

A HYBRID SYSTEM FOR DAYLIGHT-RESPONSIVE LIGHTING CONTROL

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ABSTRACT

This paper argues that analytical approaches (i.e., simulation) and inductive learning methods (i.e., neural networks) can cooperate to facilitate a daylight responsive lighting control strategy. Multiple hybrid controllers are designed to meet four control goals: enriching the informational repertoire of systems control operations for lighting (by inclusion of performance indicators for glare and solar gain), reducing the number of sensing units necessary for capturing the states of building's visual performance indicators in real time, enhancing the accuracy of predictions necessary for the identification of the best control option, and maximizing the searches in the lighting system control state space within a limited time. HISSTO (Hybrid Intelligence for System State Transition Operation), the resulting pilot control system, is capable of regulating target lighting systems effectively through a web-based interface.

INTRODUCTION

With the advances in computation and DDC (Direct Digital Control), the model-based building control becomes an attractive option. A model may not always precisely capture the actual system behavior due, in part, to the difficulty of acquiring exact descriptions of building system properties, such as materials and construction features. Some simulation programs are computationally too intensive to be effective for real-time control purpose. An example is a lighting simulator that uses ray-tracing in the modeling process. Such simulation programs cannot be used for control purposes unless the pertinent control state search space is dramatically reduced.

Machine learning can address this problem. However, it often requires large amount of data for training. Before a neural network is trained, or if it encounters unexpected conditions, it is not able to predict accurately (Curtiss 1996). The need for retraining makes it difficult for a machine learner to respond quickly to the system retrofit and/or seasonal weather pattern changes. Its sensor-dependency represents an additional difficulty, especially when placing and/or maintaining sensors are costly or otherwise not desirable.

HISSTO is a pilot control system applied to an intelligent daylight responsive lighting control task. In general, performance criteria such as human comfort and cost minimization are the driving forces behind building systems control efforts. For a successful daylight responsive lighting control, HISSTO must minimize the lighting and thermal energy consumption and maximize occupants' visual comfort in the target control zone. Visual comfort in a space is not sufficiently captured just based on illuminance distribution. Therefore, additional performance indicators must be introduced. HISSTO makes use of simulation-assisted machine learner training as well as machine learning supported simulation calibration and tuning. Its architecture is meant to capture seasonal changes in outdoor conditions while adapting to changes in building configuration (e.g. due to retrofit). There are also some non-functional requirements (Bruegge and Dutoit 1999) for developing HISSTO. Prediction of the performance indicators and evaluations for all control options should be made within a certain time frame (e.g., 10 minutes). It is not feasible to use a large number of sensors in a target space beyond a certain period of data collection. Therefore, the sensors necessary during HISSTO's operation stage need to be confined to a small number (i.e. one or two).

DESIGN OF THE PREDICTORS

Figure 1 shows how HISSTO's model components are combined and related to each other. The focus here is on a virtual building which interacts with the environment and a real building. The virtual building consists of building descriptor, sensor, predictor (a simulator and/or a machine learner), and tester. Predictor receives the building description and the real-time sensor values (for both the environment and the controlled systems) to generate control options along with performance indicators for each of them. Hybrid predictors are special types of predictors developed for HISSTO. They use both simulator and machine learner components. Tester evaluates generated control options to select the most desirable system state.

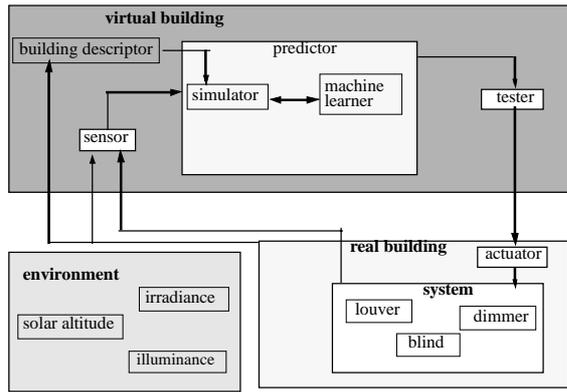


Figure 1: Conceptual structure of HISSTO

Table 1 shows the list of all variables selected for HISSTO's predictor design. For most of predictors, environmental variables (E1-E5) and system variables (S1-S2) are the inputs, reference variables (R1-R6) and performance indicators (P1-P7) are the outputs.

Table 1: Input and output variables of HISSTO's predictors

ID	Var.	Description
E1	ϕ_{sol}	Solar azimuth
E2	θ_{sol}	Solar altitude
E3	I_{dh}	Diffuse (horizontal) irradiance
E4	I_{gh}	Global (horizontal) irradiance
E5	E_{gh}	Global (horizontal) illuminance
S1	θ	Light redirection louver angle
S2	P_{Ln}	Power level of the n^{th} luminaire
R1	E_n	Indoor illuminance at n^{th} reference point ($n = 1... 12$)
R2	$bDGI$	Background luminance for DGI calculation
R3	L_{bCRT}	Background luminance for G_{CRT} calculation
R4	E_d	Illuminance on the eye ball by direct light component
R5	E_i	Illuminance on the eye ball by indirect light component
R6	L_{max}	Value of maximum luminance patch on the CRT screen
P1	E_m	Average illuminance of indoor reference points
P2	U_E	Uniformity factor in the indoor space
P3	DGI	Daylight Glare Index
P4	CGI	CIE Glare Index by electrical light
P5	G_{CRT}	Glare on the computer screen
P6	Q	Total solar radiation through the glazing area
P7	P	Total power consumption by electrical lights

Any hybrid predictor that uses feed-forward neural networks, needs training. In HISSTO, training of a machine learner is done with either the measurement data or the data generated by a simulator. HISSTO uses the light simulation tool LUMINA (Pal and Mahdavi 1999). When the measurement data is used, training is done with the most recent data set to predict the outcome of the systems actions for the next control interval. This "sliding time window" approach is scalable because it uses a constant volume of data to train neural nets on a regular basis (cp. Figure 2). One of the important issues here is to determine the size of the time window to maximize neural network's learning capability and to minimize computational load without ignoring environmental changes. Experiments show that a 4-5 days long time window results in a reasonable performance of the hybrid predictors designed for HISSTO.

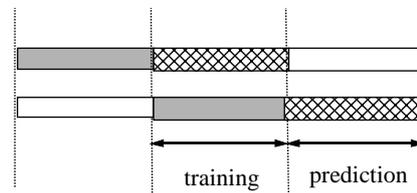


Figure 2: Neural network training and prediction scheme

Five different hybrid predictors have been designed and implemented for HISSTO. They vary mainly by their input and output as well as the source of knowledge for training - either simulation or measured data (Barto 1990). Neural network's lack of access to a certain group of performance indicators (such as glare index and solar radiation through glazing) represents a challenge. Since obtaining actual sensor readings for these variables is difficult, a neural network has to rely on the simulator to predict such variables. HISSTO uses a special neural network designed to solve this problem. This "bridge network (BRIDGENN)" obtains average illuminance and uniformity values calculated from the predicted indoor illuminance values by other neural nets, then predicts the rest of outputs (both reference variables and performance indicators) based on the training done with weather file-based simulations.

To identify each of the different hybridization schemes of the predictors, a special naming convention is established. The First two capital letters indicate the source of neural networks' training patterns. (MS: MeaSurement, SM: SiMulation, WF: Weather File). The character C after that denotes the calibration routine with the number of illuminance sensors used for the calibration process. If this number is 0, then there is no calibration. The last two letters NN indicates that the final form of the predictor is a neural network.

The five hybrid predictors are:

1) MSC0NN: This hybrid predictor is trained with measurement data only. Once a measurement-based neural network predicts the indoor illuminance distribution, both average illuminance and uniformity are calculated and fed to the bridge network as a part of its input. This bridge network then predicts all other reference variables and performance indicators.

2) SMC0NN: This hybrid predictor is trained solely based on training patterns derived from simulation inputs and outputs. This predictor doesn't need a bridge network because each training pattern includes all output variables to be predicted.

3) WFC0NN: Instead of performing simulations with the measured sky conditions, this hybrid predictor uses training patterns generated by the simulator based on the local weather file data.

4) SMC12NN: This predictor uses a neural network to calibrate simulation output by learning to predict the simulator's prediction error. Once a simulation output (i.e. indoor illuminance profile) is calibrated, average indoor illuminance and uniformity are calculated and the bridge network (BRIDGENN) completes the rest of prediction task.

5) WFC1NN: This hybrid predictor uses a neural network trained with weather file-based hourly simulations to predict the indoor illuminance profile. It then calibrates itself using only one real-time indoor illuminance sensor. In the calibration process, the relative difference between the measured sensor value and the predicted one is applied uniformly to the other sensor locations. After calibration, it uses a bridge network to generate the final prediction output (cp. Figure 3). Identifying a proper reference point is, therefore, crucial for this hybridization scheme.

Figure 4 shows how hybrid predictors are structured along with their relationships with other entities such as sensors and testers.

DESIGN OF THE TESTER

An attractive feature of a model-based control strategy is the diversity of the performance variables that can be considered. Furthermore, these performance variables need not be limited to strictly visual criteria (such as illuminance levels), but can address other performance criteria (such as energy use and thermal comfort). Currently, HISSTO predicts the values of the following performance indicators: average illuminance on any actual or virtual plane in the space, uniformity of illuminance distribution on any plane in the space (U_E , cp. Mahdavi and Pal 1998), Glare due to daylight (DGI, cp. Hopkinson 1971), Glare due to electrical light (CGI, cp. Einhorn 1979), solar gain (Q), and power consumption (P). The glare on a CRT screen (G_{CRT})

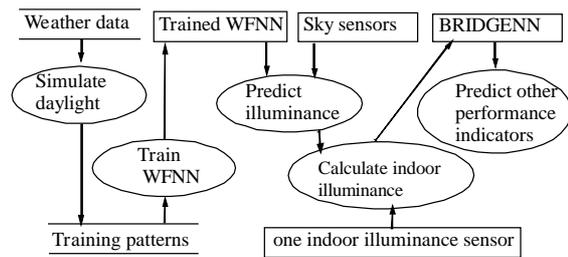


Figure 3: Training and operation of WFC1NN predictor : functional model

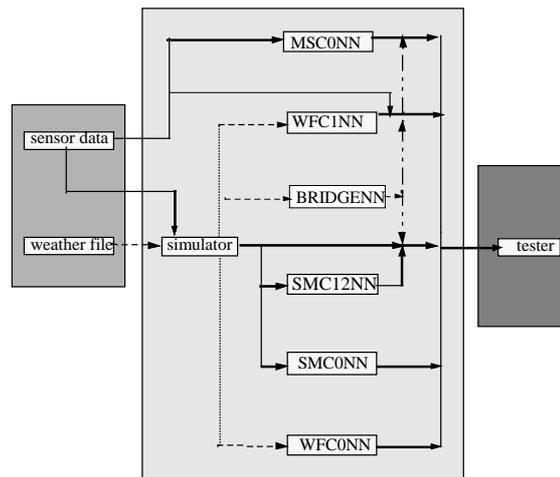


Figure 4: Structure of HISSTO's hybrid predictors

is also considered and is taken as the ratio of the luminance of the screen to the background luminance.

User's preference for the desired attributes of such performance indicators may be expressed to the control system using preference functions (Mahdavi et al. 1999).

These functions express numerically the user-preference for the range of values a performance indicator can assume. They may be derived from applicable codes and standards, results of pertinent comfort research, or subjective user input. A control objective function may be based on a single performance indicator, or a weighted aggregate of two or more performance indicators.

The weight for each performance indicator can be dynamically changed depending on the control objective and user preference. For example, if the weight factor of solar gain is zero, solar gain is not going to be considered in deciding system control action at all. The control objective function currently used in HISSTO is given in equation 1.

Maximize utility U, where:

$$U = W_E \cdot P_E + W_{UE} \cdot P_{UE} + W_{DGI} \cdot P_{DGI} + W_{CGI} \cdot P_{CGI} + W_{GCRT} \cdot P_{GCRT} + W_Q \cdot P_Q + W_P \cdot P_P \quad \text{Eq. 1}$$

Herein:

W_x = Weight for performance indicator x
 P_x = Preference index for performance indicator x

CONTROL SYSTEM DEVELOPMENT

HISSTO's system test is performed in the Intelligent Workplace (IW), a recently established laboratory at the Carnegie Mellon University campus. The daylight station, on the roof-top of the IW, measures diffuse and global horizontal irradiance. The daylight station also measures incident vertical irradiance and illuminance values for four orientations. In terms of electrical lighting, the test space can be illuminated using four indirect luminaires. The western section of a south bay in IW is dedicated to lighting studies. This

test area is partitioned from the rest of IW using white-colored partitions. About 60% of the external wall of the space consists of glazing. Glazing has low emission coating for heat loss control and a 0.3 shading coefficient with high visible transmission. The facade system includes a set of three parallel external moveable louvers which can be used for shading purposes and – to a certain degree – for light redirection purposes. An array of 12 illuminance sensors is located in the central axis of this space at a height of about 0.8 m above the floor. Illuminance measurements have been performed intermittently in this test space since December 1997. For demonstration purposes, HISSTO focuses on the two control target systems: light redirection louver and four luminaires in the IW test bay. Figure 5 shows

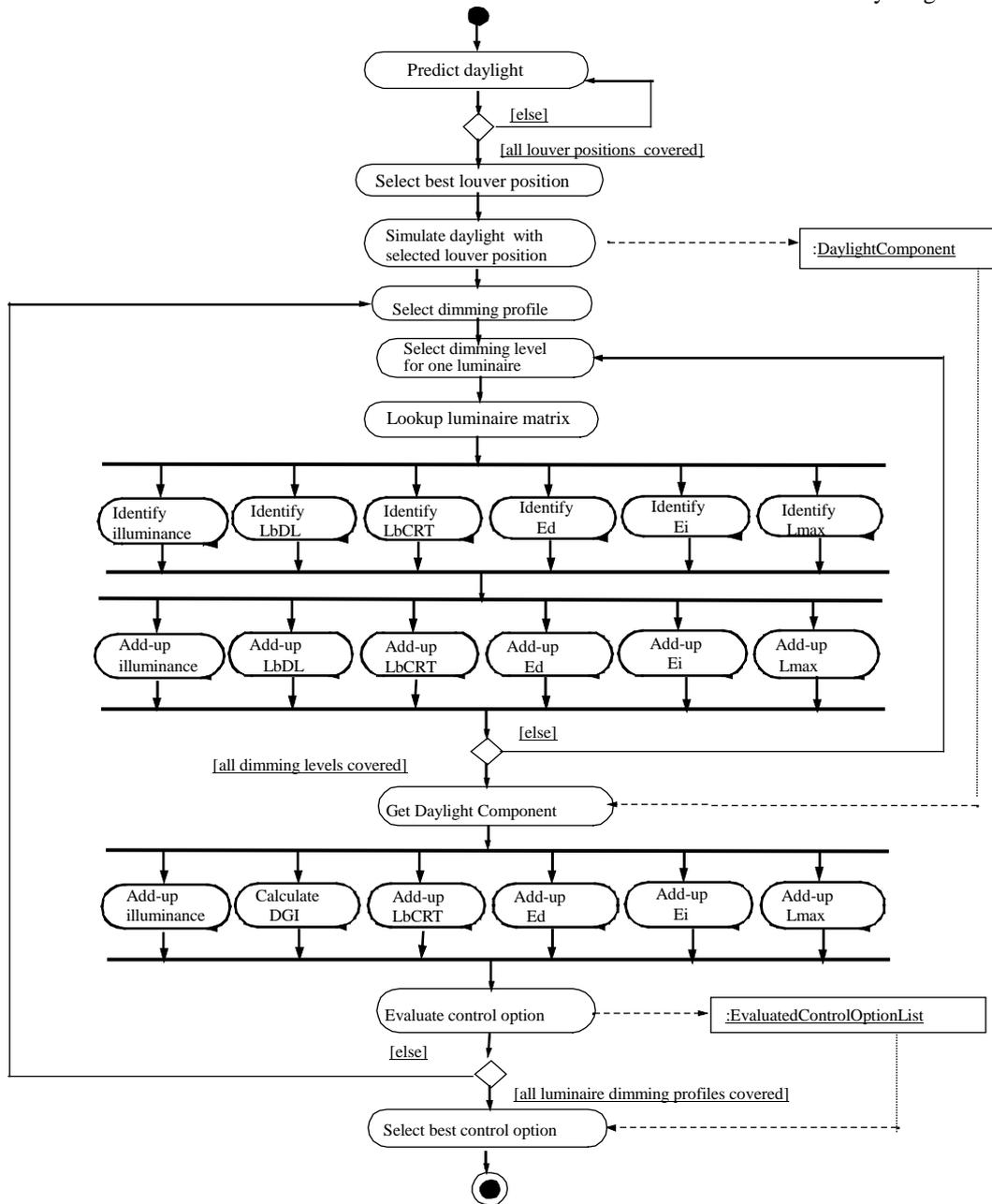


Figure 5: HISSTO's lighting control option selection process (UML notation)

HISSTO's internal process to derive a control decision. The names of variables used in the diagram are depicted in Table 1. Instead of evaluating all possible instances of combined daylight and electrical light control options, HISSTO assesses those separately and combines the results later to avoid an explosion of the control state search space. HISSTO predicts the values of daylight performance indicators using either the simulator or any of the hybrid predictors. Then, it selects the best louver angle based on the given criteria described in Equation 1. One more daylight simulation is done to generate the daylight glare components for the selected louver angle. This is necessary because only the simulator can calculate intermediate variables (R2-R6 in Table 1) to predict glare indices for a specific louver position. Then, HISSTO uses the Light Matrix (LMX), a look-up table that contains measured indoor illuminance distribution (for each luminaire at different electric power levels) as well as pre-calculated electrical light glare components, to calculate illuminance and glare components due to the electrical lights. In this process, HISSTO only considers a subset of possible luminaire power level combinations (i.e., two levels up and down plus current state for each luminaire). For each alternative luminaire power level combination, the system calculates the final values of the indoor illuminance and glare based on the additive nature of daylight and electrical light components. The overall evaluation of each control option relies on the same criteria depicted in Equation 1. Certain performance indicators address only daylight (i.e., Heat gain), some are derived only from electrical light (i.e., Power consumption), and others need to consider both. Finally, the identified desirable control option is passed to the actuator to update systems states, accordingly.

EXPERIMENT

TEST OF PREDICTION ACCURACY

Figure 6 shows the accuracy of HISSTO's hybrid predictors in predicting indoor illuminance profile using a part of the collected data set (Mar. 3 - Apr. 18, 1998). The selected test data set is divided into ten partitions based on the sliding time windows within the test period. For each time window and for each reference point in space, mean relative errors and their RMSs (Root Mean Square) of illuminance level predictions are calculated. All hybrid predictors demonstrate reasonably good prediction performance (within 3-13% RMS error range) across all sensor locations except the ones close to the window. MSC0NN and SMC12NN perform very well (within 3-7% RMS error range) in their predictions of the indoor illuminance profile. The simulator and those hybrid predictors trained by the simulator such as

SMC0NN and WFC0NN show a similar prediction trend even though they are slightly less accurate than the ones trained based on the measurement data. Among the predictors in this group, WFC0NN is clearly capable of copying the simulator with a sufficient number of training patterns based on the local weather data. It is also implied that using actual sky scanner data (instead of using synthetic sky models) can potentially increase the simulator's predictive performance as well as those of other simulation-based hybrid predictors. WFC1NN which is trained by local weather data based simulations and calibrated by one indoor illuminance sensor shows an overall performance improvement depending on the location of the reference point. Selection of the sensor position for calibration is an important factor for the performance of this predictor.

Figure 7 shows the relative prediction performances of all hybrid predictors tested in this research. Predictions are made for the target variables (both reference variables and performance indicators) to calculate their mean relative deviations from the simulator's predictions. Simulator's output is used as the base case because other hybrid predictors are dependent on the simulator's predictions for most of the target variables except average illuminance and uniformity.

The predictive patterns of both SMC0NN and WFC0NN are similar to what simulator predicts. Even though calibrated only by one sensor, WFC1NN's predictive pattern is similar to those of other simulation-based hybrid predictors. MSC0NN and SMC12NN display higher deviations for certain target variables. Because these two hybrid predictors are heavily sensor dependant (either for training or for calibration), they are affected by the difference between the measured indoor illuminance profile and the simulated one (propagated to other target variables by BRIDGENN). Certain predicted target

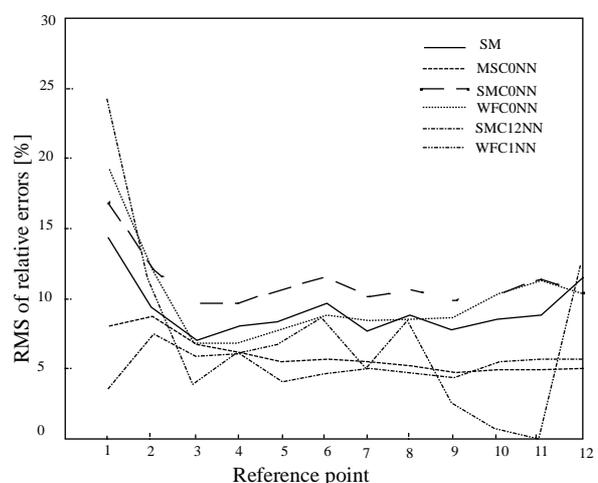


Figure 6: Hybrid predictors' indoor illuminance prediction accuracy evaluation expressed in RMS of relative errors

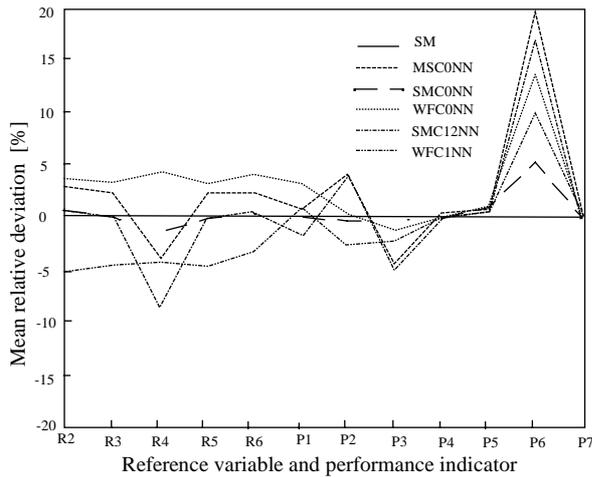


Figure 7: Hybrid predictors' performance in predicting reference variables and performance indicators expressed as mean relative deviation from simulator's prediction.

variables (i.e., P6 : solar gain) deviate significantly from simulator's predictions. This needs to be further investigated. For meaningful comparisons between multiple hybrid predictors, validation of the simulator becomes important, especially for those target variables whose values can be predicted only by the simulator.

Despite its high predictive accuracy, the need for ever-present multiple indoor illuminance sensors as well as the difficulty of systematic measurement for neural network re-training during a building's operation time makes MSC0NN less practical. This is also true with SMC12NN, a simulator calibrated by a neural network trained to predict simulation errors. It also requires multiple simulations every time it is to be deployed. Even without a measurement-based calibration process, the pure simulator performs reasonably well. The simulator's memory-intensive and time consuming process is greatly improved in other simulation-based hybrid predictors. For example, SMC0NN approximately inherits simulator's prediction capability.

Among the simulation-based hybrid predictors, both WFC0NN and WFC1NN emerge as promising ones. They are almost sensor-independent (WFC1NN uses only one indoor illuminance sensor for calibration purpose) while maintaining enhanced prediction capability. This measurement-free architecture also makes them immediately available for operation once they are trained with the data set generated by simulator based on the local weather file. The necessary training patterns can be acquired by performing daylight simulations at the beginning of each month. Obviously, all simulation-based hybrid-predictors need a sufficiently accurate simulator to maintain their desirable predictive performance level.

TEST OF SIMULATOR AND MATRIX-BASED CONTROLLER

A test is performed based on the combined daylight and electrical light control using the simulator and a pre-calculated Light Matrix. A specific time of a day from the test data set is chosen for the experiment (Table 2). Daylight simulations for eight different louver positions (from 0 to 105 degrees in 15 degrees intervals) are performed based on the current system state. After the most promising louver position is identified by the system (Table 3), all 625 different luminaire power level combinations (five discrete power levels for each luminaire) are evaluated to obtain the ten best dimming options (see Table 4).

Table 2: Initial environment and system states for test

Yr.	Mth	Day	Hr.	Igh	Idh	Egh	θ_n	PL1 [%]	PL2 [%]	PL3 [%]	PL4 [%]
1998	3	14	9	81	78	9,379	0	30	30	30	30

Table 3: Order of best louver positions (SM+LMX) (weight factors: $w_{Em}=0.45$, $w_{UE}=0.2$, $w_{DGI}=0.05$, $w_{CGI}=0.03$, $w_{GCRT}=0.1$, $w_Q=0.12$, and $w_P=0.05$)

θ_{n+1} [degree]	E_m [lx]	U_E	DGI	CGI	G_{CRT}	Q[W]	P[W]	U
105	324	0.69	7.29	0	0.97	2.27	0	0.540
90	314	0.69	6.76	0	0.93	2.11	0	0.530
0	259	0.71	5.86	0	0.98	2.30	0	0.513
75	297	0.68	6.70	0	0.94	2.02	0	0.512
45	253	0.70	6.00	0	0.95	2.00	0	0.506
15	238	0.71	5.90	0	0.99	2.10	0	0.502
30	240	0.69	5.99	0	0.99	2.00	0	0.498
60	274	0.67	6.61	0	0.95	1.99	0	0.493

Table 4: 10 best luminaire power level combinations (SM+LMX)

P_{L1}	P_{L2}	P_{L3}	P_{L4}	E_m	U_E	DGI	CGI	G_{CRT}	Q	P	U
20	10	20	10	454	0.80	8.91	0	0.84	2.27	35	0.8376
20	20	20	10	468	0.80	8.81	0	0.84	2.27	41	0.8375
20	10	20	20	468	0.80	8.83	0	0.83	2.27	41	0.8371
20	20	20	20	482	0.81	8.72	0	0.82	2.27	46	0.8369
10	10	20	10	439	0.79	9.00	0	0.85	2.27	29	0.8368
10	20	20	10	453	0.79	9.92	0	0.85	2.27	35	0.8368
10	10	20	20	453	0.79	8.82	0	0.83	2.27	35	0.8364
10	20	20	20	467	0.80	9.04	0	0.83	2.27	41	0.8363
20	10	10	10	438	0.79	8.94	0	0.87	2.27	29	0.8362
20	20	10	10	453	0.79	9.04	0	0.86	2.27	35	0.8362

TEST OF CONTROLLERS WITH EMBEDDED HYBRID PREDICTORS

With the same initial system states, the alternative control schemes with various hybrid predictors are tested. Table 5 shows WFC12NN-based controller's valuation of each louver position based on seven performance criteria. Table 6 illustrates 10 best daylight-responsive lighting control options

suggested by the controller using WFC12NN and Light Matrix.

As is shown in Table 7, most controllers that use various hybrid predictors suggest similar system states for both the louver and the luminaires. The controller that uses only the simulator comes up with different control recommendations due to its limited system control state search space (necessary to reduce computation time).

Table 5: Order of best louver positions (WFC1NN)

θ_{n+1} [degree]	E_m [lx]	U_E	DGI	CGI	G_{CRT}	Q[W]	P[W]	U
105	355	0.75	6.68	0	0.91	2.28	0	0.530
90	353	0.75	6.63	0	0.91	2.28	0	0.528
75	351	0.75	6.59	0	0.91	2.27	0	0.527
60	347	0.75	6.52	0	0.91	2.26	0	0.524
45	344	0.75	6.46	0	0.92	2.25	0	0.522
30	338	0.74	6.37	0	0.92	2.23	0	0.517
15	329	0.74	6.22	0	0.92	2.20	0	0.510
0	295	0.74	5.77	0	0.93	2.08	0	0.488

Table 6: 10 best luminaire power level combinations (WFC1NN)

P_{L1}	P_{L2}	P_{L3}	P_{L4}	E_m	U_E	DGI	CGI	G_{CRT}	Q	P	U
20	10	20	10	485	0.84	5.95	0	0.80	2.28	34.8	0.8257
20	20	20	10	499	0.84	5.86	0	0.80	2.28	40.6	0.8253
10	20	20	10	484	0.83	5.95	0	0.80	2.28	34.8	0.8250
20	10	20	20	499	0.84	5.87	0	0.79	2.28	40.6	0.8249
10	10	20	20	484	0.83	5.96	0	0.79	2.28	34.8	0.8246
20	20	10	10	484	0.83	5.98	0	0.82	2.28	34.8	0.8245
10	20	20	20	498	0.84	5.87	0	0.79	2.28	40.6	0.8242
20	10	10	20	483	0.84	5.99	0	0.81	2.28	34.8	0.8241
20	20	10	20	497	0.84	5.90	0	0.81	2.28	40.6	0.8237
10	20	10	20	482	0.83	5.98	0	0.81	2.28	34.8	0.8233

Table 7: Suggested systems states and evaluated utilities

Predictor type	θ_{n+1} [degree]	P_{L1}	P_{L2}	P_{L3}	P_{L4}	U
SM	0	20	20	20	20	0.68
MSC0NN	105	20	10	20	10	0.92
SMC0NN	105	20	20	20	10	0.59
WFC0NN	105	20	20	20	10	0.77
SMC12NN	105	20	10	20	10	0.78
WFC1NN	105	20	10	20	10	0.79

TEST OF CONTROL DECISION SPEED

In this particular research, the time necessary for finishing one instance of simulation was found to be approximately 38 seconds while a neural network took only 0.25 second to perform the same task. The exact numbers could vary depending on various parameters, yet this clearly shows that prediction speed could be enhanced by using neural networks trained by the simulator. The average time for an

instance of Light Matrix lookup and control option evaluation was approximately 0.33 second. Table 8 shows the time needed to identify the desirable system control states for three controllers using different predictor types. Eight louver positions and five discrete power levels for each luminaire are considered in the test. When only the simulator is used, it has to simulate each of 5,000 combined louver-luminaire states. The system still has to evaluate each of the control options to identify the best. When the simulator and Light Matrix are used jointly, a total of nine simulations are necessary to finish the same task including one for calculating daylight glare components after the louver position is determined. The controller using a hybrid predictor performs best in this test.

Table 8: Performance comparison among different controllers

Predictor	Prediction [sec]	Evaluation [sec]	Total [sec]
SM	190,000	1,650	191,650
SM+LMX	304	244	548
WFC1NN	2	244	246

GENERAL OBSERVATIONS

The evaluation of HISSTO is based on three different criteria. The speed toward a control decision is significantly increased by using neural networks across all controllers that use hybrid predictors. In terms of prediction accuracy, any hybrid predictor calibrated by the measurement shows an improvement in accuracy compared with the pure simulator. All hybrid predictors cover seven visual performance indicators for the evaluation of the control options by inheriting the modeling capability of the simulator. In terms of fulfilling non-functional requirements, the hybrid predictors can predict within $\pm 20\%$ relative error range with approximately 400 training patterns. Most tested hybrid predictors finish the control option selection process under 5 minutes with eight louver positions and 625 luminaire power level combinations. Trained with weather file-based simulations, a hybrid predictor such as WFC1NN can perform well with only one indoor illuminance sensor for real-time on-line calibration. Web-based remote access to HISSTO is demonstrated by using LabviewTM and ComponentWorksTM (based on ActiveX technology) off-shelf development environments (National Instrument).

CONCLUSIONS

The approach introduced in this paper can extend simulation's role from a building design evaluation tool to a building operation support tool while enriching the informational repertoire of systems control operations for lighting. Simulation's capability in building control is proven to be enhanced by being combined with machine learning technique. The hybrid system turns out to be a promising option especially to reduce computational load and sensor dependency. Reducing computational load in prediction also enables an extended search in the system control state space within a limited time window. Future research must address some of the remaining issues. Controlling lighting systems across multiple zones in a building requires extended work. The design and the training scheme for each machine learner in HISSTO needs to be further improved. A machine learner, possibly based on reinforcement learning technique, could learn how to customize personal preference functions for a specific occupant. HISSTO's applicability to the different building control domains also needs to be explored.

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REFERENCES

- Barto, A. G. (1990). "Connectionist Learning for Control, Neural Networks for Control", The MIT Press., Cambridge, MA
- Bruegge, B., Dutoit, A. (1999). "Object-Oriented Software Engineering: Conquering Complex and Changing Systems", Prentice Hall, New Jersey
- Einhorn, H. D. (1979). "Discomfort Glare: A Formula to Bridge Difference", Lighting Research & Technology, Vol. 11, No. 2, p. 90.
- Hopkinson, R. G. (1971). "Glare from Window", Construction Research and Development Journal, Vol. 2, No. 4, pp. 169-175; Vol. 3, No. 1, pp. 23-28.
- Mahdavi, A., Chang, S. J., Pal, V. (1999). "Simulation- based Integration of Contextual Forces into Building Systems Control", Proceedings of the IBPSA '99 Conference. Kyoto, Japan.
- Mahdavi, A., Pal, V. (1998). "Toward an Entropy-based Light Distribution Uniformity Indicator", Proceedings of the 1998 IESNA Annual Conference, New York.
- Pal, V., Mahdavi A. (1999). "Integrated Lighting Simulation within a Multi-domain Computational Design Support Environment", Proceedings of the Illuminance Engineering Society, New Orleans, MS.