solutions, instead of one optimal solution. The reason for the optimality of many solutions is that no single solution can be considered to be better than the other(s) with respect to all objective functions. These optimal solutions have a special name: *Pareto-Optimal Solutions*. The two-objective genetic algorithm optimization method investigated here is an elitist non-dominated sorting genetic algorithm (NSGA?) developed by Deb (Deb, 2001). A number of genetic algorithm methods with adjustments made to their control parameters were first investigated for use in solving different mathematical problems in the first step of our research. Our results showed that the real coded NSGA? performed better than the others, with respect to two performance metrics: (i) metrics evaluating closeness to the Pareto-optimal front-*Generation Distance* method (Veldhuizen, 1999), and (ii) metrics evaluating diversity among non-dominated solutions-*Spread* metric (Deb, 2001). The results are presented in reference (Nassif, 2002). Two-objectives genetic algorithm optimization (real coded NSGA) was run for 800 generations. The following genetic algorithm control parameter values are chosen: crossover probability $p_c=0.9$, mutation probability $p_m=0.04$ and population size $p_z=100$. The simulated binary crossover (SBX) was also implemented in this study (Deb, 2001).

RESULTS

All the monitoring data used in optimization process are averaged here for 30 minutes. Energy use was calculated during the optimization process using the VAV simulation model illustrated in Figure 6. Since zone monitoring data was not available for last winter, the investigation only covers the summer of 2002. However, the optimization is ongoing, in order to cover this winter (2002-2003). Figure 8 shows the optimal solutions after 800 generations at 16:00 h.

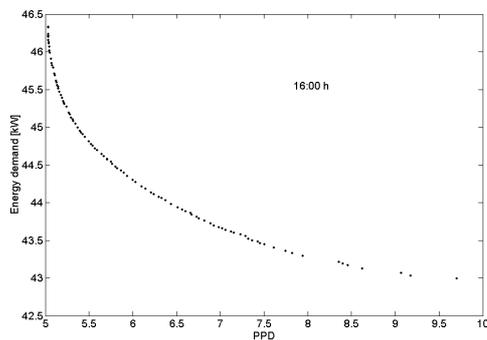


Figure 8 Optimal solutions obtained after 800 generations at 16h00 for July 29

The PPD presented here is the mean value of the PPD determined for each of 70 zones (AHU-6 PPD). An increase in thermal comfort (decrease in PPD)

requires an increase in energy use. Actual energy use was also determined for each 30-minute periods using monitoring data and appropriate models as shown in Figure 7. To compare the optimal and actual energy use of a VAV system (AHU-6), only one solution for each 30-minute periods was selected among the set of solutions presented in Figure 8. This solution has the same PPD as the PPD obtained from monitoring an AHU-6 system. Figure 9 shows this comparison of the actual and optimal energy use for the same PPD on July 29, 2002. Three optimal cases are presented, corresponding to minimum zone airflow rate constraint, as mentioned in the section above. As shown in Figure 9, the optimal energy use for “100% constraint” of the minimum airflow rate is less than the actual energy use. The energy saved by optimization is 18.8% for July 29 and 19.5% for July 25 to 31. It is clear that the energy saving is higher for the other cases with “90% constraint” and “without constraint.”

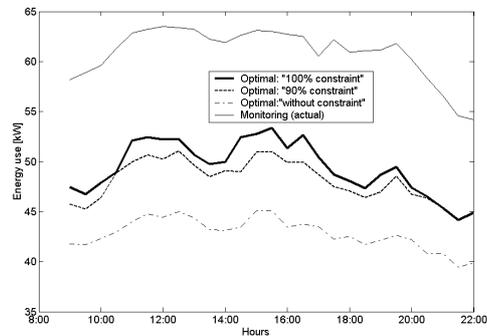


Figure 9 Energy demand for July 29

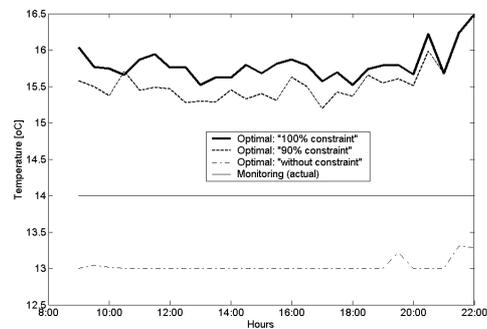


Figure 10 Set points of air supply temperature for July 29

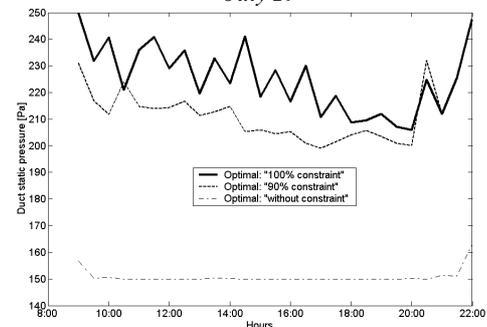


Figure 11 Set points of static pressure for July 29

Figures 10 and 11 show the optimal supply air temperature and duct static pressure set points, respectively (always for three cases of minimum zone airflow rate constraints). The actual AHU-6 system supply duct static pressure set point is fixed, and is equal to 250 Pa. In our case, the temperature and static pressure decrease when the constraint applied to the minimum zone airflow rate decreases from “100%” to “without constraint.” The figure 10 and 11 show the results relating to the AHU-6 system. Concerning the zone variables obtained by the monitoring and by the optimization, it could be noted that:

- the optimal zone airflow rates stay above 30% in the case of “100% constraint,” stay above 27% in the case of “90% constraint,” and may be 0 in situations “without constraint” applied to the minimum zone airflow rate. As it was previously mentioned, the option “without constraint” is closed to the actual system operation because it is observed through monitoring that the minimum zone airflow rates are not respected in some zones.
- the most optimal zone temperatures are between 23-24.5°C and the optimal zone PPDs are within the [5-10] range.
- the actual zone temperatures are within 19-25°C and the most zone PPDs are within the [5-15] range. Some zone PPD values are as high as 30.

These results and the energy saving (Figure 9) show that optimization improves the performance of HVAC system.

Thermal comfort is presented in this paper as the objective function (two-objective approach), but the optimization problem could be solved by defining the thermal comfort criteria as constraint, with a maximum PPD determined for the occupied period (one-objective approach). From Figure 8, the energy use is 43 kW for a PPD of 9.8%, and 46.4 kW for a PPD of 5.1%. This means that to improve the building thermal comfort from a PPD of 9.8 to a PPD of 5.1% at 16:00 h, the ratio of energy required “*e*” is 3.33 kW/PPD. In the morning, this energy ratio “*e*” is only 1.6 kW/PPD. The results show that the ratio “*e*” varies with PPD and with the time. This means that it may be better to control the mean daily thermal comfort (daily PPD) instead of the instantaneous PPD, but that is only possible with the two-objective approach. In this case, the required mean daily PPD of 7.5%, for example, is respected even if it is optimal for daily energy use to operate at 5% PPD in the morning and at 10% PPD at noon. This idea leads to the development of a two-objective optimal selection tool. This tool, using the ratio “*e*” defined above, selects at each run (30 minutes, in this case) one out of a set of solutions (Figure 8), which respect

the mean required daily PPD and the minimum possible daily energy use. The lower subplot of Figure 12 shows the ratio “*e*” required for obtaining the daily PPD and the upper subplot shows the daily energy use as a function of the daily PPD. Each point “*” in the upper subplot presents the daily energy use (sum of the energy demands of each run) obtained by selecting, from the set of solutions, only the solutions having the same PPD during the day. Given that all points “*” are above the curve, it means that the control of the mean daily PPD (presented by the curve) provides more energy saving. Although this energy saving is not significant in our case, it may be higher for the other profiles of the zone load.

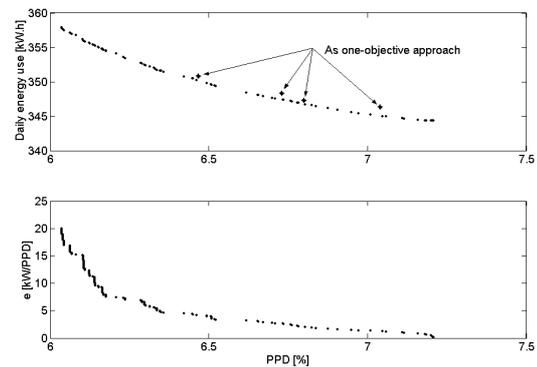


Figure 12 Optimal daily energy use obtained using the two-objective selection tool

CONCLUSION

The set points of the supervisory control strategy are optimized using a two-objective genetic algorithm optimization method. The set point values of the zone temperatures, supply duct static pressure, and supply air temperature, are optimized for existing HVAC system.

To establish the optimization procedure, the VAV system and component models were developed and validated against the monitored data of an existing system. The energy use was calculated during the optimization process using these models, while actual energy use was determined through monitoring data and appropriate validated models.

The results show that by comparing actual and optimal energy use, the on-line implementation of the genetic algorithm optimization program to determine the optimal set points of a supervisory control strategy could save energy by 19.5%, while satisfying the minimum zone airflow rates and zone thermal comfort. These results are obtained for one summer week. They also indicate that the application of a two-objective optimization problem can help control the daily energy use or daily building thermal comfort, thereby saving more energy than the application of the one-objective optimization problem.

Although this paper does not deal with the interaction between all controller set points, the research findings somewhat encourage further investigation. The interaction between the other set points, (i) zone supply temperature (reheat), (ii) chilled water supply temperature, (iii) chilled water loop differential pressure, (iv) condenser water supply temperature, and (v) ventilation requirement, will be investigated.

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