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ENERGY EFFICIENCY IN BUILDINGS: A PARAMETER FOR THE THERMAL QUALIFICATION OF OPAQUE BUILDING ENVELOPE

Mario Ciampi, Francesco Leccese and Giuseppe Tuoni
Department of Energetica *"Lorenzo Poggi"*
University of Pisa – Faculty of Engineering
56127 Pisa (Italy) – Via Diotisalvi, 2

ABSTRACT

A new parameter ε (0≤ε≤1), called coefficient of performance, is introduced, whose value quantifies the suitability of the distribution of resistive and capacitive layers in a multi-layer wall, within problems concerning thermal building-plant interaction in case of external sinusoidal or impulsive thermal variations.

After giving an integral expression for the calculation of ε, the coefficient's of performance properties are discussed; simple relations allowing to evaluate the value of ε in case of walls made of two materials only, i.e. of a capacitive layer followed by a resistive one (two-component walls), are also deduced. The developed analysis shows that the optimal ideal wall, to which the maximum value of ε corresponds (ε=1), is the lumped-parameter, three-layer, symmetrical one, obtained by lumping all capacity in the mid-plane of the wall and by distributing the resistance, split up into two equal slabs, on the faces. Finally, the coefficient's of performance values for different two-component walls, realized with materials commonly used in building, have been calculated.

INTRODUCTION

An accurate design of building's envelope is necessary, in order to assure a good level of thermal comfort and to reduce energy consumption (Directive 2002/91/CE). The design of such envelope should optimize the building passive behaviour and, at the same time, the air-conditioning plant interventions (Kossecka & Kosny, 2002; Bojic' & Loveday, 1997; Ciampi *et al.*, 2003; Ciampi *et al.*, 2002; Ciampi *et al.*, 2001). Multi-layer walls are usually used in the realization of opaque building envelopes, composed of a sequence of layers made of different materials.

Under steady conditions the thermal behaviour of a multi-layer wall, at least from an "external" point of view (heat flux, and faces' temperature), does not depend on the sequence of layers; on the contrary, the temperature distribution in the wall and then, for example, its hygrometric interstitial behaviour depend on the sequence of layers. But under unsteady conditions the sequence of layers turns out to be essential to determine the performance of a wall, in order to assure a good level of indoor thermal comfort. Generally, under unsteady conditions, the most convenient distribution of essentially resistive (I) and essentially capacitive (L) layers is considered the one with the layer I located on the outside (*outer thermal insulation*) and the worst distribution to be the one with the layer I located on the inner face of the wall (*inner thermal insulation*). In practice, the layer I is often located between two L-type slabs (*middle thermal insulation*), see Fig. 1.

An important problem is to investigate the influence of the structure of the outer walls on the type of intervention to be performed by an air-conditioning plant in order to keep the indoor air temperature constant against outdoor thermal variations. In particular, in order to facilitate the plant intervention, it might be convenient to minimize the corrective oscillations' amplitude of the power supplied by the plant in case of sinusoidal external temperature variations and to maximize the air-conditioning plant intervention length in case of impulsive external temperature excitations.

In this paper we point out that, once total heat capacity and thermal resistance of a non homogeneous wall have been assigned, the distribution of resistance-capacity within the wall itself has a considerable influence on the wall response to an external thermal variation of a given temporal trend. If we restrict our investigation to cases of oscillating (with low-frequency) or impulsive external thermal variations, it is just enough to know an integral formula of the resistance-capacity distribution for the determination of the wall response; this paper concerns the investigation of such quantity, defined as "coefficient of performance".

In the building being here assumed to be kept at constant temperature the value of ε cannot, obviously, depend on the heat capacities of the inner structures; such heat capacities have, on the contrary, a remarkable and complex influence on thermal transients relating, for example, switching-on and turning-off phases of the plant or rather when the purely passive behaviour of a building without air-conditioning plant is investigated.

PROBLEM STATEMENT

A room with an outer wall is being considered; the outdoor temperature t_e (eventually sol-air temperature) is supposed to undergo a variation of such temporal trend compared to its mean value. The room and its surroundings are assumed to be kept at the same constant temperature t_i by an air-conditioning plant. Later on the subscript i will indicate a quantity relating to the indoors, while the subscript e will indicate a quantity relating to the outdoors.

where $\vec{q}$ is the heat flux (Wm^{-2}) and τ is the time variable. In one-dimensional approximation, carrying out the Laplace transform with complex variable s, from (1) the following is obtained:

$$k\frac{dT}{dx} = \phi , \qquad \frac{d\phi}{dx} = s\mu C_p T \qquad (2)$$

In the equations (2) the notation T=£(t) and ϕ=−£(q_x) has been used, where by £ the Laplace transform operator has been indicated; later on, the quantities T and ϕ will be respectively called, for simplicity, "temperature" and "thermal flux".

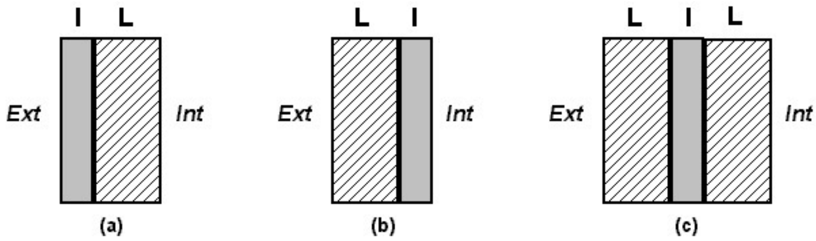

Figure 1 – Schematic representation of multi-layer walls: (a) outer, (b) inner, (c) middle thermal insulation.

Thermal resistance and heat capacity within the wall are supposed to vary continuously as function of the position. According to the distribution theory (Vladimirov, 1981), this approach is still good even in case of walls composed of layers of different materials and with inner and outer surface thermal resistances. Thermal resistance and heat capacity are supposed to be independent of the temperature, so as to consider the problem as linear.

The formalism of the Laplace temporal transform is used. Such formalism results to be particularly suitable in the case of a sudden variation of the outdoor temperature (due, for example, to a rapid variation of the solar radiation intensity owing to the varying cloudiness of the sky), but also allows calculating directly the Fourier components of the indoor temperature under a periodic variation of t_e (due, for example, to day-and-night alternating).

We consider a plane wall with thickness L; should the 0x-axis be directed from the outer face to the inner one, as shown in Fig. 2. Let c be the wall heat capacity and r the wall thermal resistance, comprehensive of the inner (r_{int}) and outer (r_{est}) surface thermal resistances; the product τ_0=rc is called "time constant" of the wall. Thermal resistance and heat capacity are expressed per surface unit: $[r]=m^2KW^{-1}$, $[c]=Jm^{-2}K^{-1}$. Let k be the thermal conductivity ($Wm^{-1}\,K^{-1}$) of the wall material, μ the density ($kg\ m^{-3}$) and C_p specific heat ($Jkg^{-1}K^{-1}$); k, μ and C_p are functions of x.

The governing equations are:

$$\vec{q} = -k \cdot \mathrm{grad}\,(t), \qquad \mu C_p \frac{\partial t}{\partial \tau} + \mathrm{div}(\vec{q}) = 0 \qquad (1)$$

Obviously, the wall thermal behaviour is determined, apart from the total resistance (r) and capacity (c), also by the resistance-capacity distribution within the wall; such distribution can be stated precisely by introducing, for 0<x<L, the following dimensionless quantities:

$$\rho(x) = \frac{1}{r}\left[r_{est} + \int_0^x \frac{1}{k(x')}dx' \right],$$

$$\gamma(x) = \frac{1}{c}\int_0^x \mu(x')C_p(x')dx' \qquad (3)$$

These quantities represent, respectively, the resistance and capacity fractions seen between the outside and the x-section. The Eqs. (3) define, in parametric form, the function ρ=ρ(γ) characterizing the resistance-capacity distribution. The definition of such function can be completed by assuming ρ(0)=0 and ρ(1)=1, so that it is still 0≤ρ≤1, 0≤γ≤1; notice that, in presence of inner and outer surface thermal resistances, the function ρ=ρ(γ) shows some discontinuities at the extremes of its field of definition. The derivative ρ′=ρ′(γ)=(dρ/dγ)= $c(rk\mu C_p)^{-1}$ showing, for γ=0 and for γ=1, several singularities that can be expressed, respectively, by $r_{est}\delta(\gamma)$ and $r_{int}\delta(\gamma\text{-}1)$ (where δ is the Dirac delta function), turns out to be of great importance.

It is convenient to visualize the trend of the function ρ=ρ(γ) in a graph (ρ, γ), it proves to be very useful for comparing a discrete (lumped-parameter) structure with a continuous (distributed-parameter) or rather two different discretizations of the same continuous structure.

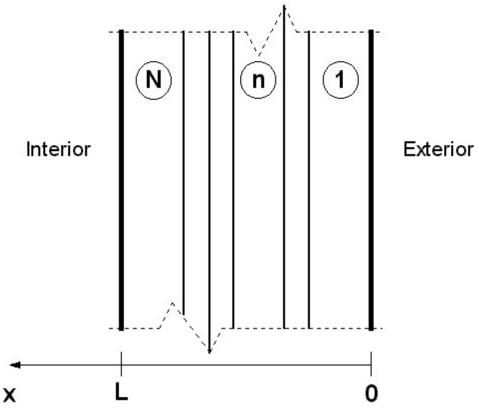

Figure 2. – N-layer wall and system of coordinates

Using the definition of γ to perform a change of variable, the Eqs. (2) can be rewritten in the following form:

$$\frac{dT}{d\gamma} = r\rho'\phi, \qquad \frac{d\phi}{d\gamma} = scT \tag{4}$$

or rather, in matrix notation:

$$\frac{d}{d\gamma}\begin{pmatrix} T \\ \phi \end{pmatrix} = m(\gamma)\cdot\begin{pmatrix} T \\ \phi \end{pmatrix}$$

with $$m(\gamma) = \begin{pmatrix} 0 & r\rho' \\ cs & 0 \end{pmatrix} \tag{5}$$

where $m(\gamma)$ is the matrix peculiar to the infinitesimal transformation from γ to γ+dγ. The solution of the linear system (5) can be written introducing the matrix $M(\gamma)$ operating the finite transformation from γ=0 to the generic value γ

$$\begin{pmatrix} T \\ \phi \end{pmatrix} = M(\gamma)\cdot\begin{pmatrix} T_e \\ \phi_e \end{pmatrix} \tag{6}$$

where T_e and ϕ_e are temperature and flux in γ=0, i.e. temperature and flux relating to the outdoors. A formal solution of the Eq. (5), obtainable, for example, for further approximations, leads to render the matrix M explicit as an infinite sum:

$$M(\gamma) = \sum_{r=0}^{\infty} M^{(r)}(\gamma) \tag{7}$$

where $M^{(0)}$ is the unit matrix and $M^{(r)}$ can be expressed by an r-dimensional integral containing the matrix m:

$$M^{(0)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$M^{(r)} = \int_0^{\gamma} d\gamma_1 \ldots \int_0^{\gamma_{r-1}} d\gamma_r \left[m(\gamma_1)\ldots m(\gamma_r)\right] \tag{8}$$

Notice that, owing to the structure of the matrix m, the matrix elements of $M(\gamma)$ result to be expressed in function of powers of s; in other words, the Eq. (7), represents, once rearranged, the power series of M in the complex variable s.

THE TRANSMISSION MATRIX

From the Eq. (6) for γ=1 we obtain:

$$\begin{pmatrix} T_i \\ \phi_i \end{pmatrix} = M^*\cdot\begin{pmatrix} T_e \\ \phi_e \end{pmatrix} \tag{9}$$

where $M^*=M(1)$ and T_i and ϕ_i are, respectively, temperature and flux in γ=1, i.e. temperature and flux relating to the indoors. The matrix M^* is called transmission matrix of the wall; later on its matrix elements will be indicated by the letters E, F, G, H. In the case of a multi-layer structure, the matrix M^* results to be the ordered product of the transmission matrices of the single layers. Referring to Fig. 1, we can, therefore, write:

$$M^* \equiv \begin{pmatrix} E & F \\ G & H \end{pmatrix} = \begin{pmatrix} E_N & F_N \\ G_N & H_N \end{pmatrix} \times\cdots\times \begin{pmatrix} E_n & F_n \\ G_n & H_n \end{pmatrix} \times\cdots\cdots\times \begin{pmatrix} E_l & F_l \\ G_l & H_l \end{pmatrix} \tag{10}$$

As the matrix of the infinitesimal transformation m, defined in Eq. (5), has spur equal to zero, the transmission matrix has unitary determinant:

$$\det(M^*) = EH - FG = 1 \tag{11}$$

Within problems concerning building-plant interaction the matrix element F turns out to be of the greatest importance. In fact, from the Eq. (9) we have:

$T_i = ET_e + F\phi_e, \qquad \phi_i = GT_e + H\phi_e$

hence we obtain, considering that $T_i=0$ (indoor environment is kept at constant temperature) and using the Eq. (11), the following equation:

$$\phi_i = -\frac{T_e}{F} \tag{12}$$

The heat flux $Q(\tau)$ the plant should provide in order to keep the indoor temperature constant against any outdoor temperature temporal variation is obtained by taking the Laplace antitransform of the Eq. (12): $Q(\tau)=\pounds^{-1}(\phi_i)$.

The series expansion for F is very interesting for all those situations in which the long-term temporal trend $Q(\tau)$ is significant, since just the first terms can be considered. Using the notation $f(\gamma_k)\equiv f_k$, it is possible to write the products of matrix in Eq.(8) as

$$m(\gamma_1) = \begin{pmatrix} 0 & r\rho'_1 \\ cs & 0 \end{pmatrix}$$

$$m(\gamma_1)\cdot m(\gamma_2) = \begin{pmatrix} rc\rho'_1 s & 0 \\ 0 & rcs\rho'_2 \end{pmatrix}$$

$$m(\gamma_1)\cdot m(\gamma_2)\cdot m(\gamma_3) = \begin{pmatrix} 0 & r^2c\rho'_1 s\rho'_3 \\ rc^2s\rho'_2 s & 0 \end{pmatrix}$$

$$m(\gamma_1)\cdot m(\gamma_2)\cdot m(\gamma_3)\cdot m(\gamma_4) = \begin{pmatrix} r^2c^2\rho'_1 s\rho'_3 s & 0 \\ 0 & r^2c^2s\rho'_2 s\rho'_4 \end{pmatrix}$$

The element of matrix F (first line, second column) turns out to be, therefore, composed of a sum of terms of the following type:

$$F = r\int_0^1 d\gamma_1\rho'_1 + r^2cs\int_0^1 d\gamma_1\int_0^{\gamma_1} d\gamma_2\int_0^{\gamma_2} d\gamma_3\rho'_1\rho'_3 +$$
$$+ r^3c^2s^2\int_0^1 d\gamma_1\int_0^{\gamma_1} d\gamma_2\int_0^{\gamma_2} d\gamma_3\int_0^{\gamma_3} d\gamma_4\int_0^{\gamma_4} d\gamma_5\rho'_1\rho'_3\rho'_5+... \quad (12a)$$

For a generic function $\Phi(\gamma)$ the following identity subsists:

$$\int_0^{\gamma_k}\rho'_{k+1}\,d\gamma_{k+1}\int_0^{\gamma_{k+1}}\Phi_{k+2}d\gamma_{k+2} = \int_0^{\gamma_k}(\rho_k - \rho_{k+1})\Phi_{k+1}d\gamma_{k+1} \quad (12b)$$

with $\gamma_0 \equiv \gamma$, $\rho_0 \equiv \rho$. Using the Eq. (12b) the Eq. (12a) can be simplified into the form of a power series:

$$F = r[1 + F_a\tau_0 s + F_b(\tau_0 s)^2 + \ldots] \quad (13)$$

with real coefficient:

$$F_a = \int_0^1 d\gamma_1(1-\rho_1)\rho_1 \,,$$

$$F_b = \int_0^1 d\gamma_1\int_0^{\gamma_1} d\gamma_2(1-\rho_1)(\rho_1-\rho_2)\rho_2 \quad (14)$$

THE COEFFICIENT OF PERFORMANCE

In two cases the expansion of F proves to be of great interest: the case in which the outdoor air temperature undergoes a short-period variation and the case in which it is subjected to long-period periodic variations.
The first case occurs if the perturbation length is short compared to the wall time constant τ_0; in these conditions the effective temporal trend of the variation of T_e is not significant and only its intensity I_e [sK] is considered to be significant. So it can be written:

$$T_e(\tau) = I_e\delta(\tau) \quad (15)$$

where $\delta(\tau)$ is the Dirac delta function. Such schematization could result suitable for the radiative component of the sol-air temperature; I_e can assume both positive and negative values with regard to increases or decreases in solar radiation caused, for example, by variations of the cloudiness of the sky (Ciampi & Tuoni 1984, Suehrcke & McCormick 1988). Under an impulsive variation of the type (15) we obviously have: £(T_e)=I_e.
The plant intervention Q(t) is characterized by an energy Q_0, by a mean time lag <t> and by an average length Δt, defined by the following relations:

$$<\tau> = \frac{\int_0^\infty \tau Q(\tau)d\tau}{\int_0^\infty Q(\tau)d\tau} \,,$$

$$\Delta\tau^2 = \frac{\int_0^\infty (\tau - <\tau>)^2 Q(\tau)d\tau}{\int_0^\infty Q(\tau)d\tau} \quad (16)$$

Such quantities can be expressed in the following form (see Appendix):

$$Q_0 = -\frac{I_e}{r} \,,\ <\tau> = \tau_0 F_a \,,\ \Delta\tau^2 = \tau_0^2[F_a^2 - 2F_b] \quad (17)$$

In other words, the energy Q_0 required from the plant, depends on the intensity of the impulse I_e and the wall resistance r, while time lag <τ> and length $\Delta\tau$ depend on the first terms of the Eq. (13). Particularly interesting is the average length of the plant intervention: the longer $\Delta\tau$ is, Q_0 being equal, the easier the plant intervention is.
In case of sinusoidal periodic oscillations with angular frequency ω, the amplitude |Q| of the plant response is given, for Eq. (12), by: $|Q|=|T_e|/|F(s)|$ with $s=j\omega$ and $j=(-1)^{1/2}$. From Eq. (13), with $s=j\omega$ and disregarding the terms of the fourth degree in ω, we have:

$$|F|^2 = r^2 + r^2(F_a^2 - 2F_b)\tau_0^2\omega^2 \quad (18)$$

Introducing the dimensionless parameter ε, defined by (Ciampi *et al.* 2001):

$$\varepsilon = 4\sqrt{F_a^2 - 2F_b} \quad (19)$$

$\Delta\tau$ and |Q| can be written in the following form:

$$\Delta\tau = \varepsilon\tau_0/4, \qquad |Q|^2 = \frac{|T_e|^2}{r^2}\cdot\frac{1}{1+(\varepsilon\omega\tau_0/4)^2} \quad (20)$$

Hence it follows that, τ_0 being equal, the higher ε is the longer the length $\Delta\tau$ of the plant response to an impulsive external excitation is, facilitating, thus, the plant intervention. In the case of sinusoidal external thermal variation, the lower the frequency is the bigger the amplitude |Q| of the plant corrective response is; in other words, at the low frequencies ($\omega\tau_0 \ll 1$) the perturbation is transferred, with little dampening, through the wall and, therefore, requires a heavier intervention from the plant. In any case, from the second of the Eqs. (20) it follows that, $\omega\tau_0$ and $|T_e|/r$ being equal, the bigger ε is the smaller |Q| is. Moreover, a more precise analysis shows that, an increase of ε generally involves a decrease of |Q| even when the condition $\omega\tau_0 \ll 1$ is not fulfilled.
From Eqs. (19) and (14), we can write:

$$\left(\frac{\varepsilon}{4}\right)^2 = \int_0^1(1-\rho_1)\rho_1 d\gamma_1\int_0^1(1-\rho_2)\rho_2 d\gamma_2 +$$
$$-\int_0^1(1-\rho_1)d\gamma_1\int_0^{\gamma_1}(1-\rho_2)\rho_2 d\gamma_2 \quad (21)$$

containing two bidimensional integrals defined on plane (γ_1, γ_2), the first extended to the unitary square whose vertices are (0,0) (1,0) (1,1) (0,1), and the second extended to the triangle Θ whose vertices are (0,0) (1,0) (1,1). Observe that, in the first integral, the integrand remains unvaried owing to the exchange between γ_1 and γ_2. Such integral can be, therefore, reduced to the same integration domain as the second integral if only it could be multiplied by the factor 2. The second member of the previous relation can be, therefore, grouped into a single integral, whose integrand is the following:

$$2(1-\rho_1)\rho_1(1-\rho_2)\rho_2 - 2(1-\rho_1)(1-\rho_2)\rho_2 = 2(1-\rho_1)^2\rho_2^2$$

so that the Eq. (21) can be rewritten in the form:

$$\varepsilon^2 = 32\int_\Theta [1-\rho(\gamma_1)]^2[\rho(\gamma_2)]^2 d\gamma_1 d\gamma_2 =$$

$$= 32\int_0^1 [1-\rho(\gamma_1)]^2 d\gamma_1 \int_0^{\gamma_1} [\rho(\gamma_2)]^2 d\gamma_2 \qquad (22)$$

In other words, we have to proceed in the following way for the calculation of ε^2: once two positions within the wall have been chosen and ordered so as to have $\gamma_2<\gamma_1$, the square of the thermal resistance fraction preceding γ_2 and the square of the resistance fraction following γ_1 should be taken and multiplied between themselves; then, we should integrate on all possible ordered couples (γ_2, γ_1) and, finally, multiply by 32.

A lumped-parameter, multi-layer wall can be considered to consist of n purely capacitive layers, with heat capacity c_s and of n+1 purely resistive layers, with thermal resistance r_s; the various layers are placed in sequence, alternating a resistive layer with a capacitive one. Obviously, the extreme resistive layers have to be comprehensive of the inner and outer surface thermal resistances. In this case the graph of the function $\rho=\rho(\gamma)$ comes to a "stepped" trend with "riser" equal to the resistance fraction r_s/r and with "tread" equal to the capacity fraction c_s/c; in other words, the function $\rho(\gamma)$ can assume the values:

$$\tilde{\rho}_k = \frac{1}{r}\sum_{s=0}^{k-1} r_s$$

Therefore, the integral in Eq. (22) can be easily calculated and we obtain

$$\left(\frac{c\varepsilon}{4}\right)^2 = \sum_{i=1}^{n}\left[c_i\tilde{\rho}_i(1-\tilde{\rho}_i)\right]^2 + 2\sum_{i<k}^{n} c_i c_k \tilde{\rho}_i^{\,2}(1-\tilde{\rho}_k)^2 \qquad (22a)$$

allowing to evaluate the coefficient of performance for any multi-layer, lumped-parameter wall.

The previous considerations can be extended to the case of a multi-layer wall consisting of n homogeneous layers with thermal resistance R_k and heat capacity C_k, representable by the scheme:

$$[\text{Int}]\cdot\begin{pmatrix} r_{int} \\ 0 \end{pmatrix}\cdot\begin{pmatrix} R_n \\ C_n \end{pmatrix}\cdot\cdots\cdot\begin{pmatrix} R_2 \\ C_2 \end{pmatrix}\cdot\begin{pmatrix} R_1 \\ C_1 \end{pmatrix}\cdot\begin{pmatrix} r_{ext} \\ 0 \end{pmatrix}\cdot[\text{Ext}] \qquad (23)$$

In this case the parameter ε can be expressed (Ciampi *et al.* 2003) by the following relation:

$$\varepsilon^2 = \frac{16}{r^4c^2}\sum_{i<k}^{n}\frac{C_iC_k}{6}\left[(12S_i^2+R_i^2)[V_k^2+\frac{R_k^2}{12}]\right] + \\ + \frac{16}{r^4c^2}\sum_{i=1}^{n}C_i^2\left[S_i^2V_i^2 - \frac{R_i}{3}R_i(r+\frac{R_i}{2})S_iV_i + \frac{R_i^2}{12}\left(\frac{R_i^2}{12}+r^2-\frac{rR_i}{5}\right)\right] \qquad (24)$$

where the resistance "preceding" the k-nth layer has been indicated by:

$$S_k = r_{ext} + R_1 + ... + R_{k-1} + \frac{1}{2}R_k \qquad (25)$$

and with V_k=r-S_k.

The Eqs. (22), (22a) and (23) point out the following important coefficient's of performance properties.

The coefficient of performance is an always positive dimensionless quantity.

The coefficient of performance does not depend on the total resistance and capacity values of a wall, but only on their distribution within the wall; so, for example, in absence of inner and outer surface thermal resistances, all the homogeneous walls have the same value of the coefficient of performance.

For any homogeneous wall with negligible inner and outer surface thermal resistances ($\rho=\gamma$) results to be: $\varepsilon = 2\sqrt{10}/15 \cong 0.42$.

The more resistances are located near the wall's outer faces the more they are "weighed"; from the symmetry of the integral defining ε, it follows that it is convenient to distribute the resistances uniformly on the two sides. This leads to conclude that the optimal ideal wall is the symmetric, three-layered, lumped-parameter one, obtained by lumping all capacity in the middle (of the structure) and by distributing the available thermal resistance (comprehensive of the inner and outer surface thermal ones), split up into two equal slabs, on the outer faces (theoretically optimal, sandwich-type wall). According to the Eq. (22a) the coefficient's of performance value of such wall results to be ε=1; it can be, then, concluded that: $0\le\varepsilon\le1$.

Ideal resistance-capacity distributions, showing all capacity lumped on one of the two outer faces or rather on both of them, result in a null value of ε; this explains the reason why, both real two-layered walls with outer thermal insulation (see Fig. 1a) or with inner thermal insulation (see Fig. 1b) and symmetric, three-layered ones with the insulating material located in the middle of the structure (see Fig. 1c), are characterized by low values of the coefficient of performance; only the presence of the inner and outer surface thermal resistances as well as of a resistance portion, distributed among prevalently capacitive layers cause ε to be other than zero.

TWO-COMPONENT WALLS

With the exception of possible finish layers, the materials used in the realization of multi-layer walls can be referred to two categories: materials with essentially resistive thermal properties (I), light and insulating, and materials with essentially capacitive thermal properties (L), heavy and mechanically resistant (two-component wall).

Three-layered wall, with the mid-layer made of L material, turn out to be particularly interesting. Later on, by r_I and c_I resistance and capacity of the material I will be indicated, while by r_L and c_L those relating to the material L will be indicated. We could think of using a portion x of the insulating material for the outer layer and the remaining portion (1-x) for the inner layer, realizing, thus, the wall of the type (23) with n=3:

$$[\mathrm{Int}]\cdot\begin{pmatrix} r_{int} \\ 0 \end{pmatrix}\cdot\begin{pmatrix} (1-x)r_I \\ (1-x)c_I \end{pmatrix}\cdot\begin{pmatrix} r_L \\ c_L \end{pmatrix}\cdot\begin{pmatrix} xr_I \\ xc_I \end{pmatrix}\cdot\begin{pmatrix} r_{ext} \\ 0 \end{pmatrix}\cdot[\mathrm{Ext}] \qquad (26)$$

For x=0 and x=1 we have a two-layered wall: for x=0 all the insulating material is located on the inside (Fig. 1c), for x=1 all the insulating material is located on the outside (Fig. 1a). For x=1/2 we have a symmetric wall presumably provided with a high value of ε, since not very different from the optimal, sandwich-type, lumped-parameter one. The analysis of the behaviour of walls of the type (26) turns out to be very simplified if the capacity of the insulating-material layer is considered to be negligible, so as to treat material I as purely resistive (c_I=0). This condition is widely satisfied by all cellular plastics (polyethylene, polystyrene, polyurethanes, etc.) and by mineral fibres, while it is not satisfied by some materials which, even though they show low values of thermal conductivity, are characterized by non-negligible values of density (lumbers, fibre panels or wood wool panels, fibreglass panels, plywood panels, and so on).

Enumerating the layers beginning from the outside, the first layer could be thought to be a pure resistance (C_1=0), consisting of the outer surface thermal resistance and the portion x of the insulating material ($R_1=r_{est}+x\cdot r_I$), the second to exhaust the available material L ($C_2=c_L=c$, $R_2=r_L$), finally, the third could be thought to be a pure resistance (C_3=0) too, consisting of the inner surface thermal resistance and the remaining portion (1-x) of insulating material ($R_3=r_{int}+(1-x)\cdot r_I$); obviously, $r=r_{est}+r_I+r_L+r_{int}$ and $S_2=r_{est}+x\cdot r_I+r_L/2$. Applying the Eq. (24) to such structure it is easy to verify that the second sum vanishes, while only the term relating to i=2 remains of the first one. We, therefore, obtain:

$$\varepsilon^2 = \frac{16}{r^4}\left[S_2^2(r-S_2)^2 - \frac{r_L}{3}(r+\frac{r_L}{2})S_2(r-S_2) + \frac{r_L^2}{12}\left(\frac{r_L^2}{12}+r^2-\frac{rr_L}{5}\right)\right] \qquad (27)$$

which shows the variation of ε with x. The Eq. (27) shows its maximum for S_2=r/2 and, then, for $x=x_0=(r_I-r_{est}+r_{int})/(2r_I)$, i.e. when the portion x is chosen so as to subdivide into two equal parts the resistance seen "from the centre plane" of the layer L (sandwich-type wall configuration). In these conditions the maximum value ε_0 of the coefficient of performance results to be:

$$\varepsilon_0 = \sqrt{1-\frac{4p}{3}+\frac{2p^2}{3}-\frac{4p^3}{15}+\frac{p^4}{9}} \cong 1-\frac{2}{3}p \qquad (28)$$

with $p=r_L/r$. In the Eq. (28) a series expansion of ε_0, truncated at the first order in p (p is generally very small and the error so introduced comes to about 0.1% for p<0.1), has been also reported. As r_L decreases the value of ε_0 increases and for r_L=0 the optimal situation, indicated in the previous section, occurs and it results to be ε_0=1. Indicating by k_L, μ_L, c_{pL} the thermophysical properties of the material L and introducing the thermal effusivity ξ of the material L, defined by: $\xi=\mu_L k_L c_{pL}$, we can be write:

$$p=\frac{r_L}{r}=\frac{1}{\xi}\cdot\frac{c}{r}$$

It, therefore, follows that, c and r being equal, the bigger the effusivity of the L material is, the higher the optimal value of the coefficient of performance is.

As an example of application we have considered as material I: the polyurethane (P), the fibreglass panels (F) and the wood wool panels (W). While as material L three types of materials have been taken into account: concrete (1), brick (2), pine-wood (3) and a cellular concrete (4). Thermophysical properties of the considered building materials are given in Table I; as far as inner and outer surface thermal resistances are concerned, the following values have been assumed: r_{int}=0.13, r_{ext}=0.04 m^2K/W (EN ISO 6946).

Combining with each insulating (P, F, W) the different materials L (1, 2, 3, 4) it is possibile to obtain three groups of two-component structures; such structures will be indicated, respectively, by the following symbols: Pn, Fn, Wn with n=1, 2, 3, 4.

The structures Pn, Fn, Wn in Tab. II, have been realized with suitable thicknesses of materials I and L so that all structures could show the same thermal resistance r=1.75 m^2KW^{-1} and the same heat capacity c=140 $kJm^{-2}K^{-1}$, (τ_0=245000 s). This explains the non-usual values of the thicknesses used for the different materials.

In Tab. II thicknesses d_I (cm) and d_L (cm) relating, respectively, to material I and material L, are reported along with the portion x_0 (cm) of insulating material that should be positioned on the outside in order to optimize the three-layer, sandwich-type, wall and the corresponding value of ε_0. For comparison, the value ε_M of the coefficient of performance relating to a three-layer, sandwich-type, structurally symmetric (x=0.5) wall is also reported (wall M).

In Tab. II the values ε_A and ε_B of the coefficient of performance are also reported for two-layer walls of the type showed, respectively, in Figg. 1a and 1b; the value ε_C of the coefficient of performance, relating to a wall with the insulating layer located between two equal layer of material L (Fig. 1c), is reported for comparison. Such disposition is widespread in building practice. The values of ε_0, ε_M, ε_A, ε_B and ε_C have been calculated by using the Eq. (24).

Table I. - Properties of the building materials used in the sample calculations

		μ kg m^{-3}	**K** W $m^{-1}K^{-1}$	**c_p** kJ kg^{-1} K^{-1}	**ξ** $WkJm^{-4}K^{-2}$
1	Concrete	1600	0.70	0.88	986
2	Brick	1000	0.47	0.84	395
3	Pine-wood	550	0.15	1.66	137
4	Cellular concrete	600	0.20	0.88	106
P	Polyurethane	35	0.035	1.60	1.96
F	Fibreglass panels	100	0.044	0.80	3.52
W	Wood Wool panels	300	0.085	1.30	33.2

Table II. – Coefficient of performance of various structures

	d_L	d_I	x_0	ε_o	ε_M	ε_A	ε_B	ε_C
P1	9.74	5.0	0.53	0.94	0.93	0.35	0.19	0.17
P2	16.4	4.3	0.54	0.86	0.86	0.46	0.33	0.22
P3	15.2	2.0	0.58	0.64	0.64	0.58	0.54	0.36
P4	26.4	0.9	0.67	0.54	0.54	0.54	0.52	0.41
F1	9.58	6.3	0.53	0.93	0.92	0.35	0.19	0.17
F2	16.1	5.4	0.54	0.85	0.85	0.45	0.33	0.22
F3	15.1	2.5	0.58	0.64	0.64	0.58	0.53	0.36
F4	26.3	1.2	0.67	0.54	0.54	0.54	0.52	0.41
W1	6.44	12.6	0.53	0.75	0.75	0.31	0.22	0.22
W2	11.4	11.4	0.53	0.72	0.72	0.37	0.28	0.25
W3	12.7	6.25	0.56	0.61	0.61	0.53	0.48	0.36
W4	24.2	3.15	0.62	0.54	0.54	0.52	0.51	0.41

The capacitive effects of the used insulating layer (see Tab. I) increase remarkably changing from the polyurethane to the fibreglass panels and to the wood wool panels; such effects are minimal for the polyurethane but result to be significant for the wood wool panels.
In fact, the values of ε_0 calculated by the Eq. (28) turn out to coincide, in practice, with those reported in the table for the structures Pn, while little deviations can occur for the structures Fn. For the structures Wn the values of ε_0, calculated by the Eq. (24), can quite differ from those calculated by the Eq. (28).
From the values shown in Tab. I the following turns out to be evident:
The considered walls, even though characterized by the same values of thermal resistance and heat capacity, show quite different values of ε_0; the best wall is the one made of concrete and polyurethane.
The optimal wall is always well approximated by the wall M, constructively more simple and independent of the value of the wall inner and outer surface thermal resistances.
The wall with the insulating layer placed between two equal layer of material L is always the worst among all the considered ones.
For a given material L, the higher ε_0 is the greater the differences among the various distributions of insulating material are (a bad wall is almost independent from the resistance-capacity distribution).

CONCLUSIONS

In a multi-layer wall, the sequence of layers with the highest value of the coefficient of performance ε, facilitates the air-conditioning plant intervention, in order to keep the indoor air temperature constant; the physical meaning of ε is explained by the following remarks:
under an external impulsive excitation, the time length Δτ of the compensating power plant intervention is proportional to ε,
under a sinusoidal, low-frequency oscillation, the higher ε is, the smaller the oscillating component amplitude of the plant compensating power results to be.
The highest value of ε (ε=1) can be obtained by using a three-layer wall, in which the middle layer is purely capacitive and the side ones are purely resistive, with equal resistance inclusive of the wall inner and outer surface thermal resistances (theoretically optimal, sandwich-type wall). Hence it follows that the optimization of the coefficient of performance turns

out to be a valid criterion for a rational thermal design of the opaque building envelope.

In this paper, after providing an integral formula for ε of general validity, its properties are discussed, examining some simple but significant situations. Moreover, the most convenient calculation methods are indicated for the coefficient of performance evaluation in case of multi-layer walls commonly used in building.

Finally, the values of the coefficient of performance have been calculated for various two-component walls, optimally realized with materials commonly used in building; such values have been compared with those obtainable for two-component walls, realized in a traditional manner, i.e. disposing the insulating layer inside, outside or in the middle of the capacitive-material layer.

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APPENDIX

If $Y(\tau)$ is a generic function of the time τ, its Laplace transform £(Y) is given, by definition, by:

$$£(Y) = \int_0^\infty e^{-s\tau} Y(\tau) d\tau \qquad \text{(A1)}$$

where s can assume any complex value. If the Laplace transform £(Y) of the function $Y(\tau)$ is known, it is possibile to deduce directly the «moments» of the function $Y(\tau)$, i.e. the following quantities:

$$m_j = \int_0^\infty \tau^j Y(\tau) d\tau \qquad \text{with } j=0, 1, 2, \dots$$

In fact from the Eq. (A1), differentiating with respect to s, we obtain:

$m_0 = £(Y)_{s=0}$

$$m_j = (-1)^j \left(\frac{d^j £(Y)}{ds^j} \right)_{s=0} \qquad \text{for } j>0 \qquad \text{(A2)}$$

If the function $Y(\tau)$ is identified with the air-conditioning plant time response Q(t), the zeroth-order (m_0), first-order (m_1) and second-order (m_2), are particularly important. It can easily occur that:

$$Q_0 = m_0, \qquad \langle t \rangle = \frac{m_1}{m_0},$$

$$\Delta t^2 = \frac{m_0 m_2 - m_1^2}{m_0^2} \qquad \text{(A3)}$$

For this reason, in order to calculate the moments of Q(t) up to the second order, it is enough to know, from the Eq. (A2), a series expansion, truncated at the terms in s^2 of the F(s); in other words, the coefficients F_a and F_b of the expansion (13) should be known. In the case being investigated here, we have an impulsive outdoor air temperature excitation of the type (15), $£(Q) = -I_e £(1/F)$, and with simple calculations it can be demonstrated that:

$$m_0 = -I_e/r, \qquad m_1 = -\tau_0 F_a I_e/r$$

$$m_2 = -2 I_e \tau_0^2 (F_a^2 - F_b)/r \qquad \text{(A4)}$$

Substituting the Eqs. (A4) in the Eqs. (A3) the Eqs. (17) are obtained.