PREDICTION MODELS OF OPTICAL CHARACTERISTICS FOR DOMED SKYLIGHTS UNDER STANDARD AND REAL SKY CONDITIONS

Abdelaziz Laouadi and Morad R. Atif

Indoor Environment Research Program

Institute for Research in Construction, National Research Council of Canada,

1200 Montreal Road Campus, Ottawa, Ontario, Canada, K1A 0R6

ABSTRACT

This paper presents analytical models to compute the optical characteristics of domed skylights under standard sky conditions and dynamic real sky conditions. The models are based on a ray-tracing technique, and can handle domed skylights of different shapes with transparent or translucent glazing. Computed dome optical characteristics under standard sky conditions (CIE overcast and clear, and IES partly cloudy skies) compare very well with those computed under corresponding dynamic real sky conditions, particularly for translucent domes. However, the dynamic sky models under-predict by about 14% the transmittance of transparent domes under clear sky conditions as compared with the standard sky model.

INTRODUCTION

The topic of this paper is part of a project to develop software to analyse the optical characteristics and daylighting performance of conventional and tubular skylights. Skylights are found in many modern or retrofitted building types. In commercial and institutional buildings, architectural skylights are used to simulate the outdoors and to bring natural light and solar heat into the indoor space. In residential buildings and houses, conventional skylights are used mainly for illumination. Skylights have the inherent potential to save electrical lighting, cooling and heating energy, beside their psychological and physiological positive effects on building occupants satisfaction (HMG 2001; Heschong and McHugh 2000; Allen 1997; AAMA 1987, 1981). However, despite these amenities, skylights integrated in building design may result in high-energy consumption if not properly designed. Skylight manufacturers lack design tools to assess the optical and daylighting performance of skylight products owing to complex skylight shapes that change with design requirements, and large sizes that impede fitting skylight products into measurement facilities. Furthermore, while measurements on some skylight products are possible, they can not be generalised for other products. In addition, fenestration software such as VISION (CANMET 1995) and WINDOW (LBL 1992) deal with only planar geometry, such as windows and flat skylights.

In this paper, analytical models are developed to predict the overall optical characteristics of domed skylights under direct and diffuse light subject to different sky conditions. The specific objectives are: (1) to predict the overall optical characteristics (transmittance, absorptance and reflectance) of domed skylights under direct beam and diffuse light; and (2) to compare the overall optical characteristics under standard sky (luminance-based) conditions with those under real (illuminance-based) sky conditions.

MATHEMATICAL FORMULATION

A domed surface is a hemispheric cap, defined by its truncation angle (σ_0) and its radius (R). Domes receive beam solar light as well as sky and ground-reflected diffuse light. The transmitted and absorbed light fluxes are dependent on the dome geometry and the beam and diffuse light source.

Beam Light Transmission

Figure 1 shows a schematic description of the beam light transmission process through a horizontal domed surface. The mathematical model has been developed to predict the optical characteristics of transparent domes (Laouadi and Atif 1998, 1999).

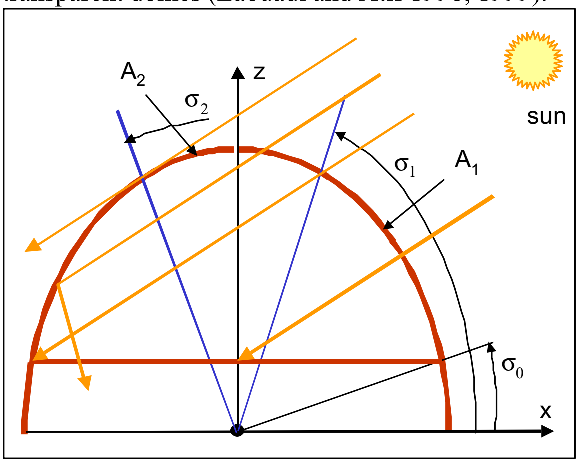

Figure 1 Transmission of beam light.

The incident beam light flux on a dome is given by:

$$Q_{dome} = \int_{A_1+A_2} E_b \cos\theta \cdot ds \qquad (1)$$

A portion of the incident beam light flux is transmitted directly to the interior space through the surface A_1 ($\sigma_0 \le \sigma \le \sigma_1$). The other portion is transmitted through the surface A_2 ($\sigma_1 \le \sigma \le \sigma_2$), and then reflected by the dome interior surface to the interior space. The transmitted and absorbed beam fluxes for transparent domes are given by:

$$QT_{dome} = \int_{A_1} E_b \cdot \tau(\theta) \cdot \cos\theta \cdot ds + \int_{A_2} E_b \tau(\theta) \rho_b(\theta) \cos\theta \cdot ds \qquad (2)$$

$$QA_{dome} = \int_{A_1+A_2} E_b \cdot \alpha_f(\theta) \cdot \cos\theta \cdot ds + \int_{A_2} E_b \tau(\theta) \alpha_b(\theta) \cos\theta \cdot ds \qquad (3)$$

The dome optical characteristics are expressed as:

$$\tau_{dome} = \frac{QT_{dome}}{Q_{dome}}; \qquad \alpha_{dome} = \frac{QA_{dome}}{Q_{dome}}; \qquad (4)$$

$$\rho_{dome} = 1 - \tau_{dome} - \alpha_{dome}$$

Equivalent Optical Characteristics

Introducing the concept of the optically equivalent flat skylight that has the same aperture surface area and yields the same transmitted, absorbed and reflected fluxes as the domed skylight, the equivalent optical characteristics are expressed as follows:

$$\tau_{eq} = \tau_{dome} \cdot \varepsilon; \; \alpha_{eq} = \alpha_{dome} \cdot \varepsilon; \; \rho_{eq} = \rho_{dome} \cdot \varepsilon \qquad (5)$$

where:

$$\varepsilon(\theta_z) = \frac{3}{4} + \{G_1 + G_2 \tan\theta_z\} / \{4\pi F_{12} F_{11}\} \qquad (6)$$

$$G_1 = \{\varphi_0(\sigma_2) - \pi/2\}\cos\sigma_2 - 2F_{12}F_{11}\,\mathrm{asin}\left(1 - 2\tan^2\sigma_0 / \tan^2\sigma_2\right) \qquad (7)$$

$$G_2 = \{\pi/2 - \varphi_0(\sigma_2)\}\sin\sigma_2 - \sin\sigma_0 \cos\varphi_0(\sigma_2) \qquad (8)$$

$$\varphi_0(\sigma_2) = \mathrm{asin}(\sin\sigma_0 / \sin\sigma_2) \qquad (9)$$

$$\sigma_1 = \min(\sigma_0 + \pi - 2\theta_z, \pi - \sigma_0); \; \sigma_2 = \min(\pi - \theta_z, \pi - \sigma_0) \qquad (10)$$

For translucent domes, the dome optical characteristics (equation 4) are independent of the incidence angle (θ_z). The beam incident flux on the dome surface is still given by equation (1). Equation (5) also holds for translucent domes by substituting the values of (τ_{dome}, α_{dome}, ρ_{dome}) by the values for the diffuse light ($\tau_{d,dome}$, $\alpha_{d,dome}$, $\rho_{d,dome}$), given by equations (15)-(17).

Diffuse Light Transmission

Consider a given diffuse light source S. Incident rays emanating from the source S undergo direct transmission and a series of inter-reflection from the dome interior surface. The directly-transmitted and inter-reflected components are both dependent on the source itself and the opacity of the glazing. For translucent glazing, incident rays undergo both direct transmission and inter-reflection. However, for transparent glazing, incident rays may undergo both direct transmission and inter-reflection, or only inter-reflection. The inter-reflection for both transparent and translucent glazing may be assumed diffuse. Figure 2 shows the diffuse light transmission through a domed surface.

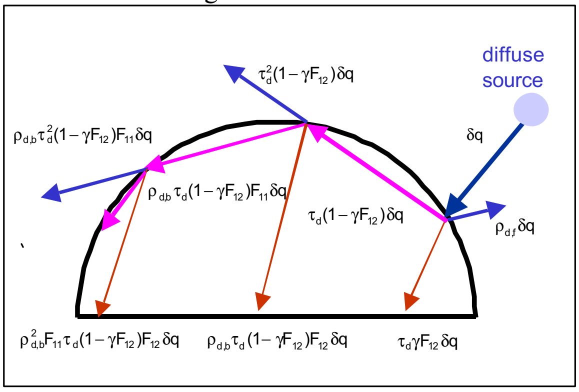

Figure 2 Transmission of diffuse light.

The incident diffuse light flux on a domed surface is expressed as follows:

$$Q_{d,dome} = \int_{A_{dome}} E_{d,t} ds \qquad (11)$$

Using the ray-tracing method, the transmitted, absorbed and reflected diffuse fluxes are expressed as follows:

$$QT_{d,dome} = \int_{A_{dome}} E_{d,t}\tau_d F_{12} \left\{ \begin{array}{l} \gamma + (1-\gamma \cdot F_{12}) \cdot ?_{d,b} \cdot \\ \left[1 + ?_{d,b}F_{11} + (?_{d,b}F_{11})^2 + \ldots\right] \end{array} \right\} ds \qquad (12)$$

$$QA_{d,dome} = \int_{A_{dome}} E_{d,t} \left\{ \begin{array}{l} \alpha_{d,f} + (1-\gamma F_{12}) \cdot \tau_d \alpha_{d,b} \cdot \\ \left[1 + ?_{d,b}F_{11} + (?_{d,b}F_{11})^2 + \ldots\right] \end{array} \right\} ds \qquad (13)$$

$$QR_{d,dome} = \int_{A_{dome}} E_{d,t} \left\{ \begin{array}{l} \rho_{d,f} + (1-\gamma F_{12})\tau_d^2 \cdot \\ \left[1 + ?_{d,b}F_{11} + (?_{d,b}F_{11})^2 + \ldots\right] \end{array} \right\} ds \qquad (14)$$

where γ is a coefficient, which depends on the light source, glazing opacity and surface inclination angle. For translucent glazing, $\gamma = 1$ for diffuse sky and ground-reflected light. For transparent glazing, γ has to be determined. The Appendix presents a method to estimate the coefficient γ.

The dome optical characteristics for diffuse light are expressed as follows:

$$\tau_{d,dome} = \overline{\gamma}F_{12} \cdot \tau_d + (1 - \overline{\gamma}F_{12}) \frac{\rho_{d,b}\tau_d \cdot F_{12}}{1 - \rho_{d,b}F_{11}} \qquad (15)$$

$$\alpha_{d,dome} = \alpha_{d,f} + (1 - \overline{\gamma}F_{12}) \frac{\tau_d \cdot \alpha_{d,b}}{1 - \rho_{d,b}F_{11}} \qquad (16)$$

$$\rho_{d,dome} = \rho_{d,f} + (1-\bar{\gamma}F_{12})\frac{\tau_d^2}{1-\rho_{d,b}F_{11}} \quad (17)$$

where $\bar{\gamma}$ is the average value of γ over the dome surface. For translucent glazing, $\bar{\gamma} = 1$ independently of the light source. However, for transparent glazing, $\bar{\gamma} = 0$ for the ground-reflected light, and is given by equation (46) for the uniform diffuse sky light (Appendix).

Diffuse Equivalent Optical Characteristics

The diffuse equivalent optical characteristics are evaluated for two types of sky conditions: (1) luminance-based sky conditions, where the sky luminance pattern is known, and (2) illuminance-based sky conditions, where the illuminance on a horizontal surface is known.

Luminance-Based Sky Models

This approach treats each luminous point on the sky as a beam source. The equivalent optical characteristics for beam light are given by equation (5) for transparent or translucent glazing. The hemispherical values of the equivalent optical characteristics are then obtained by integrating over the sky vault. However, the ground-reflected light is treated as in the upcoming section, equations (28)-(30). The total incident light flux on the dome surface from the sky and ground is given by:

$$Q_{d,dome} = \frac{A_h}{2}\int_{\eta=0}^{\eta=\pi/2}\int_{\phi=-\pi}^{\phi=\pi}\varepsilon(\eta)L(\eta,\theta_z,\phi)\sin 2\eta\, d\eta\, d\phi + \int_{A_{dome}} E_{gr} ds \quad (18)$$

The horizontal diffuse illuminance is given by:

$$E_{dh} = 1/2\int_{\eta=0}^{\eta=\pi/2}\int_{\phi=-\pi}^{\phi=\pi} L(\eta,\theta_z,\phi)\cdot\sin 2\eta\cdot d\eta\cdot d\phi \quad (19)$$

The diffuse equivalent optical characteristics are:

$$\tau_{eq,d} = \frac{1}{2E_{dh}}\int_{\eta=0}^{\eta=\pi/2}\int_{\phi=-\pi}^{\phi=\pi}\tau_{eq}(\eta)\cdot L(\eta,\theta_z,\phi)\cdot\sin 2\eta\cdot d\eta\cdot d\phi + \tau_{d,dome}(\bar{\gamma}_{gr})\cdot\varepsilon_{d,gr} \quad (20)$$

$$\alpha_{eq,d} = \frac{1}{2E_{dh}}\int_{\eta=0}^{\eta=\pi/2}\int_{\phi=-\pi}^{\phi=\pi}\alpha_{eq}(\eta)\cdot L(\eta,\theta_z,\phi)\cdot\sin 2\eta\cdot d\eta\cdot d\phi + \alpha_{d,dome}(\bar{\gamma}_{gr})\cdot\varepsilon_{d,gr} \quad (21)$$

$$\rho_{eq,d} = \frac{1}{2E_{dh}}\int_{\eta=0}^{\eta=\pi/2}\int_{\phi=-\pi}^{\phi=\pi}\rho_{eq}(\eta)\cdot L(\eta,\theta_z,\phi)\cdot\sin 2\eta\cdot d\eta\cdot d\phi + \rho_{d,dome}(\bar{\gamma}_{gr})\cdot\varepsilon_{d,gr} \quad (22)$$

Illuminance-Based Sky Models

Domed surfaces receive sky diffuse light as well as surrounding/ground-reflected light. The incident total flux on the dome surface is expressed as follows:

$$Q_{d,dome} = \int_{A_{dome}} E_{gr} ds + \int_{A_{dome}} E_{sky} ds \quad (23)$$

The sky luminous flux may be decomposed into three components: a background uniform flux, circumsolar flux and horizon brightening flux. For translucent glazing, the surrounding/ground-reflected flux and the three component of the sky luminous flux undergo both direct transmission and inter-reflection. For transparent glazing, however, the ground-reflected and horizon brightening fluxes only undergo inter-reflection. The circumsolar flux is treated as beam light. Equation (23) reads as follows:

$$Q_{d,dome} = \int_{A_{dome}}(E_{gr}+E_{su}+E_{hb})ds + \int_{A_1+A_2} E_{cs}\cos\theta\cdot ds \quad (24)$$

The total transmitted, absorbed and reflected fluxes read as follows:

$$QT_{d,dome} = \tau_{d,dome}(\bar{\gamma}_{gr})\int_{A_{dome}}(E_{gr}+E_{hb})ds + \tau_{d,dome}(\bar{\gamma}_{su})\int_{A_{dome}} E_{su} ds + \tau_{dome}\int_{A_1+A_2} E_{sc}\cos\theta\cdot ds \quad (25)$$

$$QA_{d,dome} = \alpha_{d,dome}(\bar{\gamma}_{gr})\int_{A_{dome}}(E_{gr}+E_{hb})ds + \alpha_{d,dome}(\bar{\gamma}_{su})\int_{A_{dome}} E_{su} ds + \alpha_{dome}\int_{A_1+A_2} E_{sc}\cos\theta\cdot ds \quad (26)$$

$$QR_{d,dome} = \rho_{d,dome}(\bar{\gamma}_{gr})\int_{A_{dome}}(E_{gr}+E_{hb})ds + \rho_{d,dome}(\bar{\gamma}_{su})\int_{A_{dome}} E_{su} ds + \rho_{dome}\int_{A_1+A_2} E_{sc}\cos\theta\cdot ds \quad (27)$$

where for translucent glazing $\bar{\gamma}_{gr} = \bar{\gamma}_{su} = 1$ for the ground-reflected and sky diffuse fluxes. For transparent glazing, $\bar{\gamma}_{gr} = 0$ for the ground-reflected and horizon-brightening fluxes, and $\bar{\gamma}_{su}$ is given by equation (47) for the background sky diffuse light.

The diffuse equivalent optical characteristics for the combined sky and ground-reflected light are given by:

$$\tau_{eq,d} = \tau_{d,dome}(\bar{\gamma}_{gr})\cdot\{\varepsilon_{d,gr}+\varepsilon_{d,hb}\} + \tau_{d,dome}(\bar{\gamma}_{su})\cdot\varepsilon_{d,su} + \tau_{eq}\cos\theta_z E_{cs}/E_{dh} \quad (28)$$

$$\alpha_{eq,d} = \alpha_{d,dome}(\bar{\gamma}_{gr})\cdot\{\varepsilon_{d,gr}+\varepsilon_{d,hb}\} + \alpha_{d,dome}(\bar{\gamma}_{su})\cdot\varepsilon_{d,su} + \alpha_{eq}\cos\theta_z E_{cs}/E_{dh} \quad (29)$$

$$\rho_{eq,d} = \rho_{d,dome}(\bar{\gamma}_{gr})\cdot\{\varepsilon_{d,gr}+\varepsilon_{d,hb}\} + \rho_{d,dome}(\bar{\gamma}_{su})\cdot\varepsilon_{d,su} + \rho_{eq}\cos\theta_z E_{cs}/E_{dh} \quad (30)$$

where ε_d is the ratio of the incident diffuse light on the dome surface from a given source to the one incident on a planar horizontal surface having the same aperture surface area. This is expressed as follows:

$$\varepsilon_{d,x} = \frac{1}{A_h} \int_{A_{dome}} \frac{E_x}{E_{dh}} ds \quad (31)$$

where E_x stands for the illuminance on a tilted surface from a given light source (x).

Now the incident light flux on a tilted surface is evaluated for different sky conditions:

For ground reflected light:

$$E_{gr} / E_{dh} = \rho_g \cdot E_{gh} / E_{dh} \cdot (1 - \cos\beta)/2 \quad (32)$$

For isotropic overcast skies (IESNA 1993):

$$E_{su} / E_{dh} = (1 + \cos\beta)/2 \quad (33)$$

For CIE standard overcast skies (Wilkinson 1992; Muneer and Angus 1993):

$$E_{su} / E_{dh} = \frac{3}{14}(1 + \cos\beta) + \frac{4}{7\pi}\{\sin\beta + (\pi - \beta)\cos\beta\} \quad (34)$$

For anisotropic diffuse skies (Perez et al. 1990):

$$E_{su} / E_{dh} = (1 - F_1)(1 + \cos\beta)/2 \quad (35)$$

$$E_{cs} / E_{dh} = F_1 / \max(0.087 , \cos\theta_z) \quad (36)$$

$$E_{hb} / E_{dh} = F_2 \cdot \sin\beta \quad (37)$$

where F_1 and F_2 are coefficients for the circumsolar and horizon brightness, respectively.

The incident flux ratio (ε_d) is now evaluated for the above-mentioned sky conditions:

For ground-reflected light,

$$\varepsilon_{d,gr} = \rho_g \frac{E_{gh}}{2E_{dh}}(1/F_{12} - 1) \quad (38)$$

For isotropic overcast skies ($E_{cs} = E_{hb} = 0$),

$$\varepsilon_{d,su} = (1/F_{12} + 1)/2 \quad (39)$$

For CIE standard overcast skies ($E_{cs} = E_{hb} = 0$),

$$\varepsilon_{d,su} = 9/(28\pi F_{12} F_{11}) \cdot \begin{Bmatrix} 19\pi/18 - 2\sigma_0/9 - \pi/3 \cdot \sin\sigma_0 - \\ (7\pi + 8\sigma_0)/18 \cdot \sin^2\sigma_0 - 1/3 \cdot \sin 2\sigma_0 \end{Bmatrix} \quad (40)$$

For anisotropic skies,

$$\varepsilon_{d,su} = (1 - F_1)(1/F_{12} + 1)/2 \quad (41)$$

$$\varepsilon_{d,hb} = F_2\{\pi/2 - \sigma_0 - \cos\sigma_0 \sin\sigma_0\}/(4F_{12}F_{11}) \quad (42)$$

RESULTS AND ANALYSIS

The previously developed models are applied to predict the equivalent optical characteristics of fully hemispheric domes ($\sigma_0 = 0°$). A comparison of the equivalent optical characteristics is made between luminance-based and illuminance-based models during typical summer and winter days in the Ottawa region (latitude = 45°), Canada. Two glazing options are selected: (1) transparent glazing with double clear float glass (at $\theta_z = 0°$, $\tau = 0.87$ and $\rho_f = \rho_b = 0.08$) (Pilkington, 1988), and (2) translucent glazing with similar diffuse total optical characteristics as the double clear glazing ($\tau_d = 0.66$ and $\rho_{d,f} = \rho_{d,b} = 0.23$). Sky conditions include: (1) uniform overcast skies, (2) CIE overcast skies, (3) IES partly cloudy skies (IESNA 2000), (4) CIE clear skies with pollutant air (turbidity = 5.5), (5) CIE clear with clean air (turbidity = 2.45) (CIE 1995) and (6) dynamic real skies that change with daytime (Perez et al. 1990).

Figure 3 shows the profiles of the equivalent transmittance and absorptance for beam light as a function of the incidence angle on a horizontal surface for translucent and transparent domes. The transmittance and absorptance profiles of a transparent flat skylight (with double clear glass) are also plotted in the figure. Contrary to flat skylights, transparent domes transmit and absorb substantially more beam light at high incidence angles (i.e., low sun altitudes). For instance, at an incidence angle $\theta_z = 70°$ (e.g., winter days at noontime), transparent domes may transmit up to 78% and absorb up to 150% more beam light than similar flat skylights. At near normal incidence angles (e.g., summer days at noontime), transparent domes may transmit, however, about 13% less beam light than similar flat skylights. Transparent domes transmit up to 78% more beam light than translucent domes for incidence angles $\theta_z < 70°$. Translucent domes absorb about 36% more beam light than transparent domes, particularly at normal incidence angles.

Table 1 compares the diffuse equivalent optical characteristics under uniform and CIE overcast skies using the luminance-based model (equations 20-22) with those under the same conditions using the illuminance-based model (equations 28-30). Both luminance-based and illuminance-based models yield approximately the same results, particularly for domes with translucent glazing. The models predict that translucent domes transmit about 15% under uniform overcast skies and 23% under CIE overcast skies less diffuse light than flat skylights with similar glazing. However, transparent domes transmit about 18% under uniform overcast skies and 11% under CIE overcast skies more than similar flat skylights. Transparent domes transmit up to 44% more than translucent domes under overcast skies.

Figures 4 and 5 show the daily profile of the ratio of the diffuse equivalent transmittance (absorptance) to that of a flat glazing for translucent and transparent domes during a typical summer day (June 24) in Ottawa, Canada. The luminance-based models (partly cloudy and CIE clear skies) predict that transparent domes transmit about 28% under standard CIE clear skies and 15% under IES partly cloudy skies more than under standard CIE overcast skies, particularly at low sun altitudes. Transparent domes absorb about 47% under standard CIE clear skies and 25% under IES partly cloudy skies more than under standard CIE overcast skies, particularly at low sun altitudes. Translucent domes transmit about 36% under standard CIE clear skies and 19% under standard IES partly cloudy skies more than under standard CIE overcast skies, particularly at low sun altitudes. Translucent domes absorb about 37% under standard CIE clear skies and 20% under standard IES partly cloudy skies more than under standard CIE overcast skies, particularly at low sun altitudes. The dynamic sky model predicts the equivalent transmittance and absorptance close to that under the IES partly cloudy skies, which was the case on June 24.

Figures 6 and 7 show the daily profile of the ratio of the diffuse equivalent transmittance (absorptance) to that of a flat glazing for translucent and transparent domes during a typical winter day (December 24) in Ottawa, Canada. The luminance-based models (partly cloudy and CIE clear skies) predict that transparent domes transmit about 36% under standard CIE clear skies and 18% under IES partly cloudy skies more than under standard CIE overcast skies. Transparent domes absorb about 59% under standard CIE clear skies and 32% under IES partly cloudy skies more than under standard CIE overcast skies. Translucent domes transmit and absorb about 45% under standard CIE clear skies and 25% under standard IES partly cloudy skies more than under standard CIE overcast skies. For translucent domes, the dynamic sky model predicts the equivalent transmittance and absorptance close to that under the CIE clear skies, which was the case on December 24. However, for transparent domes under clear skies, the dynamic sky model overpredicts (underpredicts) by about 14% the equivalent transmittance (absorptance) as compared with the luminance-based model.

ACKNOWLEDGEMENTS

This work was conducted as part of the project B3211, funded by PERD, the Institute for Research in Construction of the National Research Council Canada, and Natural Resources Canada. The authors are very thankful for their contribution.

CONCLUSION

Analytical models based on the ray-tracing technique were developed to predict the optical characteristics of transparent and translucent domed skylights under standard (luminance-based) and dynamic, real (illuminance-based) sky conditions. Predictions of the luminance-based models compare very well with those of the illuminance-based models for either transparent or translucent domes under overcast and partly cloudy skies. However, the illuminance-based models under-predict the transmittance of transparent domes by up to 14% under clear skies as compared with the luminance-based models. Laboratory validation is further recommended.

REFERENCES

AAMA, Skylight Handbook, Design Guidelines. American Architectural Manufacturers Association, 1987.

AAMA, Design for Energy Conservation with Skylights. Architectural Aluminium Manufacturers Association, 1981.

Allen T., Conventional and tubular skylights: An Evaluation of the daylighting systems at two ACT commercial buildings, Pro. 22nd Nat. Pass. Solar Conference, Washington DC, pp. 97-129, 1997.

CANMET, VISION 4 Reference Manual, Canadian Centre for Mineral and Energy Technology, 1995.

CIE, Spatial Distribution of Daylight -Luminance Distributions of Various Reference Skies, Technical Report, 1995.

Heschong L. and McHugh J., Skylights: Calculating illumination levels and energy impacts, Illuminating Engineering Society, pp. 90-1000, Winter 2000.

HMG, Skylighting design guidelines (www.h-m-g.com), 2001.

IESNA, Lighting Handbook, Reference And Application Volume. New York: Illuminating Engineering Society of North America, 2000.

Jensen T, Skylights. Pennsylvania: Running Press, 1983.

Laouadi A., and Atif M.R., Predicting optical and thermal characteristics of transparent single-glazed domed Skylights. ASHRAE Transactions, 105(2), 325-333, 1999.

Laouadi A., and Atif M.R., Transparent domed Skylights: Optical Model For Predicting Transmittance, Absorptance And Reflectance, Lighting Research and Technology, 30(3), 111-118, 1998.

LBL, WINDOWS 4.0 User Manual. Berkeley: Lawrence Berkeley Laboratory, 1992.

Pilkington, Glass and Transmission Characteristics of Windows, 7th edition. England, Harwills, 1988.

Perez R, Ineichen P, Seals R, Michalsky J, and Stewart R, Modeling Daylight Availability and

Irradiance Components from Direct and Global Irradiance, Solar Energy 44(5) 271-289, 1990.

Muneer T, and Angus RC, Daylight Illuminance Models for the United Kingdom. Lighting Res. Technol. 25(3) 113-123, 1993.

Wilkinson MA, Natural Lighting under Translucent Domes, Lighting Res. Technology 24(3) 117-126, 1992.

NOMENCLATURE

A_1, A_2: dome surface portions that correspond to the directly-transmitted and transmitted-reflected beam components, respectively.

A_{dome} : dome surface.

A_h : area of the dome base surface.

ds : elementary surface.

F_{11}, F_{12}: view factors of the dome interior surface to itself, and to its base surface, respectively.

E_b : beam illuminance (lux).

E_{dh} : diffuse horizontal illuminance (lux).

E_{gh} : global horizontal illuminance (lux).

$E_{d,t}$: diffuse illuminance on a tilted surface (lux).

E_{gr} : ground-reflected illuminance on a tilted surface (lux).

E_{su}: background sky illuminance on a tilted surface (lux).

E_{cs} : circumsolar sky illuminance on a tilted surface (lux).

E_{hb} : horizon brightening sky illuminance on a tilted surface (lux).

L : sky luminance (cd/m^2).

Greek Symbols

α, ρ, τ: total absorptance, reflectance and transmittance of a flat glazing, respectively.

β : surface inclination angle from the horizontal.

ε : ratio the beam incident flux on a dome surface to that on horizontal planar surface.

γ : coefficient indicating the fraction of the directly transmitted flux to the incident one.

η, ϕ : position and relative azimuth angles of a luminous point on the sky.

θ : incidence angle on surface ds.

θ_z : sun zenith angle, or incidence angle on a horizontal surface.

ρ_g : ground reflectance (albedo).

σ : inclination angle of a plane perpendicular to the plane of the sun's rays.

σ_0 : dome truncation angle.

σ_1, σ_2 : angles that delimit the surfaces A_1 and A_2.

Subscripts

b, d : beam and diffuse light.

f, b : front and back surfaces of a glazing.

dome : dome surface.

eq : dome optically-equivalent planar surface.

APPENDIX

Figure 8 shows a method to estimate the coefficient γ for a uniform diffuse sky. The directly transmitted portion of the incident flux on a given point on the dome surface is proportional to the angle B. The coefficient γ is then given by the following equation:

$$\gamma = \frac{B / F_{12}}{A + B + C} = \frac{1 + 2\sigma_0 / \pi}{1 + 2\sigma / \pi} / F_{12} \qquad (43)$$

The average value of γ over the dome surface is:

$$\bar{\gamma} = \frac{1}{A_{dome}} \int_{A_{dome}} \gamma \cdot ds = \frac{1/2}{F_{12}F_{11}} \int_{\sigma_0}^{\pi/2} \frac{1 + 2\sigma_0 / \pi}{1 + 2\sigma / \pi} \cos\sigma \cdot d\sigma \qquad (44)$$

Equation (45) may be simplified as follows:

$$\bar{\gamma} = \frac{1 + 2\sigma_0 / \pi}{2F_{12}F_{11}} \int_{\sigma_0}^{\pi/2} (1 - 1.7\sigma / \pi + 1.45\sigma^2 / \pi^2) \cos\sigma \cdot d\sigma \qquad (45)$$

which reduces to:

$$\bar{\gamma} = \frac{1 + 2\sigma_0 / \pi}{2F_{11}F_{12}} \begin{Bmatrix} 2F_{11} - 1.7 / \pi(\pi / 2 - \cos\sigma_0 - \sigma_0 \sin\sigma_0) + 1.45 / \\ \pi^2 \cdot \left(\pi^2 / 4 - 2 - 2\sigma_0 \cos\sigma_0 - (\sigma_0^2 - 2)\sin\sigma_0\right) \end{Bmatrix} \qquad (46)$$

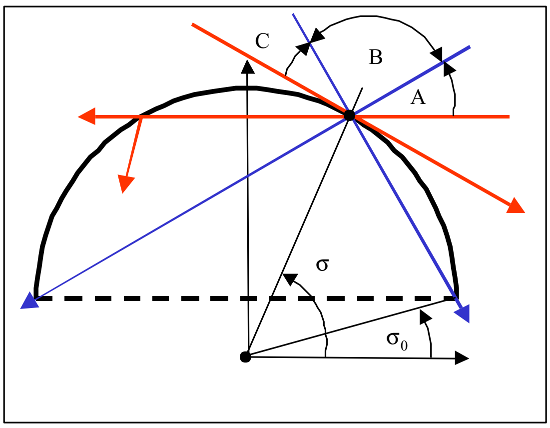

Figure 8 Method to estimate the coefficient γ

Table 1 Diffuse equivalent optical characteristics of domes with transparent (double clear glass) and translucent glazing (τ_d=0.66, ρ_d=0.23) - comparison between luminance-based and illuminance-based sky models.

	Luminance-based skies				***Illuminance-based skies***			
	Translucent Glazing		Transparent Glazing		Translucent Glazing		Transparent Glazing	
	uniform overcast	CIE overcast	uniform overcast	CIE overcast	uniform overcast	CIE overcast	Uniform overcast	CIE overcast
$\tau_{eq,d} / \tau_d$	0.85	0.77	1.18	1.11	0.85	0.77	1.18	1.07
$\alpha_{eq,d} / \alpha_d$	2.06	1.86	1.80	1.58	2.06	1.86	1.77	1.61
$\rho_{eq,d} / \rho_d$	3.10	2.81	2.27	2.09	3.10	2.81	2.28	2.21

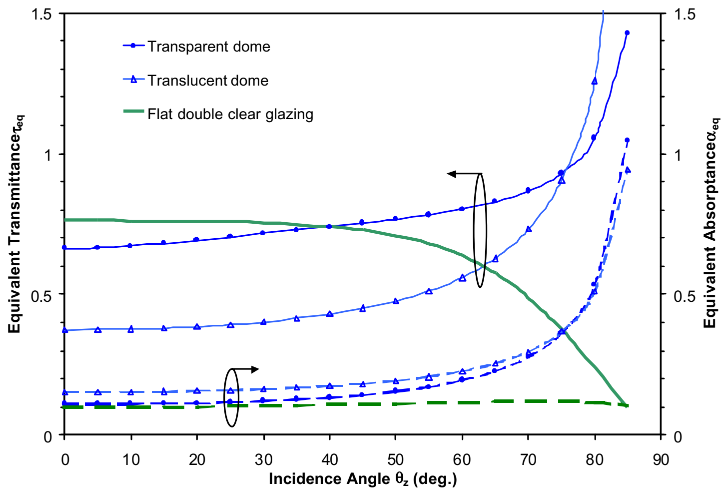

Figure 3 Profiles of the equivalent transmittance and absorptance for beam light as a function of the incidence angle on a horizontal surface.

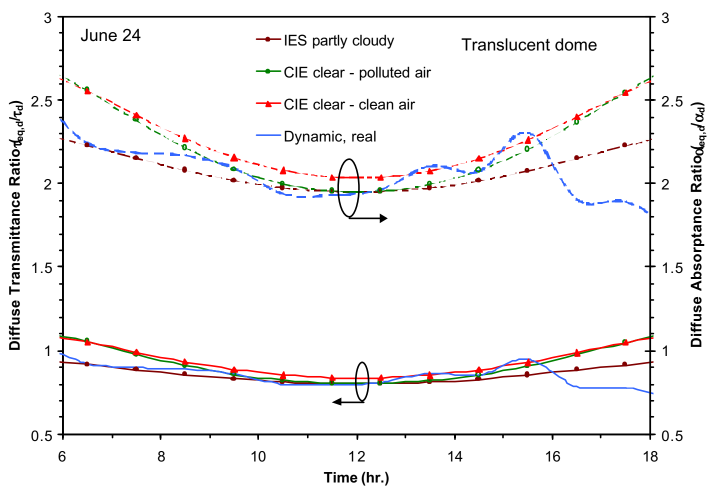

Figure 4 Daily profiles of the diffuse transmittance and absorptance ratios for translucent domes under standard and dynamic sky conditions during a summer day.

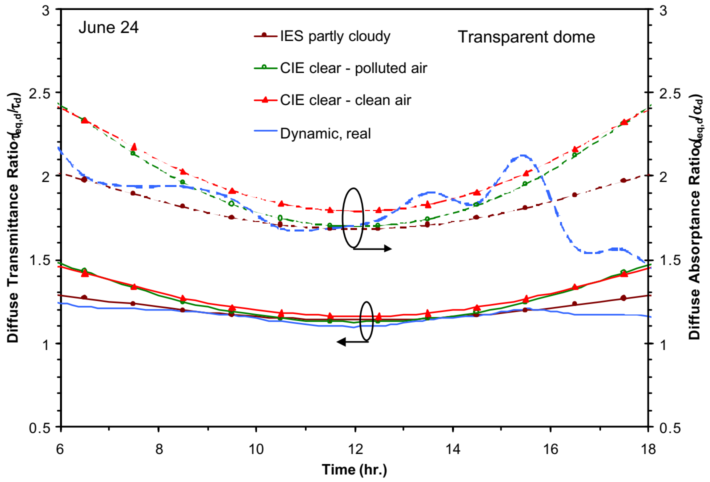

Figure 5 Daily profiles of the diffuse transmittance and absorptance ratios for transparent domes under standard and dynamic sky conditions during a summer day.

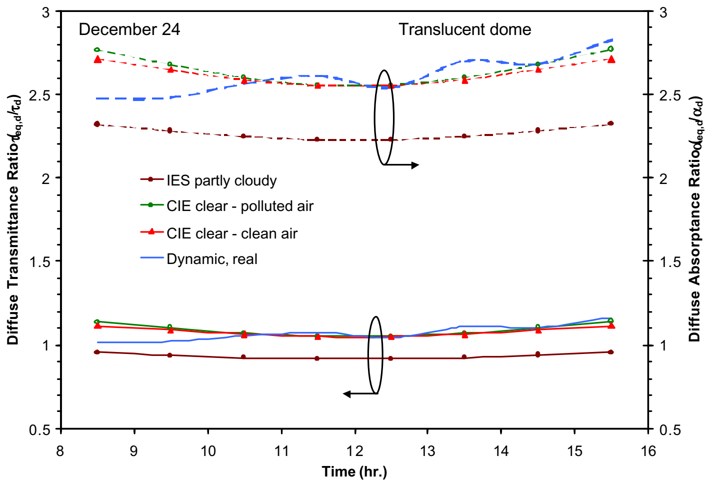

Figure 6 Daily profiles of the diffuse transmittance and absorptance ratios for translucent domes under standard and dynamic sky conditions during a winter day.

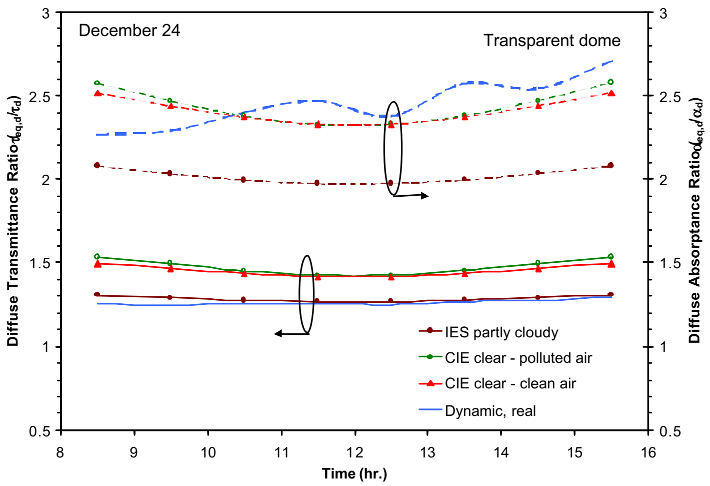

Figure 7 Daily profiles of the diffuse transmittance and absorptance ratios for transparent domes under standard and dynamic sky conditions during a winter day.