MODELLING OF AN ADVANCED INTEGRATED MECHANICAL SYSTEM FOR RESIDENTIAL APPLICATIONS

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ABSTRACT

This paper describes the development and validation of a simulation model for Advanced Integrated Mechanical Systems (AIMS) destined for residential applications. AIMS are defined as mechanical systems that integrate the functions of residential space heating (optional cooling), heat recovery ventilation and hot water heating. Efficiency advancements of these systems are expected through the use of intelligent controls, high efficiency fans and motors,and the use of computer models to optimize the performance.

The model was developed as a stand-alone application for testing and validation. The code of this stand-alone application was then incorporated into the ESP-r computer program. Validation of the model against monitored house data with commercially available integrated heating and hot water units ("combos") shows good agreement. Since there are no commercially available AIMS systems at present, further testing and validation will be required.

INTRODUCTION

The Combo/AIMS model serves to evaluate the performance of combination space and water heating systems, which may also incorporate a Heat Recovery Ventilator (HRV). When a HRV is used, the system thus modelled is referred to as an Advanced Integrated Mechanical System (AIMS).

The current model is based on a classical water-heater design connected to a fan-coil space heating system. The space heating is provided by circulating the DHW using a pump through a finned-coil. The model considers single stage, natural gas-fired burners. The HRV considered are sensible-heat-only models that always use the house distribution system to supply the fresh air. The user can select from 3 HRV configurations: no-fan not ducted system; 1-fan exhaust ducted system; and 2-fan not ducted system.

The HRV is always considered as having a temperature activated timer-delay defrost mechanism. Two types of defrost strategies are considered: blocked fresh air intake and warm air recirculation. The house simulation program can indicate to the Combo/AIMS model if outside air ventilation is allowed for any given hour. During outside air ventilation calls, the house blower is considered to be running at low speed. During heating calls, the blower is running at high speed while the outside air ventilation remains active but unaffected by the change of blower speed.

The model can treat either condensing or non-condensing water-heaters. Both direct vent and natural draft models are available. Direct vent models are always assumed to be using outside air for combustion and do not require draft control. Natural draft units are assumed to be using house air for combustion and for dilution through a barometric damper.

The model can estimate the amount of fuel required to meet the space heating load, the DHW load and the HRV-related load (fresh air). The amount of air required by the combustion and flue gas venting systems is also calculated. Electric energy use for operating the Combo/AIMS is available on an hourly basis.

The model can be adapted to fit specific Combo/AIMS systems that meet the general system layout adopted. This layout is shown in Figure 1.

Figure 1: Combo/AIMS layout – shown with no-fan not ducted HRV

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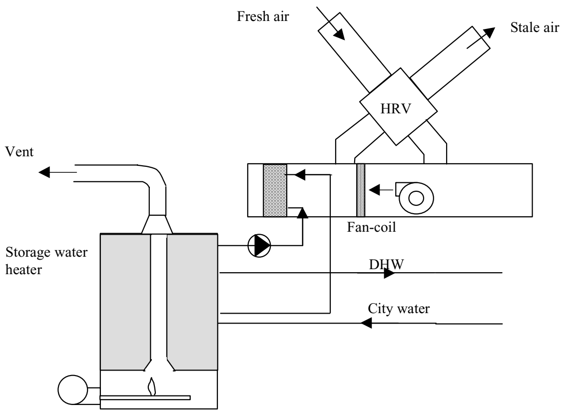
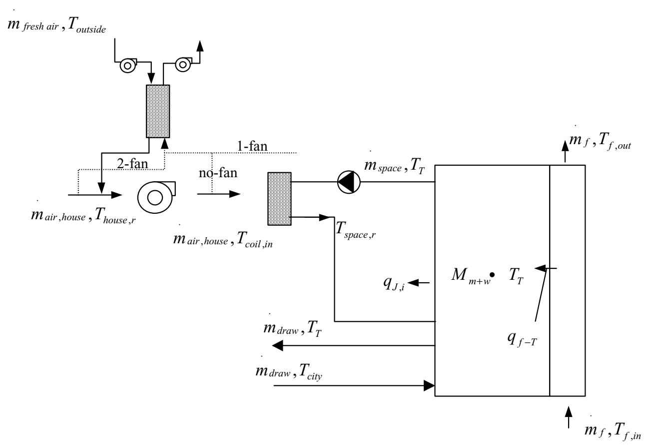


MODEL DEVELOPMENT

The model adopted is based on a lumped model of the tank and considers only its transient term. The mass of the space-heating coil, HRV, piping and other transient phenomena are neglected. Figure 2 illustrates the system considered. The approach adopted for the water heater model is adapted from Bourdouxhe, 1998. The remainder of the system, namely the space heating coil and HRV, are modelled using conventional steady state heat exchanger relations. In both cases, a NTU type approached was adopted.

Figure 2 : Schematic view of the model



The model uses a semi-empirical approach which considers only the transient phenomenon associated with the water and water heater internal mass. The energy balance on the system is represented by the following ordinary differential equation :

$$C_{w+m}\frac{dT_T}{dt} = q_{in} - q_{stk} - q_J - q_{space} - q_{draw} \qquad (1)$$

It is assumed that $T_w = T_m = T_T$ (w = water, m = metal, T = Tank). The current model considers only a 1-node tank, where the temperature represents the average temperature of a perfectly mixed tank (no stratification).

The power input, q_{burner}, is an input to the model. It corresponds to the burner's nominal capacity. From the nominal capacity, the following auxiliary variables are derived:

Fuel flow rate:

The mass flow rate of natural gas is evaluated as:

$$\dot{m}_{ng} = \frac{q_{bunrer}}{HHV} \qquad (2)$$

Combustion air:

For natural gas, the stoichiometric air requirement, κ, is 16.5 lb/lb$_{ng}$ (Gaz Metropolitain). To this value, an excess air ratio is added:

$$\dot{m}_{air} = \kappa\lambda\,\dot{m}_{ng} \qquad (3)$$

Net heat input:

The absolute net heat input to the system is evaluated as (Bourdouxhe 1998).

$$q_{in} = \dot{m}_{ng}\left(HHV + c_{p,ng}T_{ng,in} + \kappa\lambda h_{air,in}\right) \qquad (4)$$

and :

$$h_{air} = c_{p,air}T_{air} + W_{air}\left(h_{vap.} + c_{p,watervapor}T_{air}\right)$$

(ASHRAE Fundamentals 1981)

The loss through the stack gases is evaluated by estimating the rate of heat transfer between the flue gas and the water. The approach adopted for calculating this energy transfer is based on that presented in Andrews, 1986, and Bourdouxhe, 1998. A classic UA*LMTD heat exchanger equation is considered to evaluate the heat tranfered from the flue to the hot water tank. The model can separate, whenever applicable, the heat exchange area between condensing and non-condensing regions (see DATA PREPARATION). One of the disadvantage of the UA*LMTD method lies on its need that all temperatures at the heat exchanger inlet and outlet be known. Therefore, this requires the model to iterate on the outlet temperature on the flue side of the heat exchanger. The form used to express the heat transfer between the flue gas and the water is:

$$q_{f-T} = (UA)_{flue}\frac{\left(T_{f,in} - T_T\right) - \left(T_{f,out} - T_T\right)}{\ln\left(\frac{\left(T_{f,in} - T_T\right)}{\left(T_{f,out} - T_T\right)}\right)} \qquad (5)$$

The value of UA was determined during the data preparation phase prior to the simulation. This equation is used for the sensible only region. In the condensing zone, the temperature of both fluid is considered to remain constant and the following simplified equation is used:

$$q_{f-T} = (UA)_{flue}\left(T_{dew} - T_T\right) \quad (5a)$$

Since the flue outlet temperature is not known, an energy balance on the flue gas is done in order to find the correct value of $T_{f,out}$. The energy balance equation is:

$$q_{f-T} = \left(T_{f,in} - T_{f,out}\right) * (1+\kappa\lambda)\dot{m}_{ng}\, c_{p,f-low} \quad (6)$$

The specific heat of combustion gases is treated in two separate parts. Below a temperature of approximately 537.8 °C (1000 °F), the specific heat used is that obtained from the mass-weighted averaged of the specific heats of its components. Above that temperature, dissociation modifies the value of the specific heat. The model uses an equivalent specific heat for temperature above 537.8 °C (Andrews, 1986, Baumeister, 1967). To insure a proper energy balance on the flue gas stream, the specific heat of the combustion products above 537.8 °C is calculated as:

$$c_{p,f-high} = \frac{q_{in} - \dot{m}_{ng}\, h_L\left(x_{H_2O} + \kappa\lambda W_{air,in}\right) - \dot{m}_{ng}(1+\kappa\lambda)(537.8 - T_{mixt.,in})}{(T_{combustion} - 537.8)(1+\kappa\lambda)\dot{m}_{ng}} \quad (7)$$

The combustion temperature, $T_{f,in}$, is estimated from tabulated data (Baumeister, 1967):

$T_{f,in}$(deg. R)

Excess air (%)	100	120	140
Natural gas	4180	3840	3520
Methane	4010	3660	3330

The stack losses are then evaluated as:

$$q_{stk} = q_{in} - q_{f-T} \quad (8)$$

If the burner is off, the same UA*LMTD approach is used to evaluate the heat transfer rate between the off-cycle air circulating in the flue and the water. However, the UA coefficient is adjusted as a function of the reduction of fluid flow in the flue. The airflow level in the flue during off-cycle operation is calculated as a fraction of the on-cycle flow (Bourdouxhe, 1998, Butcher, 1990, Johnson, 1983). The resulting $q_{f\text{-}T}$ term then represent a loss from the water to the flue instead of a gain.

Jacket loss

The heat loss from the equipment is calculated using the steady-state equation:

$$q_J = (UA)_J\left(T_T - T_{room}\right) \quad (9)$$

The radiative losses are not considered at the moment and are assumed to be included in a linearized global UA coefficient.

Space heating demand

The space heating capacity is calculated using a NTU formulation:

$$q_{space} = \varepsilon_{coil}\, \dot{m}_{house}\, c_{p,air}\left(T_T - T_{house,r}\right) \quad (10)$$

The model assumes that the air and water flow rates into the coil remains the same during all space heating call, and hence the coil effectiveness is considered constant.

Water heating demand

The water draw demand is evaluated as :

$$q_{draw} = \dot{m}_{draw}\, c_{p,water}\left(T_T - T_{city}\right) \quad (11)$$

Having calculated all the terms used in equation 1, the model is able to determine the temperature differential during a given time interval for on or off burner cycles. However, since many of the terms involved in evaluating that temperature differential are dependent on the tank temperature, an iterative solution for the system is required. The model uses a semi-explicit approach where the water temperature used in equations 5 to 11 is based on the average temperature during the time interval.

For a given time interval, if the resulting water temperature falls outside of the controller's limits, the time interval is reduced until the temperature is just at the controller's limits. This process represent the outer iteration loop used in the detailed simulation routine.

Heat Recovery Ventilator

The HRV is modeled using the same NTU approach used for the space-heating coil. In this case, the user is required to enter the HRV core effectiveness as defined in CAN/CSA-C439-88.

As for the space-heating coil, the effectiveness is considered constant since the flow rates on both sides of the heat exchanger are considered to remain identical at all time. The effect of fouling due to frost build-up is not treated in the current model but two types of defrost control are considered. The first type uses recirculation of house air in the HRV supply side to achieve defrosting. The second type considers blocking off the fresh air intake to allow the exhaust air to defrost the HRV core.

The effect of defrost cycles on the house load is treated differently depending on the defrost strategy adopted. In the first case, the load to the house is evaluated to be that of the mass of frost built-up in the HRV core. The mass of frost used in the model corresponds to a 1 mm thick ice build-up for a typical HRV core (Venmar, 1999). This value is not currently accessible to the user. Defrost operation is based on a temperature activated timer delay (Phillips 1989, Venmar 1999). Below a predetermined temperature, defrost cycles will occur at fixed time intervals for a given amount of time. The delay between defrost cycles is currently independent of outside temperature.

For blocked fresh air defrost, the additional load to the house is calculated based on an additional air infiltration, without any heat recovery, equal to the exhaust airflow rate. The defrost cycle operation is based on the same temperature activated timer cycle as for the recirculation case.

Motors

Electric energy use for fan and pump motors is accounted for by dividing it between internal gains to the mechanical room and useful heat to the air or water. Using the user-entered motor demand and efficiency, the heat rejected to the mechanical room is obtained as:

$$q_{elect.,internal} = (1 - MotorEff) * MotorCap \quad (12)$$

The remainder of the energy is attributed to the air stream for the house and HRV blowers, and to the water for the circulating pump. For HRVs with exhaust fans, the mechanical work is considered to be done after the HRV core. Thus, the energy gain is not available in the core during the heat exchange process. For a two-fan HRV, the internal gain from the motor inefficiency is attributed to the fresh air stream since the motor is located in the air stream.

Chimney Dilution Air

Whenever a natural draft chimney is used on a combustion system, a means of draft control is usually employed. The draft control has several practical roles, such as allowing safe operation during downdrafts or isolate the equipment from updrafts. The presence of a draft control mechanism usually implies that heated house air is lost as dilution air through the chimney.

In the Combo/AIMS model, the available draft control system is based on an ideal barometric damper. The barometric damper is used to adjust the negative pressure available at the flue gas outlet (chimney/stack inlet). Whenever the negative pressure at the flue outlet tends to increase due to greater stack effect (from buoyancy), the barometric damper opens up to allow more house air to flow through the chimney in order to compensate for the increase stack effect. This allows to maintain a relatively constant amount of combustion air in the combustion chamber regardless of outside temperature or wind conditions.

The amount of dilution air used has a direct impact on the system's net efficiency. The model calculates the mass of dilution air used every hour and returns it to the building simulator. The simulator is responsible for converting it back to a space-heating load that can be passed back to the Combo/AIMS model during the next hour.

A barometric damper will usually be adjusted to achieve a given amount of dilution at standard outside conditions. The ratio of the total stack flow (chimney flow) over the flow in the flue (S/F) is used to characterize the draft control system. The model uses a S/F ratio of 1.4 for a barometric damper at 5.6 °C outside temperature (ASHRAE 103).

To get the actual value of dilution air required, the theoretical draft caused by the stack effect has to be evaluated (ASHRAE 1996, p.30.3):

$$D_t = 0.03413\,BH\left(\frac{1}{T_{out}} - \frac{1}{T_{chimney}}\right) \quad \text{(Pa) (13)}$$

This theoretical draft will not be available at the flue gas inlet since the pressure losses from the chimney air flow will reduce its value. The chimney pressure losses can be expressed using the conventional equation (ASHRAE 1996)

$$\Delta p = k\rho_{chimney}\frac{V_{chimney}^2}{2} \quad (14)$$

For an ideal barometric damper, the available draft at the flue exit is constant and is evaluated as:

$$D_a = D_t - \Delta p \quad (15)$$

A value of 9.96 Pa (Butcher, 1990) is used in the model for the available draft. The chimney flow is then adjusted to obtain the value of available draft.

DATA PREPARATION

A global heat transfer coefficient is required for the model adopted for the Combo/AIMS, as presented in equations 5 and 5a. The method used in determining this global heat transfer coefficient (UA global) employs the same equations as those used for modelling the Combo/AIMS system. However, in this case, the tank water temperature is held constant at a user-specified temperature corresponding to the provided steady state efficiency test conditions. The heat transfer rate between the flue gas and the water is then evaluated following Equation 5 using a guessed value for UA global. The routine iterates on UA global until it can reproduce the steady state efficiency provided by the user. The procedure can account for water vapour condensation (condensing water-heaters) inside the flue-water heat exchanger. This condensation can only happen when the flue gas dew point temperature is above that of the average tank temperature. Whenever the provided steady state efficiency cannot be achieved without condensation, the model employs equation 5.a in its energy balance and corrects the heat transfer coefficient to account for the mass transfer.

However, in a real system, it is sometimes possible to obtain condensation even when the average tank temperature is above the dew point temperature. This is due to the presence of stratification in the tank and the location of the heat exchanger in the colder portion of the tank. Since the Combo/AIMS model uses a lumped-mass model for the tank, it cannot account for stratification. Therefore, if the user enters a steady state efficiency and an average water tank temperature that cannot be reproduced without condensation, while the dew point is above the water temperature, the model will artificially increases the dew point until condensation can occur and the efficiency can be reproduced. This new dew point temperature is then used throughout the simulation for this system.

VALIDATION

The main objective of the validation process was not to try to adjust any of the model's empirical parameters to reproduce exactly the experimental results but rather to demonstrate that the model could reproduce satisfactorily the same behavior observed experimentally using a limited amount of data input.

A series of validation tests were performed on detailed experimental results from Combos tested at the National Research Council of Canada. Hourly energy use and temperature profiles were available for comparison with simulation results. The required input data for the simulation model was extracted from manufacturer equipment specifications. This data provided information on tests performed on two combos, a high efficiency condensing unit and a mid-efficiency unit.

The data of interest that was available consisted of hourly measurements of air and water supply and return temperatures, hourly volume of DHW draw, hourly volume of water circulated in the space-heating coil, on-time for the water heater, space-heating circulation pump and the amount of fuel used by the system. The data was available in Excel files in its almost raw format.

Upon analysis of the data, it was decided that hourly input data would be fed to the model in order to reproduce the experimental conditions as close as possible. The validation procedure was to impose on the simulated system the same return air temperature and city water temperature as well as the same DHW draws as those measured experimentally. Since the house is not formally simulated in the model, the space heating behavior needed to be reproduced indirectly. It was decided to impose the same operating time for the space heating pump as that measured. From this imposed hourly input data, the model can then evaluate the amount of fuel that will be required and this value can be compared to the experimentally measured fuel mass.

In order to proceed with the validation, the water heater and space heating coil characteristics have to be entered in the model. The first series of validation test were done on a Condensing combo. The data for this system was obtained from the available manufacturer's specifications. Table 1 present the input data used for the water heater system and for the heating coil. The heating coil characteristics were derived from the first few hours of testing for the first week of test.

Table 1: Input data for the condensing combo.

Water Heater Input Data	
Steady state efficiency	94 %
Nominal capacity	29.3 kW
Tank volume	139 L
Set point	57.2 °C
Shell insulation	0.35 W/m2/°C
Space Heating Coil Input Data	
Design Flow Rate	0.28 L/s
Heating Capacity	10.5 kW
Air flow rate	0.702 kg/s
Air side design temperature drop	15 °C

The comparative results for this condensing unit are presented in Table 2.

Table 2: Natural gas use comparison – simulation done using same draw schedule and pump operating time, 94.0 % steady state eff.

	Operating time	Water draw	Natural gas used kg		
Week #	H	L	Model	Exp.	Diff. %
1	30.6	1948	33.3	31.6	-5.3
2	20.4	1964	26.0	27.5	5.5
3	21.0	1901	30.9	32.8	5.7
4	15.8	2249	29.2	28.4	-2.6

5	0.0	1943	6.6	7.6	12.9
6	0.0	1715	5.8	6.3	8.7

The results for this condensing system demonstrate good agreement with the experimental data. The larger differences for the water heating only tests tend to indicate that the off cycle losses through the shell and/or flue are somewhat underestimated in the model. The configuration of the flue heat exchanger in this unit, using a downward flow configuration, would tend to limit the off cycle flow to a very low value (as also indicated in the product description).

During the validation process, some unexpected differences were initially detected following simulation for week #3. Unlike the first two weeks, the difference in predicted mass was initially over 25 %, as shown in Table 3. Following investigation of this increased difference, it was discovered from the data file that the coil air flow was modified for this week and the following weeks from it low setting to its high setting. The coil characteristic under higher flow was obtained from the first few hours of week #3. These new input parameters were then used in the model. The simulation model's results using the new coil parameters were then in good agreement with the experimental data. This demonstrated the ability of this semi-analytical model to diagnostic fault or improper behavior with regard to experimental results.

Table 3: Natural gas use comparison – low speed vs. high speed results for air handler blower, 94.0 % steady state eff.

	Oper. Time	Water draw	Natural gas used Kg		
Week #	h	L	Model	Exp.	Diff. %
2 – High speed	21.0	1901	30.9	32.8	5.7
2 – Low speed	21.0	1901	24.5	32.8	25.4

Following the validation with the condensing combo, a second set of experimental data for a non-condensing unit was used for validation. The same validation procedure was used.

Table 4: Input data for the Rheem combo.

Water Heater Input Data	
Steady state efficiency	80 %
Nominal capacity	23.4 kW
Tank volume	205 L
Set point	57.2 °C
Shell insulation	0.708 W/m2/°C
Space Heating Coil Input Data	
Design Flow Rate	0.228 L/s
Heating Capacity	10.2 kW
Air flow rate	0.677 kg/s
Air side design temperature drop	15 °C

Table 5: Natural gas use comparison, 80.0 % eff.

	Load, KJ		Natural gas used, kg		
Week #	Space	Water	Model	Exp.	Diff. %
1	1 203 034	315 946	37.7	39.7	5.0
2	473 360	318 025	21.8	22.8	4.1
3	606 025	316 235	24.8	25.6	3.0
4	172 233	440 618	19.7	19.3	-2.0

These results indicate a very good agreement between the model and the experimental results for every week considered. The overall efficiency was calculated for each week investigated. The results are given in Table 6.

Table 6: Overall efficiency (%)

Week #	Model	Experim.	Diff. %
1	75.4	72.1	4.7
2	68.3	65.4	4.4
3	70.1	68.0	3.2
4	58.6	59.8	-2.0

This validation series show that the model offer good agreement for mid-efficiency units.

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CONCLUSIONS

Conclusions from this validation work are that the model is providing reasonable estimates for the performances of the units examined. Further validation of some of the empirical correlation employed in the model would be useful in increasing the confidence in the model. Off cycle flow losses are of particular interest in the present case. The validation work also demonstrated the usefulness of an analytically- based tool in identifying or diagnostic of incoherent experimental data or possible uncertainties. Further validation work with regard to a few other options in the model is required, specifically for the HRV model. The model does not account for line losses and assumes that all water leaving the tank is part of the useful load. This may point to the need of evaluating the actual impact of line losses and eventually accounting for theses losses in the model.

Other aspects of the model may also gain from further study, such as:

- potential sources of inefficiencies during on-cycle that are not considered in the model,
- more detailed treatment of the jacket heat losses,
- long term efficiency reductions (i.e. fouling),
- effect of stratification on the performance of combo units, especially condensing units.

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NOMENCLATURE

c_{pair} = specific heat of combustion air

$c_{p,f\text{-}low}$ = specific heat of combustion gases

$c_{p.ng}$ = specific heat of fuel

$c_{p,water}$ = specific heat of water

$c_{p,watervapor}$ = specific heat of water vapor

C_{w+m} = Heat capacity of water + metal mass

D_a = available draft at flue exit

D_t = theoretical stack effect draft

HHV: fuel higher heating value

h_{air} = inlet enthalpy of combustion air

h_L = latent heat of vaporization of water

h_{vap} = reference enthalpy of water vapor

m_{air} = combustion air mass flow rate

m_{draw} = DHW water draw mass flow rate

m_{house} = mass flow rate of house air

m_{ng} = mass flow rate of fuel

q_{burner} = burner capacity

q_{draw} = domestic hot water demand

$q_{elect.,internal}$ = heat loss to surrounding from motor

$q_{f\text{-}t}$ = heat transfer between flue gas and tank

q_{in} = burner net power input to the system

q_J = jacket heat losses

q_{space} = space heating demand

q_{stk} = heat loss through the stack

T_{air} = temperature of combustion air

T_{city} = temperature of city water

T_{dew} = dew point temperature of combustion Product

$T_{house,r}$ = house return air temperature

$T_{f,in}$ = inlet flue gas temperature

$T_{f,out}$ = exit flue temperature

$T_{mixt,in}$ = inlet temperature of the fuel air mixture

$T_{ng,i}$ = fuel inlet temperature

T_{room} = temperature of surrounding

T_{stk} = flue gas outlet temperature

T_T = average tank temperature

$(UA)_{flue}$ = total UA of flue

$(UA)_J$ = total UA for jacket loss

W_{air} = humidity ratio of combustion air

X_{H2O} : ratio of mass of water in combustion products to mass of fuel (for natural gas approx. 2.136 lb/lb stoichiometric)

ε = coil effectiveness

Δp = pressure drop through chimney

κ = stoichiometric air requirement

λ = excess air ratio