MOISTURE CONTENT INFLUENCE ON THERMAL CONDUCTIVITY OF POROUS BUILDING MATERIALS

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ABSTRACT

The present work deals with the determination of a mathematical correlation for conductivity in the fully water-saturated state in terms of dry-basis conductivity and porosity. In the mathematical model, the material microstructure is taken into account in a multiscale percolation system and the macroscopical conductivity is obtained with a renormalization technique. The model is presented and the obtained correlation is tested for some porous building materials. To conclude, we show how porosity can affect thermal conductivity.

INTRODUCTION

Effective thermal conductivity is an important diffusive transport coefficient to evaluate the coupled heat and moisture transfer through porous walls so that conduction heat fluxes can be precisely calculated. This heat transport coefficient in a porous material can be described in terms of the conductivity of solid matrix and fluid phases and their quantities, phase change phenomena and spatial organization of the phases.

Generally, the available data in the literature is the dry-basis thermal conductivity or for very low moisture content and the total porosity of the material. Thus, the present work deals with the determination of a mathematical correlation for conductivity in the fully water-saturated state in terms of dry-basis conductivity and porosity. In the mathematical model, the material microstructure is taken into account in a multiscale percolation system and the macroscopical conductivity is obtained with a renormalization technique.

The model is introduced and results are compared with experimental data for some common porous materials used in civil construction and with correlations obtained by the geometric mean and by Krupiczka's model (Kaviany, 1995).

We also analyse the moisture effects on thermal conductivity for some materials and observe that can be really significant and not neglectable on building thermal performance simulation.

MODEL BASED ON THE GEOMETRIC MEAN

In a first attempt to evaluate the porous medium thermal conductivity when it is fully saturated of water (λ_{sat})) and to also evaluate the thermal conductivity of the solid grains (λ_s), it was studied simplified models such as models based on the arithmetic and harmonica means or based in DeVries' theory (1952), assuming lamellar, fibrous and spherical grains, but it was noticed that all of them were unsatisfactory and that when they didn't underestimate λ_{sat} and λ_s, the results were physically inconsistent.

Therefore, it was considered a model based on the geometric mean of the medium components, as follows:

$$\lambda_{dry} = \lambda_s^{(1-\eta)}\,\lambda_{air}^{\eta} \quad \text{(I)},$$

$$\lambda_{sat} = \lambda_s^{(1-\eta)}\,\lambda_{H_2O}^{\eta} \quad \text{(II)}, \qquad (1)$$

or explicitly for λ_s:

$$\lambda_s = \left(\frac{\lambda_{dry}}{\lambda_{air}^{\eta}}\right)^{\frac{1}{(1-\eta)}} \qquad (2)$$

Thus, we see from eq. (2) that with data for porosity (η) and dry-basis thermal conductivy (λ_{dry}), we can calculate the phase solid thermal conductivity. Consequently, with the value for λ_s, we can determine by using eq. (1.II), the thermal conductivity for water-saturated medium (λ_{sat}).

Table 1 supplies experimental values of thermal conductivity for different materials in the dry and saturated conditions and respective porosities. Table 2 uses these values to calculate the conductivities of solid phase and fully-wetted mediun by using the geometric mean correlations.

The models are studied for 3 building materials which have the necessary data for validation. The Fernandes' (1990) mortar (MTR1) it is a material composed of 20% of water with fine sand, whitewash and cement in the proportions of 8:2:1, in terms of mass, with a porosity of 31% and a density (dry-basis) of 1710 kg/m³. The Perrin's (1985) cement mortar (MTR2) has density of 2050 kg/m³, porosity of 18% and with the following composition in terms of mass: 1 part of cement portland, 3 parts of sand and ½ part of water; it is constituted, predominantly, of mesopores ($20\ \mathring{A} < raio < 500\ \mathring{A}$), reflecting a highly hygroscopic behavior. The Perrin's (1985) brick has a high number of macropores that provide to it a little hygroscopic behavior. Its density is 1900 kg/m³ and the total porosity is 29%.

In Table 2, we notice that the geometric mean can provide good results, contrary to the arithmetic and harmonic ones.

Next we present a model based on the renormalization method and then the two models presented are compared at the end of the article.

MODEL BASED ON THE RENORMALIZATION METHOD FOR MULTISCALE PERCOLATION SYSTEMS

Multiscale Percolation Systems (MPS) are used to represent the microstructure of porous materials. A description of MPS models as well as their geometric properties is given by Fernandes et Al. (1996, 2000).

MPS is built in such way to keep up an imposed pore size distribution. Each MPS model scale corresponds to a random distribution of a pore size class. However, the several scales generation (pore size classes) produce a spatially correlated structure for displacements lower than the largest pores.

For a given MPS and the thermal conductivities of the 2 phases (solid and water/air), it is possible to determine, by using the renormalization method, the MPS thermal conductivity in a short computer run time.

In this article, it is shown how to determine the conductivities for solid phase and for the fully water-saturated medium, from the porosity and the dry-basis thermal conductivity, by using the renormalization technique for MPS.

Several authors have used renormalization technique for porous materials conductive properties evaluation. The term conductive property is general and it could designate the thermal, electric or hydraulic conductivity (or the intrinsic permeability) among other designations. King (1989) and Hinrichsen et Al. (1993) used the renormalization method for the intrinsec permeability determination in monoscale cubic percolation networks from the previous knowledge of elementary permeabilities of each network element. In King (1989), the system is seen as just-one scale system, represented for a mesh (squared or cubic), having a random distribution of permeabilities. Xu et Al. (1997a, 1997b) used renormalization for intrinsic permeability determination of great number of reconstructed materials in a MPS model. MPS 's structure was obtained from Mercury intrusion curves. Fernandes et al. (2000), from 2-D section binary images of petroleum reservoir rocks , went forward for the determination of pore size distribution (with mathematical morphology techniques), reconstruction in MPS and evaluation of intrinsic permeability with renormalization method.

In order to illustrate the renormalization technique idea (see King, 1989 and Hinrichsen et al.,1993), consider one system, one network, as it is shown in Fig.1 where each element or block of this system has a given conductivity (Fig. 1.a). It is considered also that the conductivity values are randomly distributed along the network. The effective thermal conductivity of a 4-block group (or 8 blocks in a 3-D case)) of the original network is explicitly evaluated before going to a higher scale.

Considering, for example, the conductivities grouping K_{a1}, K_{a2}, K_{a3} and K_{a4} we can calculate the effective conductivity K_a that represents the same heat flux for the four original blocks at the same temperature difference. This scale change process is repeated until a thermal conductivity stable result is reached. This result corresponds to the effective conductivity of the original randomly distributed network. Clearly, it is possible to directly solve the linear equations system associated to the conductivities network as a whole, however, for big networks, it is required substantial computational efforts (processing time and memory) according to Hinrichsen et al., 1993.

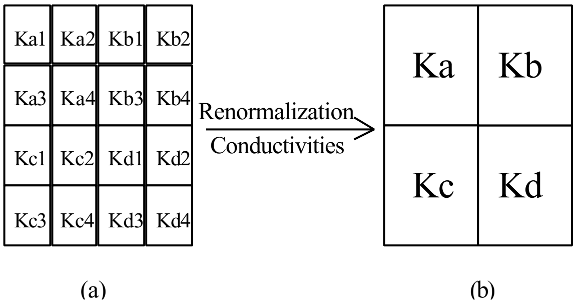

Figure 1. In (a) the original network of conductivities. In (b) the renormalized network with the effective conductivities.

SOME RESULTS WITH THE RENORMALIZATION METHOD IN MPS

In this section, renormalization method is used in random and correlated MPS structures.

The medium is called *correlated 1* when has an equally divided volume distribution for each of the 5 classes, while the medium called *correlated 2* presents volume fractions for porosity of 20% equal to:V_1=0.08, V_2= 0.02, V_3= 0.06, V_4= 0.02 and V_5= 0.02. However, for the 70% porosity medium, it was considered the folllowing volume fractions: V_1=0.20, V_2= 0.15, V_3= 0.20, V_4= 0.05 e V_5= 0.10.

Table 3 presents comparisons between 1 and 5-scale percolation systems with 2 different random number generator seeds, for thermal conductivity of both dry and fully-wetted media with porosities of 0.2 and 0.7.

Table 3 shows that thermal conductivity is slightly sensitive to the generator seed choice. The highest difference is observed between the correlated and random media, especially for high-porosity dry media. For fully-wetted media, the differences are very small since the ratio between the conductivities of grain and fluid decrease by a factor of 23.5.

Tables 4 and 5 present average values of thermal conductivity (λ_{dry} and λ_{sat}) – obtained by simulations of correlated and random media – with different generation seeds. The errors presented in those two tables are relative to the thermal conductivity value of the highest network dimension NX.

We notice from Tables 4 and 5 that errors decrease with the porosity. The increase of grain conductivity – λ_s – results in great errors on the effective thermal conductivity because the ration between the conductivities of the solid and fluid phases - λ_s/λ_f - are increased.

Figures 2 and 3 show convergence to determine the medium effective conductivity as a function of step numbers of the renormalization process for dry and wet media (media fully satureted of water). We note that for wet media, the convergence is rapidly attained with a smaller number of steps.

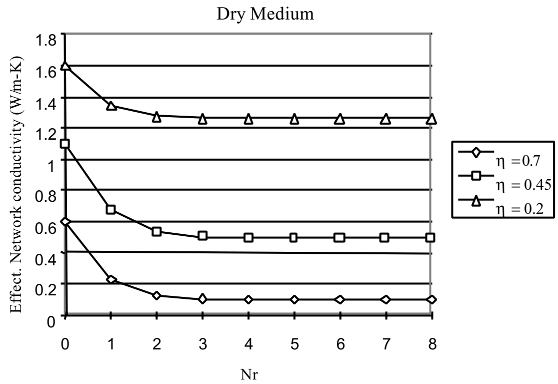

Figure 2: Effective thermal conductivity - λ_{ef} – as a function of step numbers of the renormalization process for a dry medium with λ_s=2 W/m-K.

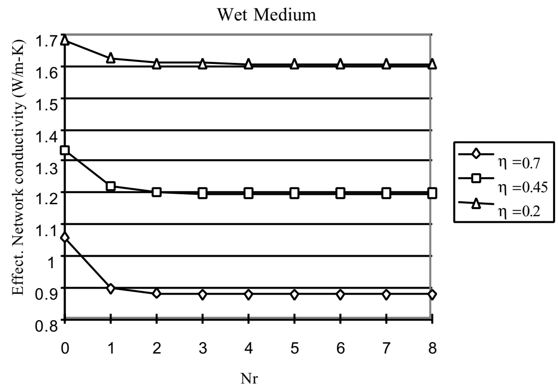

Figure 3: Effective thermal conductivity - λ_{ef} – as a function of step numbers of the renormalization process for a wet medium with λ_s=2 W/m-K.

MATHEMATICAL CORRELATIONS DEVELOPMENT

As it was seen in the geometric mean based model, we can write the medium thermal conductivity as a function of the conductivities of each phase (solid and air/liquid) and porosity. Thus, we can write:

$$\lambda_{dry}=f(\lambda_s, \lambda_{air}, \eta) \quad (3)$$

and

$$\lambda_{sat}=f(\lambda_s, \lambda_{H2O}, \eta) \quad (4)$$

Expressions (3) e (4) can be reduced to a first one by adimensionalization:

$$\pi_1 = f(\pi_2, \pi_3), \quad (5)$$

where: $\pi_1 = \lambda_{ef} / \lambda_f$;

$\pi_2 = \eta$;

$\pi_3 = \lambda_s / \lambda_f$.

The conductivity λ_{ef} can represent either λ_{dry} or λ_{sat}. On the other hand, λ_f denotes λ_{air} when the medium is dry, or λ_{H2O} when the medium is fully saturated of water.

However, it is necessary to create a reasonable quantity of data to determine mathematical correlations capable to represent Eq. 5 in space, by using renormalization group techniques. The first step was to establish the range of the ratio λ_s / λ_f as well as η.

According to Kaviany (1991), building materials that are commonly used have effective thermal conductivities between 0.1 e 1 W/m-K. We have adopted this range, assuming the medium is fully saturated of water, for the dimensionless number π_1 ranging between 3 and 14, for π_3 between 0.82 and 17.18 and finally for π_2 between 0.1 and 0.7. Applying the renormalization process to those ranges, we obtain the points plotted on Fig. 4.

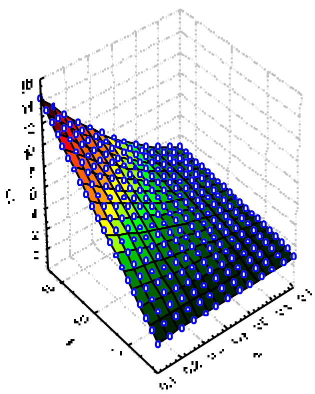

Figure 4: The dimensionless groups π_1, π_2, π_3 obtained by the renormalization process for $\pi_1 = f(\pi_2, \pi_3)$.

Those points plotted in Fig. 4 can be mathematically represented, with a correlation factor of R=0.99997, as:

$$\ln \pi_1 = -0.12578 + Z \ln \pi_2 \quad (6)$$

where:

$$Z = -5.18776 \exp\left(0.225433 \pi_3^{0.196297}\right) - 0.023289 \left(\pi_3^{0.707251}\right) \ln(\pi_2) - 0.975144 (\pi_2) \ln(\pi_3) + 6.361418$$

Krupiczka (Kaviany, 1995) presented the following correlation for a packed bed of spheres:

$$\pi_1 = \pi_3^{(0.280 - 0.757 \log(\pi_2) - 0.057 \log(\pi_3))} \quad (7)$$

In Fig. 5, we compare Eq. (6) with Krupiczka's correlation (Eq. 7). We observe they are in very good agreement for low values of π_3. Prasad et al. (1989), cited by Kaviany (1991), compared Krupiczka's correlation with empirical correlations obtained from experimental work and observed a good agreement for π_3 not greater than 2000.

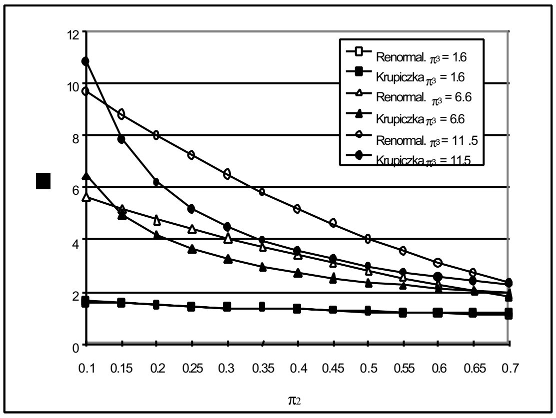

Figure 5: Comparison between Krupiczka's correlation and the correlation obtained by the renormalization method

We obtained, for the same data, an explicit function for π_3 as a function of π_1 and π_2 with a correlation factor of R=0.9999999,

$$\pi_3 = \ln\left(0.001507\pi_2 + 8.074095\pi_1\right)\left(0.306726\pi_1^{0.836455} - 0.000004\ln(\pi_2) + 0.172689\right) \quad (8)$$

Correlations (6) and (8) are valid for media entirely saturated of water (π_3 lower than 17). For values superior to 17 and lower than 200, the following correlation must be used:

$$\pi_3 = a_0 \pi_1^{(a_1 \exp(a_2 \pi_1^{a3} \pi_2^{a4}) + a_5)} + a_6 \pi_1^{a_7} \ln(\pi_2) + a_8 \pi_1 \pi_2 + a_9 \quad (9)$$

Eq. (9) is divided into 2 parts. The first one is valid for $0.1 \le \pi_2 < 0.4$ (R=0.99998), and the second one for $0.4 \le \pi_2 \le 0.7$ (R=0.99999). The coefficients (Table 6) of Eq. (9) were obtained by using 7000 points extracted from renormalization process simulations of cubic matrices with dimensions of 256x256x256.

Table 7 exhibits thermal conductivity values for 3 different materials, obtained by the renormalization method, geometric mean and Krupiczka's correlation.

CONCLUSIONS

Krupiczka's correlation – Eq. (7) – gives high values for grain (solid) thermal conductivity since it is inadequate for high values of π_3 (dry medium), contrarily to what was observed by Prasad et Al. (1989), cited in Kaviany (1991). This leads to an overstimation of λ_{sat} , as shown in Table 6, and its use must be restricted to low values of π_3 as shown in Fig. 5.

For MTR2, it was found a thermal conductivity λ_{sat} with an error of 13% by renormalization method and of 32% by the geometric mean which are considered small. For brick, it was also found reasonable results with errors of 32% by renormalization method and of 18% by the geometric mean.

However, the methods presented here did not give very good results in thermal conductivity for MTR1. The best result was the one given by the geometric mean for λ_{sat} with errors about 37% against 62% obtained with the renormalization method. We believe the error found by using the renormalization model was due to the fact that MTR1 grains do not behave as a single phase, but as a grain composed at least by 2 phases and the renormalization approach presented here can be only applied for 2-phase systems.

For a quantitative analysis, comparing the methods presented in this article, we see it is necessary to execute simulations for a greater number of porous materials before saying which method is better. The geometric mean is much simpler than the renormalization method. Nevertheless, the renormalization formulation was written in a such way that they can be easily calculated and, besides, they just need to be calculated once in building simulation codes such as the *UMIDUS* program (Mendes et Al., 1999) for prediction of Hygrothermal performance of porous building elements.

In conclusion, it was possible to see how water can increase thermal conductivity in porous building elements. It is predictable that this effect can be more important in humid climates, but surely it can not be just neglectable in drier climates and we are acquainted that more research must be done to improve mathematical models.

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Table 1: Experimental data for thermal conductivity and porosity obtained by Fernandes (1990) – MTR1 – and Perrin (1985) –MTR2 and TIJ.

	MTR1	MTR2	TIJ
λ_{seco}(W/m-K)	0.70	1.92	0.985
λ_{sat}(W/m-K)	2.95	2.57	2.08
η(%)	31.0	18.0	29.0

Table 2: Calculated values for λ_s and λ_{sat} according to the consideration of volume-weighted geometric mean.

λ (W/m-K)	MTR1	MTR2	TIJ
λ_s - calculated	3.07	4.94	4.35
λ_{sat} - calculated	2.07	3.81	2.86
λ_{sat} - measured	2.95	2.57	2.08

Table 3: Comparisons between comparisons between 2 and 5-scale 256x256x256 percolation systems with 2 different random number generator seeds.

		Seed = - 21		Seed = - 15	
η	MPS	λ_{dry}	λ_{sat}	λ_{dry}	λ_{sat}
0.2	Random	1.2623	1.6097	1.2622	1.6098
	Correl. 1	1.2956	1.6150	1.3130	1.6241
	Correl. 2	1.2904	1.6146	1.2798	1.6072
0.7	Random	0.1011	0.8786	0.1008	0.8785
	Correl. 1	0.1895	0.8934	0.1861	0.8909
	Correl. 2	0.1953	0.8971	0.1877	0.8906

Table 4: Average conductivity obtained by 3-D simulations with λ_s=2W/m-K

	η = 0.7			η = 0.2		
NX	64	128	256	64	128	256
$\bar{\lambda}_{DRY}$	0.1014	0.1020	0.1011	1.2635	1.2622	1.2624
erro (%)	0.3096	0.9289	0	0.0865	0.0163	0
$\bar{\lambda}_{SAT}$	0.8786	0.8786	0.8786	1.6099	1.6098	1.6098
erro (%)	0.0016	0.0302	0	0.0097	0.0033	0

Table 5: Average conductivity obtained by 3-D simulations with λ_s=5W/m-K

	η = 0.7			η = 0.2		
NX	64	128	256	64	128	256
$\bar{\lambda}_{DRY}$	0.1388	0.1400	0.138	3.1219	3.1184	3.1190
erro (%)	0.567	1.4689	0	0.0918	0.0206	0
$\bar{\lambda}_{SAT}$	1.1658	1.1671	1.1656	3.5645	3.5632	3.5631
Erro (%)	0.0197	0.126	0	0.0376	0.0027	0

Table 6: Coefficients of Eq. (9)

π_2	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
[0.1-0.4[	1.0110	0.9987	2.2634	-0.1971	1.8892	0.0110	0.0015	1.4345	1.4456	-0.3367
[0.4-0.7]	0.7939	0.9985	2.9175	-0.1861	2.3810	0.0000	0.1794	1.1412	4.2195	-3.4515

Table 7: Thermal conductivities obtained by renormalization, geometric mean and Krupiczka's correlation.

	MTR1	MTR2	TIJ
λ_s – renorm.	1.45	2.85	1.95
λ_s – Geometric mean	4.94	3.07	4.35
λ_s – Krupiczka	18.2	13.36	32.02
λ_{sat} – renorm.	1.12	2.23	1.42
λ_{sat} – Geometric mean	1.86	3.39	2.46
λ_{sat} – Krupiczka	4.39	6.52	6.29
λ_{sat} – measured	2.95	2.57	2.08