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THERMAL DYNAMIC MODELS USING Z-TRANSFORM COEFFICIENTS: AN ALGORITHM TO IMPROVE THE RELIABILITY OF SIMULATIONS

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ABSTRACT

The Transfer Function Method (TFM) is a tool able to solve heat transfer problems in building envelopes and environments and it is recommended by the American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE). Authors have investigated TFM mathematical features, especially concerning the reliability and the quality of the thermal dynamic simulations. Using some basilar control systems tools like Bode plots, step response, model validation, results show clearly that, for a very massive building, a simply application of TFM very often fails. We have developed a software, THELDA 2000, founded upon a new TFM algorithm, to determine the Z-transform coefficients able to perform a thermal building simulation. Target of our work is to identify an automatic procedure able to optimise the reliability of the simulation by selecting and using in a more correct way poles and the residuals of the transfer functions. Paper explains as applying to TFM some concepts coming from system theory, like choosing the only significant poles in the transfer function, it is possible to obtain a drastic improvement of the quality of the simulation. Some cases studies show that using a limited number of coefficients can give a faster and more reliable simulation and sometimes it is the only method to obtain some results. Furthermore, the analysis shows as the choice of the sampling period is crucial to carry out a reliable simulation.

INTRODUCTION

Analysis of the thermal performances of building can be done with simulation programs that are able to analyse the heat transfer phenomena. Usually dynamic modelling is required and one-dimensional heat transfer is normally assumed. Commonly used techniques are finite difference methods (ESP, APACHE-sim, K2, SERI-RES, HTB-2), convolution-based *conduction transfer function* methods (BLAST, TARP, DOE-EnergyPlus, TRNSYS). (Gough M. 1999), (Klein S. A., Beckmann W. A., Duffie J. A.. 1976).

A large part of the actually employed models are still deterministic. Their purpose is to predict the time evolution of the building plant system behaviour from a given set of driving forces, expressed as explicit time functions. Taking into account that the deterministic approach often fails due to the high degree of randomness in real systems, developers have to cope with this new challenge: to face the uncertainty. Furthermore, new techniques in building and HVAC system modelling like neural network and genetic algorithms can contribute to overcome the linearity assumption (Hanby V. I., Dil A. J.. 1995), (Palomo, E., G., Lefebvre. 1995).

Dynamics are modelled using a discretisation of the time. A sampling time-step is normally used, the main determinant of which is the time-resolution of available weather data (usually 1 hour). Many non-linearity occurs in the equations representing the thermal balance of the room. However in the type of plant model usually adopted, the control characteristics take the form of linear functions relating room heating or cooling inputs to zone temperatures, which can readily be incorporated into the linear equation sets representing the room's thermal balance. Solution techniques vary between programs. All, however, employ some combination of linearisation, matrix manipulation and iterative methods.

Research-oriented building simulation programs are created for a variety of purposes. Being designed for use by researchers with specialised knowledge, they tend to be less user-friendly than their commercial counterparts (Gough M. 1999).

This has not, however, prevented some of them – ESP-r, HTB-2, SERI-RES and TRNSYS, for example – from being used from time to time in commercial projects. The principal types of research activity involving programs of this type are:

- Developing new software and new approaches to simulation.
- Developing simulation techniques and data relating to particular physical phenomena (for example convection, conduction, solar radiation, ventilation, high thermal inertia).
- Investigating and optimising the performance of new or existing technologies (for

example transparent insulation, double-façades, controls, air conditioning systems, photovoltaic).

- Model validation.
- Development of codes, standards and simplified calculation methods.
- Education.

Although the scientific community is involved in this field since more than thirty years, the most part of the available commercial software, which are largely diffused and accepted all over the world, give some troubles when utilised to simulate very massive building as the ones different from USA buildings typology.

In previous papers the authors have showed that TFM have some trouble when applied to Mediterranean buildings, characterised by high value of thermal inertia. (Beccali G., Cellura M., Giarrè L., Lo Brano V., Orioli A.. 2002).

SOFTWARE THELDA 2000

THELDA 2000 (Thermal Elements Dynamic Analysis) is able to simulate the thermal behaviour of a single thermal zone using the Z–Transform and the TFM. It belongs to the research activities of type 2, 4 and 6 above mentioned.

THELDA 2000 is also capable to change the sampling period, and to fix the preferred number of poles and number of coefficients used for the simulation. Besides, for appraising the precision of the calculation, THELDA 2000 performs simulation even with a different mathematical approach like Fourier series expansion.

A remarkable and innovative characteristic of this software is the possibility to modify, by using user-friendly interface, any parameters linked to the application of TFM changing, by this way, the accuracy of the analysis. This feature is rarely available in other common tools for thermal building simulation and it gives to the software characteristics of flexibility and of innovation with respect to the software programs mostly diffused in this field.

We have also implemented an innovative algorithm that permits to re-order the residuals for every transfer function calculated by THELDA 2000. Results show clearly as the possibility to reorder the residuals has a strong influence in the reliability of the carried out simulations (see next sections).

Authors (Beccali G., Cellura M., Lo Brano V.. 2001), (Cellura M., Lo Brano V., Orioli A.. 2001) have utilised TH.EL.D.A. 2000 to perform a simulation on a single thermal zone belonging to a very massive ancient building in the city of Marsala, in the south of Italy. The features of this building give some troubles carrying out a thermal simulation using other software so we have decided to perform a deep analysis over the transfer functions of this building.

The room used for the analysis is composed by six walls, one bordering with the external environment. Every wall is composed by a tuff layer in the middle and inside-outside plaster. We have analysed a room where all walls have identical characteristics. With the aim to study the reliability of TFM related to the thermal inertia we have considered several cases: the thickness of the tuff layer in the middle varies from 0.4 m to 0.8 m. Such features are typical of the Mediterranean building historical heritage and they are not an exception.

Concerning the response of the system, it represents a matrix in which the first six columns represents the hourly behaviour of the temperature on the inner surface of each wall and the last column represents the hourly temperature of the air (free floating simulations).

The input signal is the air temperature and solar radiation on the external surface of the first wall calculated for the city of Marsala in July (orientation south-east). Moreover, other input signals are radiative heat flux due to a window and the air ventilation. The input signals are periodic and repeat themselves every 24 hours.

MATHEMATICAL BACKGROUND

The non–steady–state heat problems in multilayered walls are described by a set of differential equations which can be solved by mathematic operators reaching the matrix:

$$\begin{vmatrix} T_e \\ Q_e \end{vmatrix} = \begin{vmatrix} A & B \\ C & D \end{vmatrix} \times \begin{vmatrix} T_i \\ Q_i \end{vmatrix}$$

in which *T* and *Q* are, in this case, Laplace transform (LT) of the temperatures t_i and t_e and of the heat fluxes q_i and q_e in correspondence of the inside and outside surfaces of the wall, while *A*, *B*, *C* and *D* are the coefficients of the wall transmission matrix reached through the product of transmission matrixes, of each *n* layers forming the wall:

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = \prod_{m=1}^{n} \begin{vmatrix} a_m & b_m \\ c_m & d_m \end{vmatrix}$$

$a_m\ b_m\ c_m\ d_m$ are respectively:

$$a_m = \cosh\left(L\sqrt{\frac{s}{\beta}}\right)$$

$$b_m = \frac{\sinh\left(L\sqrt{\frac{s}{\beta}}\right)}{\lambda\sqrt{\frac{s}{\beta}}}$$

$$c_m = \lambda\sqrt{\frac{s}{\beta}} \cdot \sinh\left(L\sqrt{\frac{s}{\beta}}\right)$$

$$d_m = a_m$$

$$\beta = \frac{\lambda}{\rho C_p}$$

and the used symbols have, for each *m* layer, the following meaning:

λ thermal conductivity $\left[\frac{W}{mK}\right]$

ρ density $\left[\frac{kg}{m^3}\right]$

C_P specific heat $\left[\frac{kJ}{kgK}\right]$

L thickness $[m]$

The system can be described using the ZT at the LT place. The fixed time-step method for multi-layer slabs uses transmission matrix formulations coming from the Laplace Transform domain. The convolution-based 'response factor' method developed by Stephenson and Mitalas forms the basis of the ASHRAE procedures. For a time-continuous $f(t)$, with sampling period T, LT is:

$$f(0) + f(T)e^{-sT} + f(2T)e^{-2sT} + \ldots$$

Supposing $e^{sT} = z$

We obtain:

$$f(0)z^0 + f(\Delta)z^{-1} + f(2\Delta)z^{-2z} + \ldots,$$

That is the ZT of the function $f(t)$.

If a system is solicited by an input signal whose ZT is $U(z)$ and the output is $Y(z)$, the link we have to determine is $\frac{Y(z)}{U(z)}$. We briefly introduce some dynamical systems basilar concepts that are useful in the sequel.

Transfer function:

Let $u(t)$ be the input signal and $y(t)$ be the output signal. Let $U(z) = Z[u(t)]$ and $Y(z) = Z[y(t)]$ be the corresponding *z*-transformed signals. Then, the transfer function of the linear model for the system is given by $G(z) = \frac{Z[y(t)]}{Z[u(t)]} = \frac{n(z)}{d(z)}$, where $n(z)$ is the polynomial called *numerator* and $d(z)$ is a polynomial called *denominator.*

Poles and zeros:

The roots of the numerator (z_i such that $n(z_i) = 0$) are called *zeros* and the roots of the denominator (p_i such that $d(p_i) = 0$) are called *poles.* Clearly the number of poles n_p denote the *order* of the polynomial $d(z) = (z - p_1)(z - p_2)\ldots(z - p_{np})$

Sampling Period:

The sampling period T is related to the hours of data-collection, generally 1 hour. For different sampling period we get different transfer function in the *z*-domain.

In order to select the *best model* representing the real system we carry an analysis on the obtained transfer functions. The *tools* that we are using for the analysis are based on some control systems basilar concepts.

Step response:

For a given input signal $u(t)$, the corresponding output is $y(t) = Z^{-1}[G(z)U(z)]$, where $Z^{-1}[.]$ represents the inverse z-transform. The step signal is a canonical one used in control literature to get, if possible, the steady state behaviour of stable system (the output signal after the transient, for large *t*).

Frequency Response:

The response of a linear system to a sinusoidal input is referred to as the system's *frequency response.* For continuous-time systems, a stable linear system, which transfer function in the Laplace domain is $G(s)$, excited by a sinusoid will eventually exhibit a sinusoidal output with the same frequency ω of the input. The magnitude, $A(\omega)$ of the output with respect to the input is $|G(j\omega)|$ and the phase $P(\omega)$ is $\langle G(j\omega)$, letting *s* taking values along the imaginary *jw*-axis.

It is possible to define the frequency response also for discrete-time systems. If the system has transfer function $G(z)$, we define its magnitude and phase for *z* taking on values around the unit circle by $G(e^{j\omega T}) = A(\omega T)e^{jP(\omega T)}$. If an amplitude one sinusoid is applied, then in the steady state, the response samples will be on a sinusoid of the same frequency ω_0 with amplitude $A(\omega_0 T)$ and phase $P(\omega_0 T)$.

Bode Plots:

Plotting for each frequency the *magnitude* and the *phase* of the output response of each sinusoid input on a semi-logaritmic scale we get the Bode plots.

Model validation:

In order to evaluate the accuracy of the calculations carried out by using a given set of ZT coefficients, we have to compare them with a reference response coming from a procedure having a different mathematical background and even able to give the time continuous response of the system.

Thus, in order to evaluate the reference response if we want to control the reliability of the coefficients sets evaluated by mean of diverse input signals we have to:

Using input signals having the typical time variation of the physical phenomena and then avoiding steps, ramps, and other not physically possible signals.

Adopt an evaluating approach which differs from that employed to calculate ZT coefficients sets, and then exclude Laplace Transform (LT).

We carried out a comparison between the simulation data obtained from Fourier steady state algorithm and those one obtained from TFM (Bansal N. K., Bhandari M. S.. 1996),(. It has been necessary, to perform this comparison, to define for the t_e temperature or any generic input signal, a rule of interpolation able to reproduce a physic realistic behaviour starting from the sampled data.

So, the input signal it has been represented by the function $X(t)$, which is not discrete within the generic interval of sample t_n, t_{n+1} defined by the expression:

$$X(t) = at^3 + bt^2 + ct + d$$

in which the coefficients *a, b, c, d* are calculated imposing the following constraints:

1) the $X(t)$ assumes the values $X(t_n)$ and $X(t_{n+1})$

2) the tangent to the curve at the point $[t_n,\ X(t_n)]$, forms equal angles with the segments joining $X(t)$ to $X(t_{n-1})$ and $X(t_n)$ to $X(t_{n+1})$

In periodic steady state any wave $f(t)$ with period T can be analysed into the sum of sinusoidal waves of different frequencies:

$$f(t) = \frac{1}{2}a_0 + \sum_{n-1}^{\infty}\left[a_n \cos(n\omega t) + b_n \sin(n\omega t)\right]$$

Supposing that $f(t)$ is known in correspondence to the N instants in which the period T was subdivided, coefficients are:

$$\begin{cases} a_0 = \dfrac{2}{N}\displaystyle\sum_{k=0}^{N-1} f\left(\dfrac{kt}{N}\right) \\ a_n = \dfrac{2}{N}\displaystyle\sum_{k=0}^{N-1} f\left(\dfrac{kt}{N}\right)\cdot\cos\left(\dfrac{2\pi kn}{N}\right) \\ b_n = \dfrac{2}{N}\displaystyle\sum_{k=0}^{N-1} f\left(\dfrac{kt}{N}\right)\cdot\mathrm{sen}\left(\dfrac{2\pi kn}{N}\right) \end{cases}$$

Then, calculating the time response to each harmonic component and recombine all of them, one can obtain the system response to the generic $f(t)$.

In relation to the different forms of the employed signals, the calculated temperatures have been compared to those ones obtained by Fourier's analysis in periodic steady state conditions.

We founded the comparison on the percentage mean error parameter PME:

$$PME_i = \frac{1}{24}\sum_{\tau=1}^{24}\frac{|T_{Zi}(\tau) - T_{Fi}(\tau)|}{T_{Fi}(\tau)}\cdot 100;$$

$$PME_{air} = \frac{1}{24}\sum_{\tau=1}^{24}\frac{|T_{Z\,air}(\tau) - T_{F\,air}(\tau)|}{T_{F\,air}(\tau)}\cdot 100$$

where T_Z and T_F are the hourly temperatures at generic time τ for the *i-th* inner surfaces or for the air, calculated using the ZT and the Fourier analysis respectively. To exit from transient response, the calculation with ZT is repeated until signal is stabilised (Davies M. G.. 1978).

ANALYSIS

Our approach is based on the method summarised in the previous paragraph. We have built the $G(z)$ taking an increasing number of poles and the corresponding residuals. Let $G_5(z)$, $G_{10}(z)$ and $G_{15}(z)$ be the corresponding transfer function of order $n_p = 5$, $n_p = 10$, and respectively, $n_p = 15$.

In Figures 1, 2 the output response $y(t)$ of G_5, G_{10} and G_{15} to the temperature step signal $u(t) = 100step(t)$, a step signal with amplitude 100 degrees is shown for different sampling period ($T = 1,2$ hours). In Figures 3 and 4 we have reported the discrete-time Bode plots of $G_5(z), G_{10}(z)$ and $G_{15}(z)$ for sampling time $T = 1,2$ hours. It is evident that the model obtained for $T = 1$ hour which output response is shown in Figure 1 is not correct. In particular, increasing the number of poles the behaviour is getting worse.

Increasing the sampling period, $T = 2$ hours, we see in Figure 2 that most of the numerical problems are solved and the steady-state behaviour is the one expected. After the transient, the output reaches steady state values of $T = 100\ ^{\circ}C$.

Looking at Figures 3 and 4, it is evident that wl the sampling period is big enough, the Bode plots the different transfer functions are identical for l frequencies, but increasing the number of poles, frequency response is slightly different at h frequency.

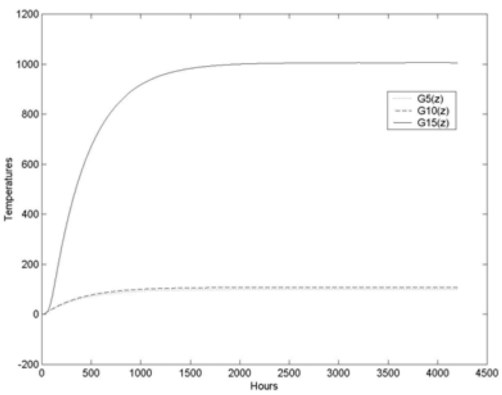

Figure 1. Step response output for the transfer functions $G_5(z)$, $G_{10}(z)$ and $G_{15}(z)$ with $T = 1$ hour.

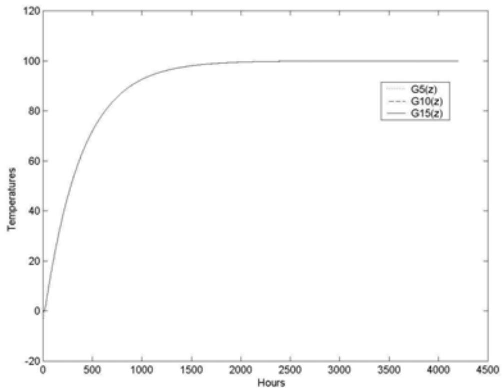

Figure 2. Step response output for the transfer functions $G_5(z)$, $G_{10}(z)$ and $G_{15}(z)$ with $T = 2$ hours.

From Figure 3 is clear that $G_{10}(z)$ and $G_{15}(z)$ are not correct for low frequencies (implying that the steady state value in the step response is completely wrong).

Comparing Figure 3 and 4 it is manifest that with $T = 1$ hours for $n_p = 15$ at low frequency the magnitude is not correct (ten times bigger than the correct one!).

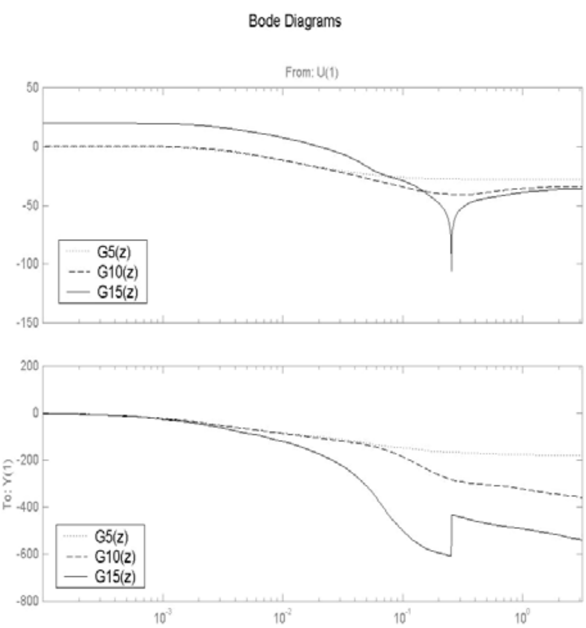

Figure 3. Bode plots for the transfer functions $G_5(z)$, $G_{10}(z)$ *and* $G_{15}(z)$ *with* $T = 1$ *hour.*

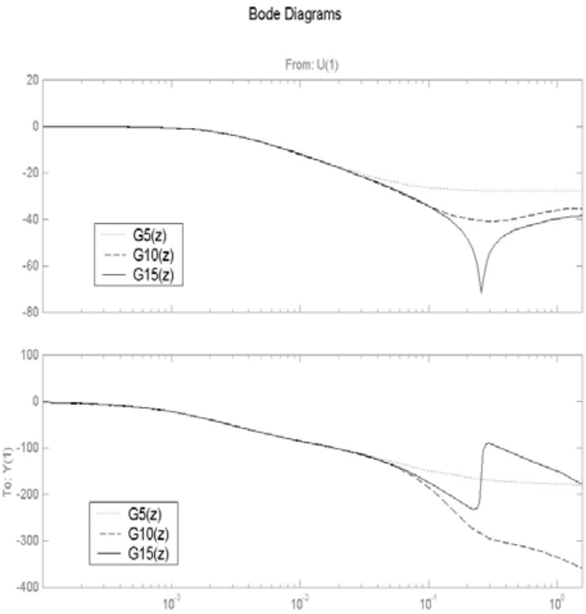

Figure 4. Bode plots for the transfer functions $G_5(z)$, $G_{10}(z)$ *and* $G_{15}(z)$ *with* $T = 2$ *hours.*

CHOICE OF SIGNIFICANT POLES

Our approach is based on the following method, named Procedure I.

Given a Laplace transfer function $G(s) = \sum_{n=1}^{N} \frac{d_n}{(s - p_n)}$, let $\hat{d}_n$ the ordered residuals be such that $|\hat{d}_{i-1}| > |\hat{d}_i| > |\hat{d}_{i+1}|$, whose corresponding poles are $\hat{p}_i$. Then $G(s) = \sum_{n=1}^{N} \frac{d_n}{(s - p_n)} = \sum_{n=1}^{N} \frac{\hat{d}_n}{(s - \hat{p}_n)}$. Let us consider the truncated $\hat{G}(s) = \sum_{n=1}^{\hat{N}<N} \frac{\hat{d}_n}{(s - \hat{p}_n)}$. This is obtained getting rid of the non significant poles corresponding to very low value of the residuals and taking into account only the $\hat{N}$ residuals (and corresponding poles) such that $|\hat{d}_i| > \sigma$, where σ is a prefixed value. It is well know from any classical control textbook that only the significant poles add information to the overall system. We have observed that is possible to eliminate the residuals and the correspondent poles when residuals present values that have value lower than $\sigma = 10^{-10}$.

Applying the new method for the choice of the only significant poles the quality of the simulation has a drastic improvment. In the following figures we show data from simulations carried out using two different procedures:

Procedure I utilises the new alghorithm, it reorders the residuals cutting off values lower than 10^{-10}, consequently eliminates the corrispondent poles.

Procedure II is the normal procedure utilised by the standard TFM and no optimal selection of residuals and poles is performed.

Once the Procedure I has selected a $\hat{G}(s)$ with only significant poles, the discrete time of fixed order $n_p = 5$, $n_p = 10$, and $n_p = 15$ has been evaluated: $\hat{G}_5(z)$, $\hat{G}_{10}(z)$ and $\hat{G}_{15}(z)$.

In Procedure II starting from $G(s)$ with no reordering and truncation, we evaluate $G_5(z)$, $G_{10}(z)$ and $G_{15}(z)$.

Figures 5–7 show that the simulations carried out using the procedure I have a very good performance in term of PME. It is evident that models obtained for T = 1 hour and using procedure II are not correct because the error index is very high. Besides increasing the number of poles the behaviour is getting worse. On the other hand the same models using the procedure I give good results with PME lower than 1%.

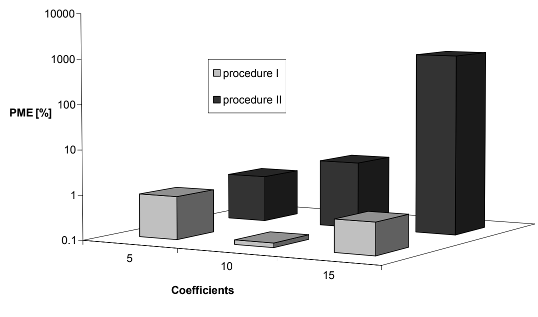

Figure 5. PME for the surface bordering with the external environment; comparison between Procedure I and Procedure II; thickness of walls = 0.8 m.

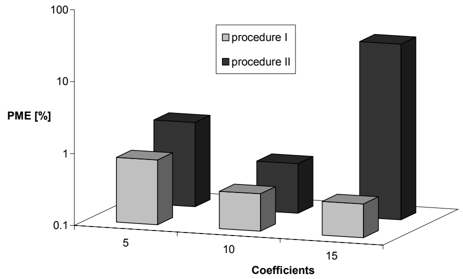

Figure 6. PME for the surface bordering with the external environment; comparison between Procedure I and Procedure II; thickness of walls = 0.6 m.

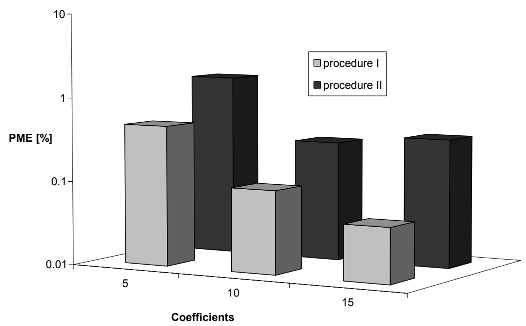

Figure 7. PME for the surface bordering with the external environment; comparison between Procedure I and Procedure II; thickness of walls = 0.4 m.

RESULTS INTERPRETATION

The widespread mathematical models adopted for the simulation of non-steady state thermal behaviour of buildings if applied just as they are in a Mediterranean context, due to the particular thermophysical, typological, structural and technological characteristics of this geographical area, could give raise to numerical problems. In particular, the high thermal inertia of the buildings belonging to the historical centres of the south European cities, along with the intrinsic characteristics of the materials employed, the arrangement of building types, the particular urban layout, do not allow to carry out simulations with the required degree of accuracy. Thus, even if there are simulation models widely used and appreciated all over the world, their application to the buildings in our historical town centres is still a doubtful matter. Calculation models largely diffused in U.S.A. (such as DOE, TRNSYS, BLAST) well suited for structures exhibiting low thermal inertia and capable of singling out the radiative part of thermal exchanges, could be hardly adapted to a European context. The European rules, in the present state-of-art, recognise these limits and uncertainties: in fact, the Technical Committee TC98 (as part of the European Committee for Standardisation (CEN)), is still debating on the accuracy level and on the applicability either of a dynamic simulation carried out using detailed models, or a steady-state model dynamically corrected taking into account the thermal mass of the building.

The aforesaid research work makes clear, in fact, the limits of the most popular simulation models in describing, with suitable accuracy and usefulness, building structures exhibiting high thermal inertia; the next step of the work will overcome these limits.

With the present analysis, some solutions to the main numerical problems connected to the use of standard software package for massive buildings have been showed. In particular, according to the authors' previous papers (Beccali G., Cellura M., Giarrè L., Lo Brano V., Orioli A.. 2002), (Beccali G., Cellura M., Lo Brano V.. 2001), (Cellura M., Lo Brano V., Orioli A.. 2001), it is evident that model validation and selection is crucial. The two main points of (Beccali G., Cellura M., Giarrè L., Lo Brano V., Orioli A.. 2002) were the following:

1) Not always the best models is the bigger in terms of number of poles

2) Increasing the sampling period is a good way to eliminate malfunctioning

In the present work an efficient way to choose the only significant poles influencing the dynamics and the behaviour of the model has been presented. Combining this new procedure that picks the optimal poles and increasing the sampling period, almost all the numerical problems disappeared. Moreover, based on the Percentage Mean Error (PME), an automatic model selection can be implemented. Clearly, the "*best*" model is the one giving raise to the lowest PME. For example, considering a thickness of the walls of 0.8m (as the one reported in Figure 5), all the models obtained with the new procedure even for T=1 hour presented a PME less than 1%, but the best model is $\hat{G}_{10}(z)$ showing a PME = 0.12. The importance of the presented model selection stands in the possibility of carrying this selection in a fully automated way, without any supervision of the user, which can be also a not-expert one.

CONCLUSIONS

DOE and TRNSYS, two of the most diffuse software for thermal building simulation use the ASHRAE TFM calculation methodology. Currently the attention is on integrated software where the user can appreciate different aspects as visual, acoustic and thermal comfort, LCA, etc, coming from various project choices. Referring to the quality of a thermal dynamic simulation there are some limits, particularly for TRNSYS, in the case of buildings characterised by the high thermal inertia, typical of traditional European building. Within this framework we think our work very actual because we point out the critical points in the quality of a thermal simulation performed by ASHRAE TFM calculation methodology. The advantage of the TFM is the speed in the case of application to a single multi-layered wall. But, when the TFM calculation methodology is applied to a room, the manual calculation becomes impossible. In the simplest case one should have to calculate the determinant of a 7^{th} order matrix several thousands times in order to calculate only few poles. Also knowing the coefficients of the transfer functions, the method should have to be applied tens of times for different Input-Output couples. In the case of thermal zones simulated with the TFM the advantage is not in the speed or the manual calculation (virtually impossible). The advantage is that it is easy to arrange a multi zone model taking advantage from the linearity of the algorithm. Furthermore, since the model is flexible in defining the boundary conditions, it can be coupled with other tools, without reducing the accuracy of the whole model (Haltrecht D, Zmeureanu R, Beausoleil-Morrison. 1999). Disadvantage is the loss of information due to the linearity assumptions, but previous studies have estimated that the linearity assumptions in TFM, for most building materials and operation strategies comport no significant difference between the results of this model and those of other numerical models (Haltrecht D, Zmeureanu R, Beausoleil-Morrison. 1999).

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