

## ARCHITECTURAL CONSTRAINTS IN A GENERATIVE DESIGN SYSTEM: INTERPRETING ENERGY CONSUMPTION LEVELS

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### ABSTRACT

This paper investigates the possibility of encoding architectural design intentions into a generative design system, using as a test bed the School of Architecture at Oporto [Portugal], designed by Álvaro Siza. Based on language constraints derived from Siza's original design, the generative system [GS], consisting of a genetic algorithm and the DOE-2.1E building simulation program, created facade solutions resulting in lower annual energy consumption, while acting simultaneously as a diagnosis mechanism for problems occurring in the existing building. Results suggest this GS may be a useful tool to architects during the design process, by identifying potentially problematic areas and suggesting ways to approach them. Experiments were also performed for other geographical locations apart from Oporto, to test the algorithm's capability to adapt solutions to different climates within the same language constraints.

### INTRODUCTION

Previous work described the development and testing of a new generative design system (Caldas and Norford, 1999, 2000). The Generative System [GS] consists of a method for creating new designs, a procedure to evaluate them, and a mechanism to evolve solutions towards an improved performance in terms of the selected criteria. It uses a genetic algorithm [GA] as a search and optimization engine and the DOE-2.1E building simulation software (SRG, 1993; Winkelman, 1983) to assess the use of natural lighting, thermal performance and yearly energy consumption.

Work by others includes the GenOpt software for optimization purposes (Wetter, 2000), which uses as a search method the Simplex method of Nelder and Mead with an extension by O'Neill. The main drawback of this search method is that it works well only in small problem sizes, up to about 10 variables. Radford and Gero (1978) suggested the use of dynamic programming to select window size and glazing materials in order to minimize energy consumption. Previous attempts to use DOE-2.1E to optimize design parameters of buildings include using regression analysis on data created by parametric DOE-2 runs (Sullivan et al. 1992). Simplified procedures like the LT method (Baker and Steemers, 2000) use energy graphs to explore the impact of a few key building-

design parameters but are unable to handle the level of detail and complexity enabled by the system presented in this paper.

A GA was chosen as a search mechanism that manipulates in parallel a population of solutions. GAs are particularly suited to be used in an architectural design context because their output is not a single solution but a number of high-performance solutions, one or more of which the architect can further develop by considering other criteria not included in the search process. A GA starts by generating a number of possible solutions to a problem, calculates their fitness [objective function value], and applies the basic genetic operators of reproduction, crossover and mutation to the initial population, in a stochastic hill-climbing process. This generates a new population with higher average fitness than the previous one, which will in turn be evaluated. The cycle will be repeated for the number of generations set by the user. For more detailed information on GAs, see Goldberg, 1989. GAs have been used in building applications related to energy consumption, mostly to optimize the sizing and control of HVAC systems (Huang et al. 1997, Wright 1996, Dickinson et al. 1995).

This paper approaches the integration of architectural design intentions into the GS, when applied to the study of building facades. Siza's School of Architecture at Oporto was used as a test bed because the clear but complex composition rules used in the elevations provided an excellent framework to work upon. Due to the large dimension of the project, the study focused solely on one of the studio buildings, tower H [see figure 1].

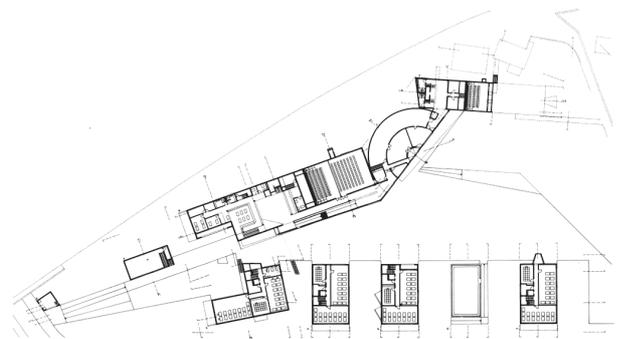


Figure 1 – School of Architecture plan. Tower H in on the right lower corner, with studios on the south and east.

The School of Architecture at Oporto was designed and constructed from 1984 to 1996 by Álvaro Siza. Faced with a challenging site steeply sloping towards the Douro River, Siza opted for distributing the academic activities among different spatial units, creating a remarkable piece of architecture. Studios and faculty rooms are housed in towers E, F, G and H; the library, auditoriums and administrative services are in the northernmost buildings. For a more detailed analysis, see Testa (1999).



Figure 2 – Southeast view of studio towers. Tower H is in the foreground

Tower H was chosen for this study for its rich spatial configurations and use of a variety of architectural light sources: fenestrations of different proportions and sizes facing distinct orientations [some including overhangs], zenithal light from roof monitors in the top floor, and a loggia in the south façade [see figure 2]. From a computational perspective, tower H also presents some challenging features. The internal relations between the different spaces and their light sources give rise to a multiplicity of interactions that are hard to predict and make the resort to computational analysis an interesting option. The fact that Tower H mainly houses studio teaching rooms also makes a strong case for the careful control of natural light in order to maintain adequate daylighting levels for drawing tasks while precluding direct sun over the drafting tables and excessive solar gains in the rooms.

## OBJECTIVES

The objectives of this research were twofold: first, to study the incorporation of language constraints into the generative design system, so that solutions generated are within certain design intentions; and, second, to examine the generative system results from the perspective of the existing design by Álvaro Siza, an architect well known for his control of light, and to analyze to what extent the inclusion of factors other than light [like the thermal performance of the building] could make solutions follow a different path. For that analysis we used energy consumption levels discriminated by end use in the building.

## SIMULATION

In this study a micro-GA was used instead of a conventional GA. The main difference between the two

methods concerns population size. Typical population sizes for GAs range from 30 to 200, based on earlier studies such as those of Grefenstette (1986), where suggestions for optimal population choices based on parametric studies are presented. Micro-GAs (Krishnakumar 1989) use a small population (in this case, only five individuals) and quickly makes it converge to a solution. Convergence is measured by comparing the chromosomes of the individual solutions. If they differ by less than 5%, it is considered the population has converged. When that happens, the micro-GA restarts a new random population while carrying over the individual with the best fitness in the previous generation, a strategy known as elitism. This way new individuals are often brought into the search, without losing track of the ones that did better until that point. An advantage of using the micro-GA procedure is that the algorithm tends to perform a local search around the best solutions during the generations prior to convergence, since at that stage solutions only differ by a few alleles. This local search is important in finding local minima around good solutions, and is usually hard to implement in conventional GAs. Another advantage is that the search procedure is faster, since the micro-GA does not have the inertia of the large populations associated with conventional GAs.

The generative system works on a complete three-dimensional description of the building, including its geometry, orientation, spatial organization, construction materials, etc. In this study, building geometry, space layout and construction materials were left unchanged, and the algorithm's search space related only to elevation design solutions. It should be noted, however, that the GS has the capability to handle many other types of problems.

For the existing building layout as designed by Siza, the GS generates a population of façade solutions that take into account the use of daylighting in the space, the subsequent use of artificial lighting, and the energy consumed to heat and cool the building. Although maximum use of natural lighting is a desirable goal, the control of heat gains and losses introduces a balance point to be achieved. It is this elusive balance point that the computer tries to locate.

As mentioned above, the DOE-2.1E program is used to calculate the fitness of each solution. For each individual space in Tower H where daylighting is available, two lighting reference points were selected [typically the furthest points from the windows where a certain light level was to be achieved] and desired illuminance values were specified according to the type of occupation and tasks performed. Generally, we used 500 lux for studios and other working spaces and 150 lux for service areas. The artificial lighting system is supposed to be continuously dimmable, even though in Siza's existing building such a system is not implemented. The

subsequent DOE-2.1E run provides the annual energy consumption of the building for that particular solution. That value represents the fitness of the individual, and is then passed into the genetic algorithm to further guide the search process.

### ANALYSIS OF EXISTING BUILDING

Due to the need to find elements that would lead to the development of a method to understand and encode Siza's design intentions [rules], a visit to the School of Architecture took place in January 2000. The analysis of the drawings and the visit to the building allowed us to infer design rules that we consider to be applicable to the existing elevations. Those rules relate both to compositional axes of the facades and to general proportions of the openings. In tower H, different rules seem to apply to each elevation, while maintaining a strong coherence in the overall design of the building and in the relations with internal spaces [for example, long horizontal windows are always used in the studios].

The south elevation presents a strong symmetry axis for the openings, but introduces other elements such as overhangs and the loggia. The north façade is also mostly symmetrical in its composition, with a single asymmetrical element. However, east and west façades obey quite different rules. We considered the east elevation to be organized by two vertical axes along which the ends of the different openings are aligned. The small openings present in the west elevation relate to the interior spaces they serve [service areas like stairs and restrooms]. As for the proportions of the openings, the majority of them tend to be long horizontal windows, with many variations in size and placement.

This interpretation of existing design rules was followed by the determination of areas of search for the generative mechanism, implemented as constraints to the algorithm. The search areas are bounded by maximum and minimum dimensions the openings can assume, and those limits were made sufficiently broad to allow for a significant search space that could promote the emergence of a rich variety of solutions. Other constraints implement the compositional axes determined during the analysis stage.

In figure 4, the upper row represents the constraints applied. Compositional axes are represented by the lighter lines. For each opening, the smaller area represents the lower bound to the algorithm, and the larger area the upper bound. For horizontal windows, the constraints are specified in a way that prevents the appearance of vertical openings. This set of constraints was proposed by us to control the generation of solutions within certain architectural intentions that we related to Siza's design. Changing the constraints would allow for the exploration of many different design solutions, a path we did not pursue in this work.

### EXPERIMENTS

The GS was run for three climates with distinct characteristics, to test its capability of adapting architectural design solutions to different environmental requirements while subject to the same language constraints. Apart from Oporto, where the existing building is located, the other climates chosen were Phoenix, Arizona, and Chicago, Illinois, both in the USA. [A note should be added here that the exercise of placing the building in a different geographical location is purely academic, since Álvaro Siza would never use the same approach for designing a building in such different geographical and cultural environments. While the experiments for Oporto illustrate a process that could have taken place in Siza's design, the other examples are a test of the generative system but do not represent an appropriate architectural design process].

Oporto's climate is mild, with average monthly temperatures in the coldest months [December and January] around 10°C. The warmest months [July and August] have average monthly temperatures of 20°C. The lowest temperature registered in the weather file used [TMY] was 2.8°C in December, and the highest was 30.4°C in August. Phoenix's climate is much hotter and dryer, with temperatures peaking at 44.4°C in June in the file used [WYEC]. Average monthly temperatures in July reach almost 34°C. Average monthly temperatures in the coldest months [December and January] are similar to Oporto's case, around 11°C. Chicago's climate is characterized by extremely low temperatures in the winter. The minimum temperature registered in the file used [TMY] was -22.8°C in January. Average monthly temperatures are -2.2°C in December and -3.3°C in January and February. The monthly average temperature in July is about 24°C.

### METHOD

Once the constraints were graphically determined, they were used as inputs to the generative system. The step size used ranged from about 30 cm for windows to 50cm for exterior shades. After the GS finished running, results from the search process could be automatically visualized using an existing visualization program, in order to relate a given design layout to a corresponding annual energy consumption level. The 2-D drawings obtained this way were exported to a CAD package and served as the basis for manually creating a 3D model of the best generated solution. That model was then exported to rendering software that allowed the production of images like the ones in figure 3. We also exported the model to a detailed lighting simulation program for better visualization of GS results and comparison with the existing Siza solutions [figure 6]. Those images together with energy end-use graphs for the different solutions are the basis for the results presented below.

## RESULTS

### OPORTO

Results from the GS ranged from an almost exact coincidence with Siza's solutions to some radical departures from the existing design. In figure 3, three-dimensional models of both Siza's and GS solutions are displayed.

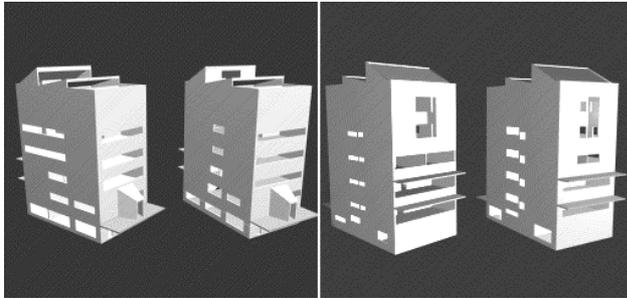


Figure 3 - Three-dimensional models of Siza's and GS solutions. The two images on the left show northeast views, with Siza's on the left and GS on the right. The two images on the right show southwest views, this time with Siza's on the right and GS on the left, as when the camera rotated around the two 3D models, the relative position of the two images was reverted.

In the north façade, the large horizontal stripes generated by the algorithm very approximately resemble those created by Siza [except for the melodic variations in height in the original design], denoting that in Oporto's mild climate the use of natural light in the studios clearly offsets the heat losses through the large glazing areas, as Siza may have predicted. Heat gains are not a significant issue in this orientation. It can be observed in figure 4 that as the quality of solutions decreases [oportobest, oportoaverage, oportoworst], window sizes decrease too.

Towards the west, the algorithm used small window sizes as Siza did, even further reducing them. This was due to the lower illuminance levels that the service areas [stairs and restrooms] require and to the reduced size of the spaces. It can be seen in figure 4 that as the openings get larger, the quality of the solutions decrease.

In the south orientation, the generative system solutions present more significant modifications in relation to the existent. In Siza's design, the second and third floors have south facing studios with long horizontal windows shaded by 2-meter deep overhangs. The algorithm solutions tend to suggest these overhangs may be too deep. When the overhang depth is kept as 2 meters [oportobest], window sizes assume the largest dimensions allowed by the constraints. The deep overhangs block the admittance of daylight into the room, and to counteract that effect the GS increases the openings size. When overhang depth is a variable [oportoshading], the algorithm reduces it to 0.5m, and also reduces window sizes to a dimension closer to that used by Siza. The shallower overhangs still block direct

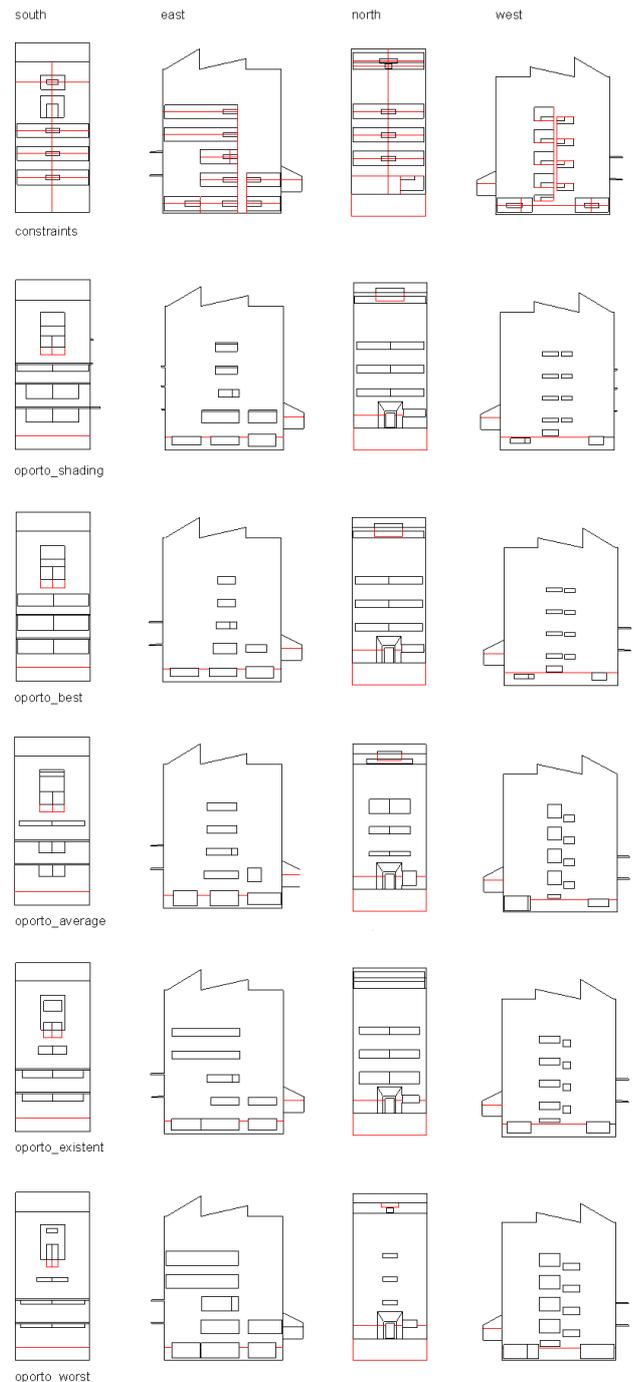


Figure 4 – Oporto solutions

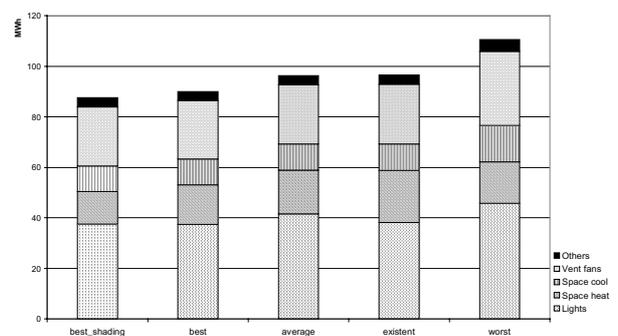


Figure 5 – Energy consumption levels for Oporto solutions

Solution	MWh
oportor_shading	87.58
oportor_best	89.99
oportor_average	96.22
oportor_existent	96.45
oportor_worst	110.55

Table 1 – Annual energy consumption for Oporto solutions

sun and high solar gains, since in the hottest months the sun is high in the south quadrant and can be controlled with smaller overhangs. On cold winter months, when the sun is lower in the sky, useful solar gains are still admitted into the rooms, reducing the need for heating. The `oportor_shading` solution has lower energy consumption than `oportor_best`.

In the sixth floor, the solution from the GS for the south facing loggia must be understood in conjunction with the roof monitors solutions [which can be seen in figure 4 or in the 3-D images of figure 3]. The sixth floor is basically occupied by a single space, lit from above by two roof monitors, from the south by a loggia window, and with blank walls in all other directions. The algorithm increases the south-facing loggia window to the maximum allowed by the constraints, and reduces the glazed area of the roof monitor that lights the space closer to the loggia. The roof monitor faces north and is a large source of heat losses in winter. Increasing the south opening permits reducing the roof monitor without losing too much daylight in the studios. On the other hand, the second roof monitor assumes the largest dimensions possible in the GS solution [as in Siza's design], since that area of the sixth floor has no other light source. This result suggests the tilt of the roof could be varied to allow for a larger roof monitor in that location.

The fourth and fifth floor south solutions must be analyzed together with east results, since in those floors the studios share both south and east openings. The GS increases south-facing windows in relation to the existing design, and simultaneously reduces east-facing ones. East orientation is unfavorable due to high solar gains during the morning in summer months and reduced daylighting levels during the afternoon for most of the year. South-facing openings perform better both in terms of natural light admission and control of heat gains. When the algorithm has the possibility of trading between the two options, it consistently favors south.

Figure 4 shows that as the size of east facing windows increase, the quality of solutions decreases. However, when the algorithm was allowed to place overhangs in the east façade too [`oportor_shading`], it significantly increased window sizes in the second floor, while placing quite deep overhangs to shade the low morning sun. It should be noted that the studio in the second floor

has only east-facing windows. For the studios on the fourth and fifth floors, which have both east- and south-facing windows, the GS kept east openings small [although slightly larger than in the unshaded case] with shallow overhangs, and again privileged south-facing openings.

For Oporto's climate, the worst solution found by the GS has about 26% higher energy consumption than the best solution with shading as a variable. Siza's design consumes about 10% more energy than the best GS solution with altered shading.

Examining results broken down by energy end-use [figure 5], it is possible to see that the existing solution by Siza performs almost as well as the best solution from the GS in terms of natural lighting use [meaning that artificial light consumption is low]. From the previous description of results, we could see that the main differences between the two solutions were in south and east facades, as well as the roof monitors. Although the GS performed many changes in the individual spaces, which may therefore have a more balanced use of daylighting, the overall artificial lighting consumption of the building did not change much. To get a better insight into what changed in each individual space it is necessary to go a step further into result analysis.

A lighting simulation software that combines radiosity and ray tracing was used to visualize the results of one particular studio room, where the algorithm could trade off between south and east orientations [fourth floor]. Figure 6 shows the renderings for the summer solstice at 9am, 10am, noon and 3pm. The top row is the existing solution, and the bottom one the GS best solution without using shading as a variable.



Figure 6 – Siza and GS solutions for fourth floor studio

From the figure it can be seen that, during the summer, the large unshaded east facing windows in the existing solution allow direct sun penetration in most of the room. In the GS solution, both windows are also unshaded but the south-facing window allows less direct sun into the room throughout the day. The small east-facing window only allows direct sun into the back of the room during the morning. In the afternoon, Siza's solution originates a much darker environment than the GS solution.

Artificial-lighting consumption levels increased about 22% from the best solution with shading to the worst solution. This number could probably be higher if some of the lighting reference points were not placed so deep into the room [about 1m from the wall most distant from a window]. The increase in space cooling in the worst solution is probably due to the large east- and west-facing windows. The existing solution does not differ much from the best one with shading. Even though some east windows decreased in size, others increased but had overhangs added. South windows had their overhang depth reduced for increased natural light, but that increased heat gains, too. For space heating, the best GS solution performs considerably better than the existing solution, as it allows more useful south solar gains in the winter, and reduces heat loss through the north-facing monitor and west and east openings.

## PHOENIX

In the hot Phoenix climate, the GS solution for the south façade significantly reduced the area of unshaded windows [fourth floor] relative to Oporto, even though shaded ones remained quite large. East windows were made much smaller, too. This reflects the effect of high heat gains in that geographical location. West windows remained small, as they did for Oporto, to avoid heat gains. The north façade suffered almost no alteration, since it is only marginally affected by direct solar gains.

In terms of energy end-use, the main differences between the several solutions occur, as expected, in space-cooling energy and associated ventilation-fan energy. If the existing design would theoretically be placed in Phoenix, it would consume about 20% more energy in space cooling and fans than the best solution the GS found [without using shading as a variable]. This happens because all unshaded east, south and west windows were greatly reduced, while shaded windows and north-facing ones were kept large for increasing daylighting use. Once again, the differences in artificial lighting use are not as significant as it might be expected, probably due to the extreme location of the lighting reference points.

The worst solution the GS found for Phoenix consumes about 55% more energy in space cooling and ventilation fans than the best one, within the same language constraints.

Solution	MWh
phoenix_best	119.02
phoenix_existent	131.09
phoenix_worst	162.74

Table 2 – Annual energy consumption for Phoenix solutions



Figure 7 – Phoenix solutions

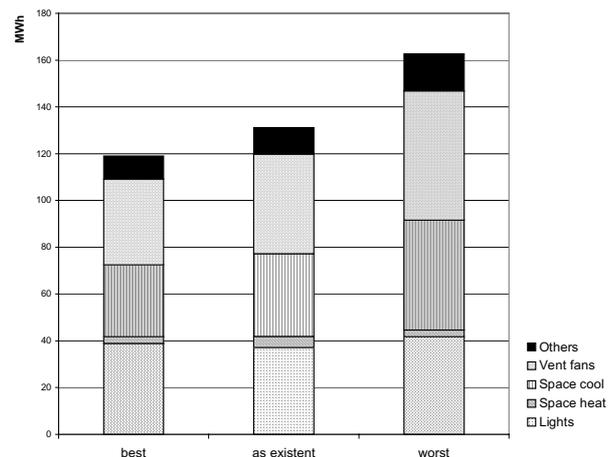


Figure 8 – Energy consumption levels for Phoenix results

## CHICAGO

The extremely cold Chicago climate originated some interesting changes in relation to Oporto and Phoenix solutions. For north-facing studios, the windows were reduced to the minimum dimensions allowed by the constraints, due to high heat losses through the glazing and to the absence of solar gains that would be beneficial in Chicago's cold climate. This façade-level solution may allow for an extrapolation in terms of spatial organization, suggesting that north-facing studios should be avoided in this type of climate. It is also interesting to observe in figure 4 that the best solution for Chicago is very similar to the worst solution for Oporto.

Towards the south, unshaded windows were made quite large, since they couple daylight admission with useful solar gains. However, shaded windows were reduced to minimum dimensions, as both natural light and solar gains are blocked, and heat losses prevail. When overhang depth was used as a variable, the algorithm reduced it to the minimum allowed and simultaneously increased window sizes [this result is not shown in the images]. It can be concluded that south shading may be undesirable in this climate. Towards the east, rooms that have only east-facing windows received average-sized openings [first and second floors], a compromise between such positive factors as daylight admission and morning solar gains, and such negative factors as high heat losses through the glazing. For studios with both east- and south-facing windows, east openings were made quite small because once again south was preferred. West fenestration received minimum dimensions.

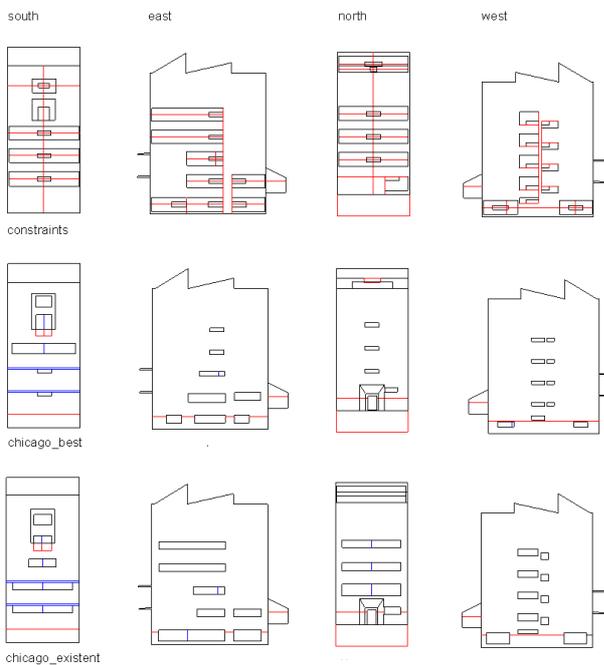


Figure 9 – Chicago results

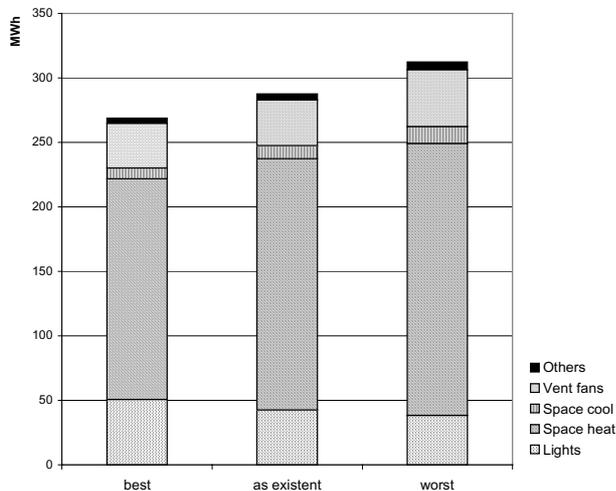


Figure 10 – Energy consumption levels for Chicago results

Solution	MWh
chicago_best	268.72
chicago_existent	287.36
chicago_worst	312.24

Table 3 – Annual energy consumption for Chicago solutions

In terms of energy end use, space heat dominates in the Chicago solutions, as expected. The high energy-consumption levels could be reduced using such strategies as properly insulating the building and using better glazing types, which are not variables considered in this study. But even using only opening sizes as variables, the existing design consumes about 14% more heating energy than the best GS solution, and the worst solution consumes 23% more heating energy. Because those savings in heating energy imply reducing opening size and thus reducing natural light availability, the overall performance between the different solutions is that the existing design consumes about 7% more energy than the best GS solution, and the worst design consumes about 16% more energy annually.

### GS SEARCH PROGRESSION

Plotting the progression of search results from the GS along the 100 generations [figure 11], it can be observed that the first generations correspond to a steep decrease in the minimum function value, which then stabilizes and slowly decreases during the remaining search trials. The elitism strategy keeps the best solution along the entire search progress, while the micro-GA is responsible for always introducing new points in the search space, preventing early convergence of the solutions to a local minimum and promoting diversity into the search.

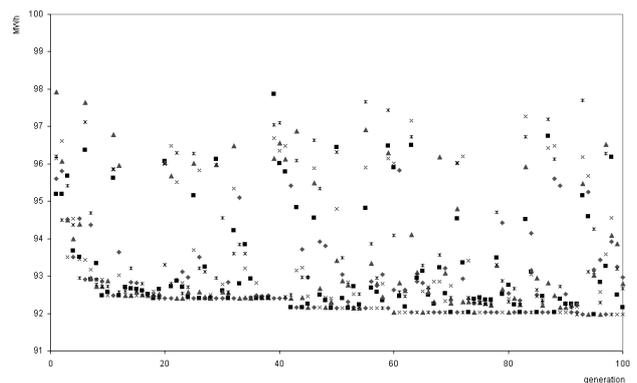


Figure 11 – Progression of GS search

### CONCLUSIONS

The Generative System proved to be flexible enough to incorporate constraints that allow the user to manipulate certain architectural design intentions. The close coincidence between GS and Siza's solutions in some situations was of particular interest. On the other hand, the departures from the existing design proposed by the

algorithm suggests that this generative system may be a useful tool in exploring multiple paths during the design process.

Another interesting dimension of the GS is its capability to account for interactions between different elements of the building and make the design for each specific element dependent on its integrated role on the architectural whole. The relations between the solutions for the loggia and the roof monitors, or between south and east facing windows in some of the studios, are a demonstration of that capability. The possibility of extrapolating from the algorithm's results to other dimensions like building geometry or spatial organization suggest new directions for further work where these aspects may also be manipulated by the generative system.

The GS was able to generate, within language constraints, solutions that have low energy consumption levels. This result can be analyzed from two perspectives: first, reducing energy consumption adds to a building's sustainability, an issue of current concern to the architecture discipline; and, second, high consumption levels indicate problems in the architecture of the building, showing that it might be poorly adapted to the climate. In situations where mechanical systems are not installed to offset those deficiencies, users will eventually suffer from discomfort inside the building. In this case, the School of Architecture has a heating system but no air-conditioning, so overheating may be an important issue for students. High artificial lighting use also works as an indicator of poor daylight control in the spaces but, as could be seen in the analysis section, this does not seem to be a problem in the existing building. Finally, the range of solutions the GS offered for the different geographical locations showed that the system is able to adapt the architectural design to the climate where it is located, even within the same language constraints.

The ultimate objective in the development of this software is its inclusion as a generative system operating in early conceptual phases of the design process. Solutions must not be interpreted as definite or optimal answers, but as diagnoses of potential problems and as suggestions for further architectural explorations, thereby building an innovative and promising interaction between architecture and computation.

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