

AUTOMATING THE SELECTION OF FENESTRATION SYSTEMS TO BEST MEET DAYLIGHTING PERFORMANCE GOALS

Luís Fernandes^{1,2}, Konstantinos Papamichael¹

¹Building Technologies Department, Environmental Energy Technologies Division, Ernest Orlando Lawrence Berkeley National Laboratory, Berkeley, California, USA

²Building Systems Program, Civil Architectural and Environmental Engineering Department, University of Colorado, Boulder, USA

ABSTRACT

Traditional selection of fenestration systems follows a trial-and-error approach, i.e., iterative selection of fenestration systems and evaluation of their performance through the use of simulation tools and techniques. This paper is about the development of a new method, which inverts traditional practice, i.e., aims at automated selection of fenestration systems that best meet specific lighting performance goals.

The new method is based on manipulation of matrices that represent the optical properties of fenestration systems, the propagation of light in interior spaces, and the outdoors luminous conditions. The method follows two steps: 1) the determination of the luminous flux distribution originating from the location of the fenestration that best meets the desired lighting performance goals and 2) the selection of the fenestration system that comes closer to producing the desired flux distribution from the outdoor daylight distribution.

INTRODUCTION

The selection of fenestration systems for daylighting is traditionally done by trial and error. Designers try to find fenestration systems that will meet lighting performance goals by determining and evaluating the performance of various alternatives and then selecting the one that best meets the performance goals. This is a time-consuming process that limits the number of fenestration systems that can be considered. Moreover, the selection and performance assessment of fenestration systems is based mostly on intuition, rather than a comprehensive performance analysis.

INVERTING THE PROCESS

This paper is about a new method that aims to invert traditional practice by automating the selection of fenestration systems based on specific daylighting performance goals. This new method is based on mathematical operations on matrices that represent the bi-directional optical properties of

fenestration systems, the propagation of light in interior spaces and the outdoors luminous conditions (figure 1).

The method involves two major steps. The first step is focused on determining the desired directional distribution from the fenestration that best meets a set of specified lighting performance goals, e.g., daylight work plane illuminance. The second step of the method is focused on determining the light transmission characteristics of a fenestration system that will best match the desired distribution from the exterior lighting conditions.

Step 1: Determining the desired directional distribution

A fenestration system can be considered as a light source whose candlepower distribution, $I(\theta, \phi)$, varies over time [Papamichael, 1990]. Dividing the hemisphere into n solid angles and assuming uniform luminous intensity within each solid angle, the distribution of transmitted flux, Φ_T , through a fenestration system is specified by:

$$\Phi_T = \begin{bmatrix} \phi_{T1} \\ \vdots \\ \phi_{Ti} \\ \vdots \\ \phi_{Tn} \end{bmatrix} \quad (1)$$

where ϕ_{Ti} is the flux emitted into the i th solid angle.

The effect of an arbitrary flux distribution on the illuminance and/or luminance at an arbitrary point within a space is the sum of the effects of the light emitted by the fenestration system into each of the solid angles considered in the discretization of the emitted intensity distribution. If illuminance at point P is being considered, then

$$E_{Ptot} = \sum_{i=1}^n E_{Pi} \quad (2)$$

where E_{Pi} is the illuminance at point P due to the light emitted by the fenestration into the i th solid angle.

E_{Pi} can be expressed as

$$E_{Pi} = c_i \phi_{Ti} \quad (3)$$

where c_i is a factor representing how many units of illuminance are produced at P for every unit of flux emitted into the i th solid angle.

E_{Ptot} then becomes

$$E_{Ptot} = \sum_{i=1}^n c_i \phi_{Ti} \quad (4)$$

The objective of the first step of the method is to invert equation 4, i.e., to compute the Φ_T that will meet a target value for E_{Ptot} . Equation 4, however has an infinite number of solutions, i.e., there is an infinite number of distinct values of Φ_T that will meet a target value for E_{Ptot} . This changes, however, if goals are satisfied at n or more points, i.e., when illuminance goals are specified over a surface or several surfaces. Fortunately, this is consistent with daylight design! The mathematical representation then becomes a system of n or more equations. If m equations are defined, then:

$$\begin{cases} E_{Ptot,1} = \sum_{i=1}^n c_{i1} \phi_{Ti} \\ \vdots \\ E_{Ptot,m} = \sum_{i=1}^n c_{im} \phi_{Ti} \end{cases} \quad (5)$$

This can also be put in matrix form:

$$\mathbf{E}_{Ptot} = \mathbf{C} \cdot \Phi_T \quad (6)$$

where \mathbf{C} is a m -by- n matrix that contains the coefficients c_{ij} .

Assuming that $m > n$, the system of equations is over-determined and a solution can only be found in the least-squares sense, i.e., computation of the distribution flux that will come as close as possible to the meeting the set of illuminance performance goals over one or more surfaces.

Step 2: Determining the desirable fenestration transmittance

Just like the transmitted distribution through the fenestration system, the distribution of flux coming from the exterior onto the fenestration system can be discretized into p solid angles:

$$\Phi_I = \begin{bmatrix} \phi_{I1} \\ \vdots \\ \phi_{Ii} \\ \vdots \\ \phi_{Ip} \end{bmatrix} \quad (7)$$

The transmission of luminous flux through the fenestration system can then be described by the following equation:

$$\Phi_T = \mathbf{T} \cdot \Phi_I$$

$$\begin{bmatrix} \phi_{T1} \\ \vdots \\ \phi_{Tn} \end{bmatrix} = \begin{bmatrix} t_{11} & \dots & t_{1p} \\ \vdots & \ddots & \vdots \\ t_{n1} & \dots & t_{np} \end{bmatrix} \cdot \begin{bmatrix} \phi_{I1} \\ \vdots \\ \phi_{Ip} \end{bmatrix} \quad (8)$$

where ϕ_{Ii} is the flux incident from the i th component of the exterior distribution into the system and ϕ_{Tj} is the flux transmitted by the system into the j th component of the interior. The coefficients t_{ij} represent the fraction of flux that comes from the i th component of the exterior that is transmitted into the j th component of the interior. The matrix \mathbf{T} is a representation of the bi-directional transmittance distribution function (BTDF) [IESNA, 1999] of the fenestration system. Each column of \mathbf{T} corresponds to a solid angle of the incident (i.e. exterior) distribution and each row to a solid angle of the transmitted (i.e., interior) distribution.

The coefficients of \mathbf{T} must obey conservation of energy, which results in the following constraints:

$$\begin{aligned} 0 \leq t_{ij} \leq 1 \\ 0 \leq t_{1k} + \dots + t_{nk} \leq 1, \end{aligned} \quad (9)$$

for any column k .

The objective of the second step of the method is to invert equation 8, i.e., to compute \mathbf{T} based on Φ_T and Φ_I . Equation 8, however, has an infinite number of solutions, i.e., there is an infinite number of \mathbf{T} s that can produce Φ_T from Φ_I . This changes, however, if we consider multiple values for Φ_I , i.e., when Φ_I is specified for different times and/or sky conditions. Fortunately, this is again consistent with daylight design! It is then possible to have enough equations to be able to solve for the coefficients of \mathbf{T} , again in the least-squares sense:

$$\begin{cases} \Phi_T = \mathbf{T} \cdot \Phi_{I1} \\ \vdots \\ \Phi_T = \mathbf{T} \cdot \Phi_{Iq} \end{cases} \quad (10)$$

EXPERIMENTATION

A simulation-based experiment was conducted in order to test and demonstrate the application of the method. The objective of the experiment was to determine the BTDF of the fenestration system that best met specific illuminance goals in a typical small office space.

The performance goal was a horizontal illuminance of 500 lx over two desktop areas, as shown in figure 2. The outdoor conditions considered were CIE clear sky for 38°N latitude, South window orientation, on winter solstice, equinox and summer solstice, from 8:30 to 17:30 solar time, at one-hour intervals. This brought the total number of different incident distributions (q in equation 10) to 29. The ground was considered to be homogeneous and uniformly diffuse, with a reflectance of 0.2.

Step 1

The window's angular distribution of transmitted flux was decomposed into 26 solid angles, which was assumed to be a level of discretization sufficient to accurately describe fenestration systems. There is no mathematical difference in using other levels of discretization except for computational expense. The number of grid points was 50, therefore obeying the $m > n$ condition necessary for equation 5 to have a solution in the least-squares sense.

The contribution of each one of the distribution components to the illuminance at the grid of points was calculated using the Radiance lighting simulation software [Larson, 1998]. The window was set up to emit radiation only within each one of the 26 solid angles at a time, with a uniform intensity distribution, i.e. at each point of the window, $I(\theta, \phi) = \text{const.}$ for directions within the solid angle in question, otherwise $I(\theta, \phi) = 0$. For each of those 26 flux distributions, the illuminance at each of the 50 grid points was then determined by Radiance¹.

The least-squares solution was found using a non-negative least-squares (NNLS) algorithm [Lawson, 1974]. The NNLS algorithm was selected to prevent negative coefficients that would imply negative flux being emitted by the window. The desired distribution thus determined is shown in figure 3.

Step 2

The outdoor flux distribution arriving at the window was decomposed into 14 solid angles. Given the diffuse uniformity of the ground model,

all solid angles corresponding to radiation reflected off the ground onto the fenestration were lumped into one (figure 4). The flux coming from each sector was calculated using the CIE clear sky distribution. The least-squares solution for the coefficients of the BTDF matrix was found using an algorithm for least-squares with inequality constraints (LSI) [Lawson, 1974], which enabled taking into account the constraints described by the inequalities 9.

The resulting \mathbf{T} matrix is shown in figure 5 and the correspondence between columns and rows of \mathbf{T} and solid angles of incident and transmitted distributions, respectively, is given in figure 4. The non-zero elements are at rows 1, 10, 14, 18, 23, 24, and 26, which correspond to the non-zero flux solid angles shown in figure 3. Elements of these rows smaller than 10^{-5} were neglected, since their contribution to light transmission is insignificant.

The incident solid angles with the highest transmittance are 2, 3, and 6, whose radiation is fully transmitted into solid angle 1. These incident solid angles comprise the regions of the sky that are outside of the sun's trajectory and whose luminance is more constant throughout the day and the year. This is probably so because the indoor illuminance goals, and hence the transmitted flux requirements, are constant.

To check if a fenestration system described by the \mathbf{T} matrix obtained above would attain the specified illuminance targets, the illuminance at the desktop grid points was calculated by:

$$\mathbf{E}_{\text{Tot}} = \mathbf{C} \cdot \mathbf{T} \cdot \Phi_I \quad (11)$$

The average illuminance for each of the two desktops is shown in figure 6. On the desktop closest to the window illuminance oscillates around the 500 lx target between 8:00 and 16:00. Later in the day it decreases, eventually to below 100 lx at sunset. On the other desktop the behavior is similar, but around a lower value, closer to 400 lx. This lower illuminance is probably due to the desk's position, farther from the window, and to the fact that some points on its surface are significantly shadowed by the overhead cabinet.

CONCLUSIONS & FUTURE WORK

The work described in this paper indicates that:

- It is possible to determine an intensity distribution for an area source of light that comes the closest to meeting lighting design goals, under the following assumptions: 1) the lighting design goals are linearly additive, e.g. luminance and illuminance (but not some glare indices) and 2) they are specified for a

¹ Using the `rtrace -i` command.

sufficient number of points. This part of the method is also applicable to electric lighting.

- It is also possible to determine the optical transmission properties of a fenestration system that would come closest to meeting daylighting performance goals, if a sufficient number of different outdoor conditions are considered.

Fortunately the above assumptions are consistent with daylighting design, i.e. in realistic daylighting design problems, lighting performance goals must be met at multiple reference points and times.

It should be possible to use a BTDF determined through this new method to select a fenestration system from a library of fenestration systems that contains BTDF data. Such libraries are already being developed [IEA, 2000; Breitenbach, 2001; Smith, 2001; Andersen, 2001, 2003].

Another potential, and perhaps more immediate, use of the method presented in this paper is in the development of fenestration systems for daylighting. The ability to calculate the desired properties of fenestration systems for a variety of situations may serve as guidance in both the development of new fenestration systems as well as in the improvement of existing ones.

Issues to be addressed in future work include: 1) the development of a method for searching efficiently within a library of BTDF data of fenestration systems; 2) the use of other performance requirements, such as glare indices or view through the fenestration; 3) the applicability of this method to dynamic fenestration systems and 4) the feasibility of using this method in realistic design cases.

ACKNOWLEDGEMENTS

This work was supported by Ph. D. Fellowships of the Ministry of Science and Technology, Lisbon, Portugal, and of the Calouste Gulbenkian Foundation, Lisbon, Portugal, and also by the Assistant Secretary for Energy Efficiency and

Renewable Energy, Office of Building Technology, State and Community Programs, Office of Building Research and Standards of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

The authors would like to acknowledge Judy Lai for assistance with Radiance modeling.

REFERENCES

Andersen, M., Michel, L., Roecker, C., Scartezzini, J.-L., 2001, *Experimental assessment of bi-directional transmission distribution functions using digital imaging techniques*, Energy and Buildings, 33(5), pp. 417-431.

Andersen, M., Rubin, M., Scartezzini, J.-L., 2003, *Comparison between ray-tracing simulations and bi-directional transmission measurements on prismatic glazing*, Solar Energy, In Press.

Breitenbach, J., Lart, S., Längle, I., Rosenfeld, J. L. J., 2001, *Optical and thermal performance of glazing with integral Venetian blinds*, Energy and Buildings, vol. 33, pp. 433-442.

Illuminating Engineering Society of North America (IESNA), 1999, *Lighting handbook: reference and application*, 9th ed., IESNA, New York.

Larson, G. W., Shakespeare, R., 1998, *Rendering with Radiance: the art and science of lighting visualization*, Morgan Kaufman, San Francisco.

Lawson, C. L., Hanson, R. J., *Solving least squares problems*, Prentice-Hall, 1974.

Papamichael, K., 1990, *Fenestration systems as luminaires of varying candle power distribution*, Proceedings of the 25th IEEE Annual Meeting, Seattle, WA.

Smith, G. B., Green, D. C., McCredie, G., Hossain, M., Swift, P. D., Luther, M. B., 2001, *Optical characterisation of materials and systems for daylighting*, Renewable Energy, 22(2001) pp. 85-90.

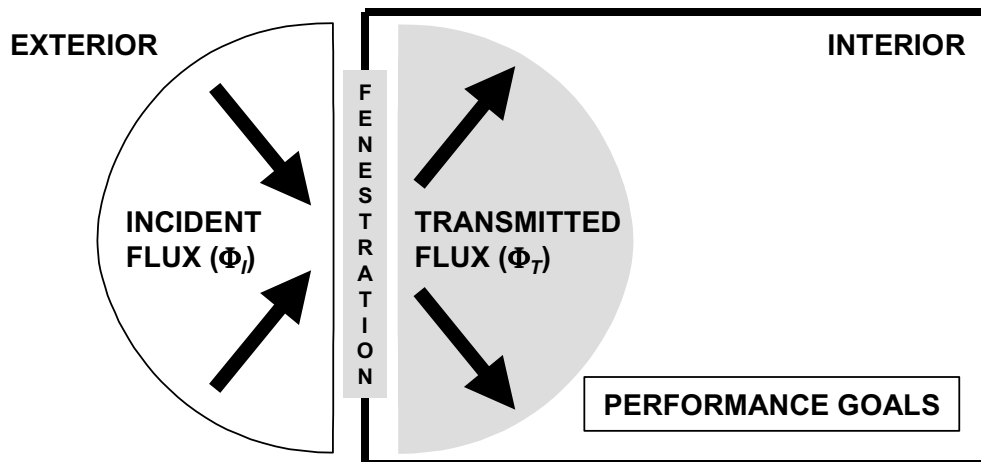


Figure 1. Schematic representation of the method. Shaded boxes represent the quantities to be determined; clear boxes represent known quantities.

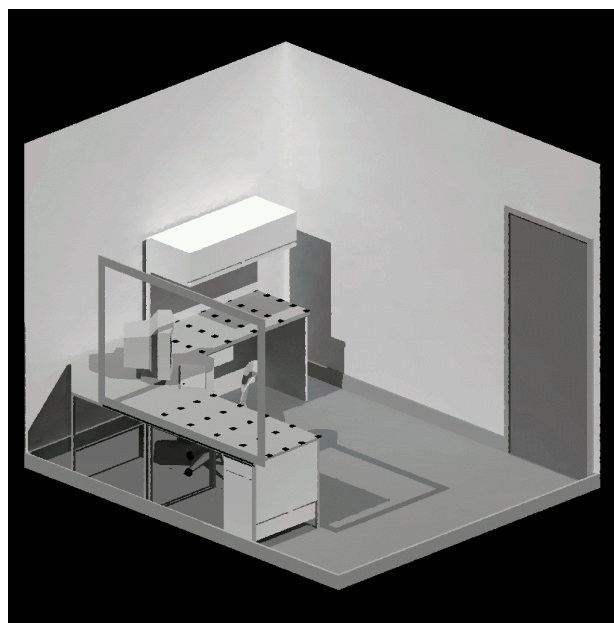


Figure 2. Office configuration. Dimensions are (L x W x H) 3.7 x 3 x 2.9 m. The 50 points where the illuminance goal was specified are represented by small black squares on the desks. The position of the window is represented by the window frame.

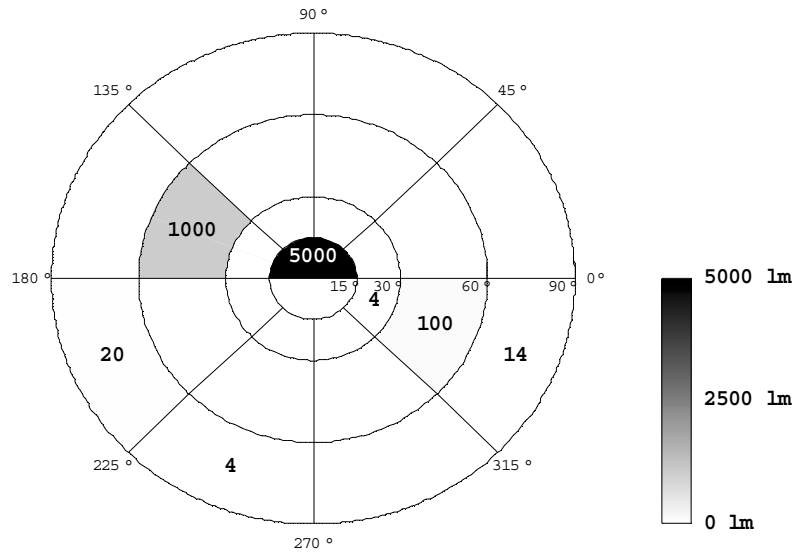


Figure 3. Desired transmitted flux distribution.

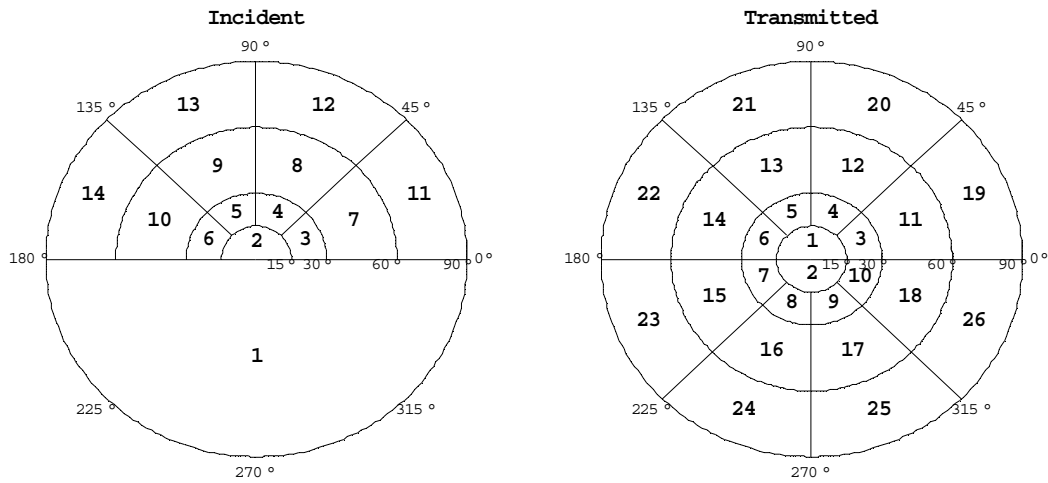


Figure 4. Numbering of the solid angles for the incident and transmitted flux distributions. These correspond, respectively, to the columns and rows of the transmission matrix.

0.12	1.	1.	0.0014	0.0014	1.	0.025	0.00079	0.0008	0.024	0.045	0.00044	0.00049	0.03
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.00011	0	0	0.000014	0.000014	0	0.000035	0	0	0.000035	0.00005	0	0	0.000035
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.026	0	0	0.0032	0.0032	0	0.0081	0.0023	0.0023	0.008	0.012	0.0017	0.0017	0.0081
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.0026	0	0	0.00032	0.00032	0	0.00081	0.00023	0.00023	0.0008	0.0012	0.00017	0.00017	0.00081
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.00051	0	0	0.000064	0.000064	0	0.00016	0.000044	0.000044	0.00016	0.00023	0.000033	0.000033	0.00016
0.00012	0	0	0.000014	0.000014	0	0.000036	0.00001	0.00001	0.000036	0.000051	0	0	0.000036
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.00037	0	0	0.000046	0.000046	0	0.00012	0.000032	0.000032	0.00011	0.00016	0.000024	0.000024	0.00012

Figure 5. Transmittance matrix obtained in Step 2. Columns correspond to incident flux and rows to transmitted flux.

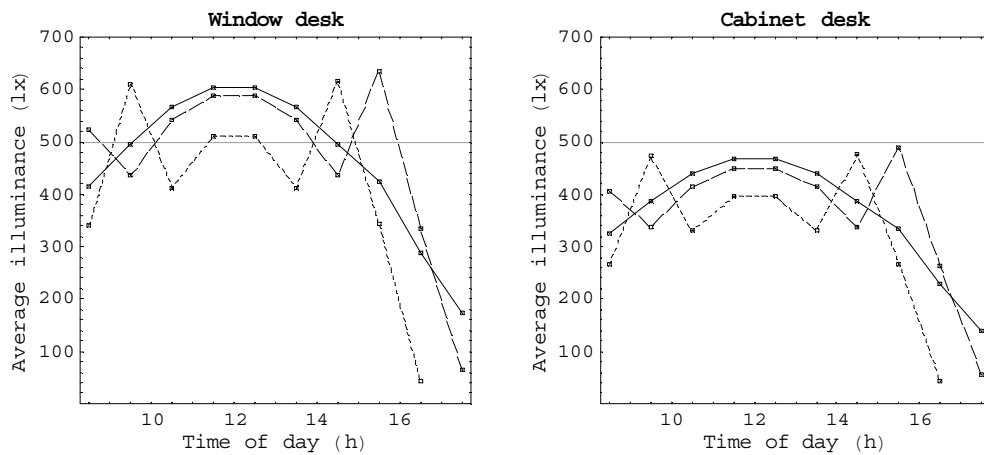


Figure 6. Average illuminance on desks for winter solstice (short dash), equinox (long dash), and summer solstice (solid).

