NUMERICAL SIMULATION ON SIMULTANEOUS CONTROL PROCESS OF INDOOR AIR TEMPERATURE AND HUMIDITY

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ABSTRACT

In order to evaluate the air conditioning system performance in terms of control, comfort and energy conservation, this paper presents an approach to modeling automatic control process in a typical conditioned space of an office building. As air-conditioning system, constant volume single-duct system is adopted for interior zone and FCU system for perimeter zone. As for simulation model, we adopted dynamic calculation model expanded by state transition method [1]. The changes of indoor ambient temperature and relative humidity, the PMVs and the energy consumption are investigated by these simulations. The comparison was made between two cases, one is controlled by temperature only and the other is simultaneously controlled by temperature and humidity.

INTRODUCTION

The simulation of air conditioning system is classified into static simulation and dynamic simulation. The static simulation is often used to analyze indoor environment and energy consumption of air-conditioning system at 1-hour intervals. The dynamic characteristics of each component and control system are usually neglected in the simulation process, except for load calculation of the building with high thermal capacity and large time lag. By static simulation, it is impossible to know about influence given to indoor environment and energy consumption by the dynamic characteristics of them. On the other hand, by dynamic simulation using dynamic calculation model of each component, the characteristics of buildings, air conditioning facilities and control system can be simulated more accurately. By the dynamic simulation computing at short intervals of several seconds or several minutes, indoor air temperature and humidity change, controllability of control system and heat extraction rate of coil can be analyzed more according with actual movement.

The study about simultaneous control process of indoor air temperature and humidity in air conditioning system is carried out currently. Simulation program HVACSIM$^+$ developed by U.S. National Bureau of Standards [2] utilizes research result of Elmahdy and Mitalas [3] on chilled water coil to simulate simultaneous control process of indoor air temperature and humidity. Because heat balance and mass balance about tubes and fins are neglected as for calculation model of chilled water coil in HVACSIM$^+$, the effect of the thermal capacity on dynamic characteristic of the coil can't be reflected by calculation result. Zaheer-uddin [4] showed an approach of simultaneous control process in winter about air conditioning system which consists of a boiler, a hot water coil, a humidifier and a control system, but there is not a reference about the simulation of the summer when use chilled water coil. Yamashita et al. [5] showed fundamental investigation about indoor air temperature/humidity control and the interior static pressure calculation for a variable air volume air-conditioning system. In the research of Yamashita et al., the static model of chilled water coil is used however.

This paper presents a dynamic simulation by state transition method on PID control process for air-conditioning system. The object space is an office room with constant volume single-duct system for interior zone and FCU system for perimeter zone. The simulation is carried out from June 1st to August 31st.

SYSTEM DESCRIPTION

Schematic of air conditioning system is shown in Fig.1. The building is located in Fukuoka of South Japan. The object space is a typical office room on typical floor and faces south. The heat gain (loss) through partitions with the adjacent conditioned room is neglected. A dimension of the room is 6m width, 13m depth and 3.69m floor height. The space is divided into two conditioning zones, 3m window side for perimeter zone and the rest 10m for interior zone. The building data are shown in Table 1. The perimeter zone is conditioned by two fan coil units (FCUs) and the interior zone by an air handling unit (AHU). The specifications of FCUs and AHU are shown in Table 2 and Table 3. Humidity control of both perimeter and interior zone depends on this interior AHU. The constant volume single duct system is adopted in interior zone. The supply air is distributed through underfloor plenum and the return air is returned through ceiling plenum. The feedback control system of perimeter zone is shown in Fig.2. The temperature sensor measures

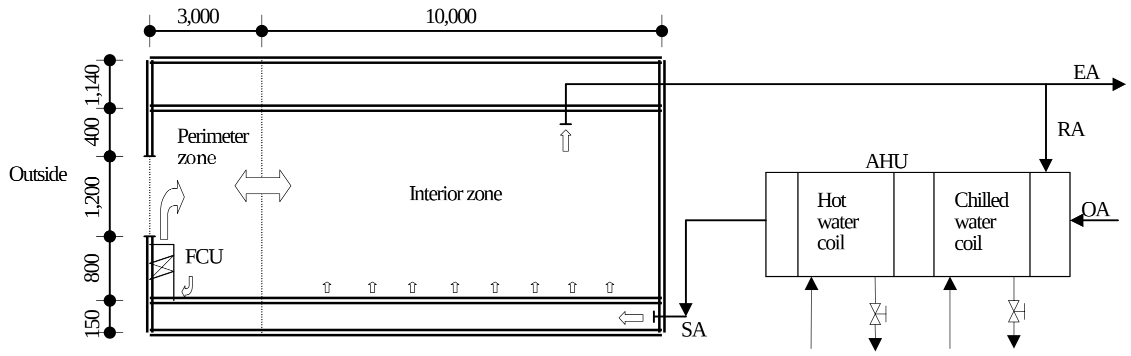

Figure 1 Temperature / relative humidity control system (unit : mm)

Table 1 The constructions of building element

Building Elements	Materials	Thickness (mm)	Building Elements	Materials	Thickness (mm)
Exterior wall (under the window)	Foaming rigid polyurethane (indoor side) Concrete Tile (outdoor side)	15 150 10	Interior wall	Plaster (side of the office) Mortar Concrete Mortar Plaster (side of adjacent room)	3 15 100 15 3
Exterior wall (above the window)	Gypsum board (indoor side) Concrete Tile (outdoor side)	25 150 10	Free access floor	Tile carpet (side of the office) Tile	7 15
Ceiling	Formed plywood of rock wood Plywood	15 9	Floor	Concrete Mortar	175
			Window	Plate glass	3

Table 2 Specification of fan coil unit

Fin material	Aluminum	Tube row spacing	20.7 mm
Fin thickness	0.2 mm	Coil air face tubes	4
Fin spacing	2.1 mm	Coil air face tube spacing	25.4 mm
Tube material	Copper	Coil face area	0.1016 m^2
Tube outside diameter	9.6 mm	Air volume	350 m^3/h
Tube inside diameter	8.4 mm	Water velocity	0 - 1.5 m/s
Tube length	1,000 mm	Inlet water temperature	7 deg. C
Coil rows	2		

Table 3 Specification of air handling unit

	Chilled water coil	Hot water coil		Chilled water coil	Hot water coil
Fin material	Aluminum	Aluminum	Tube row spacing	75 mm	75 mm
Fin thickness	0.5 mm	0.5 mm	Coil air face tubes	8	8
Fin spacing	3.2 mm	3.2 mm	Coil air face tube spacing	65 mm	65 mm
Tube material	Copper	Copper	Coil face area	0.52 m^2	0.52 m^2
Tube outside diameter	15.5 mm	15.5 mm	Air volume	1,500 m^3/h	1,500 m^3/h
Tube inside diameter	14.5 mm	14.5 mm	Water velocity	0 – 1.5 m/s	0 - 1.5 m/s
Tube length	1,000 mm	1,000 mm	Inlet water temperature	7 deg. C	40 deg. C
Coil rows	6	1			

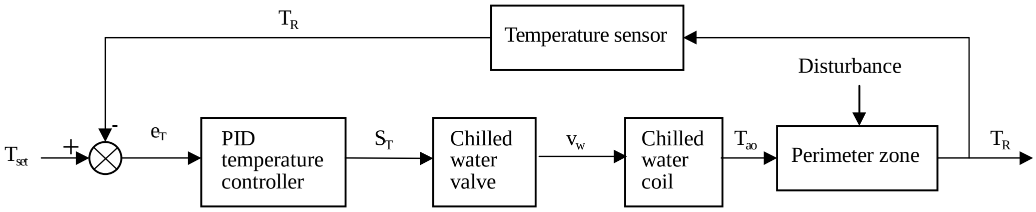

Figure 2 Schematic diagram of the feedback system in perimeter zone

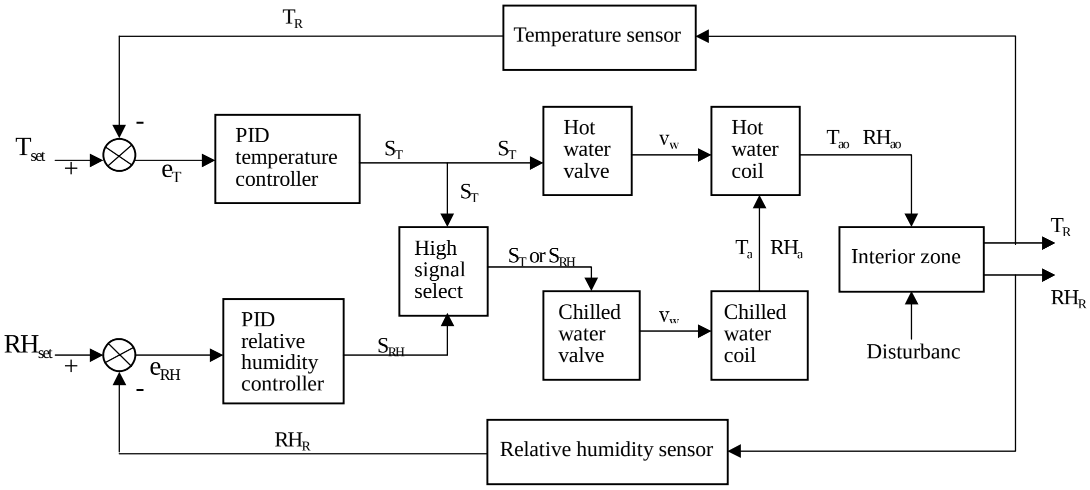

Figure 3 Schematic diagram of the feedback system in interior zone

the controlled variable (air temperature) of the perimeter zone. Output of the temperature sensor is compared with set point and deviation is calculated. The signal is conveyed to the controller and PID operation is added to the input. The controlled device (chilled water valve of the fan coil unit) reacts to signals (output of temperature controller) to vary the flow of chilled water. The feedback control system of interior zone is shown in Fig.3. Temperature and relative humidity of interior zone is measured with temperature sensor and relative humidity sensor. Outputs of the temperature sensor and humidity sensor are compared with each set point, and deviations are calculated. The temperature deviation and relative humidity deviation are input into each of the controllers and PID operation is added to the input signals. The outputs of temperature controller and relative humidity controller are input into high signal select and are compared in it. The highest signal becomes output of high signal select. Chilled water valve of air handling unit is adjusted by the output of high signal select. The hot water valve of air handling unit is regulated by output of temperature controller.

SIMULATION MODELS

The dynamic calculation models of the air conditioning system components are as follows.

1) BUILDING

The HVAC system consists of many components. The transient responses in some components occur relatively slowly and those in the others occur relatively quickly. Therefore it is desirable that the computation intervals can be freely selected in different components at any time.

In this paper, the unsteady state heat conduction through the walls is calculated by the state transition method [1] in which the computation intervals can be freely selected. Detail of the state transition method is shown by appendix 1, which is included in the electronic proceedings (CD-ROM). For simplifying the calculation, we assume that the office has a window without a solar shading device and the transmitted solar radiation projects on the floor of perimeter zone evenly. The heat exchanges of the long wavelength radiation and the short wavelength radiation among the inside surfaces of the walls are calculated by the Gebhart's method [6]. As for the temperature distribution and humidity distribution, non-uniformity in the space is neglected.

2) CHILLED/HOT WATER COIL

Heat balance and mass balance are applied to a finned tube, and governing differential equations can be obtained. The equations can be expressed in terms of the s-plane variable rather than time when the Laplace transform is used. Using the aforementioned state transition method, we can obtain the relationship of inputs and outputs in time domain. Based on the relationships of inputs and outputs about a finned tube, the relationships of inputs and outputs about the coil can be obtained by solving the simultaneous equation [7]. Detail of dynamic model on chilled/hot water coil is shown by appendix 2, which is included in the electronic proceedings (CD-ROM). Because saturation relative humidity line originally has non-linear property, repetition calculation becomes indispensable to calculate air relative humidity. The saturation relative humidity line is calculated with linear expression to shorten computation time of cooling dehumidifying coil in this paper. The coefficient of the linear expression is decided using current calculation results by the least square method, and it is employed in the calculation of the next step.

3) CONTROLLER

A general equation that describes proportional, integral and derivative control is as follows;

$$S=K_P(e+\int e\,dt/T_I+T_D\,de/dt) \quad (1)$$

where

S : the controller output

e : error, the difference of indoor air temperature (or air relative humidity) and set point

t : time, [s]

K_P: the proportional gain

T_I :integral time, [s]

T_D:derivative time,[s]

In DDC system, the PID algorithm must be represented in discrete form as follows;

$$S(n)=K_P e(n)+K_I \triangle t \sum_{i=1}^{n} e(i)+K_D[e(n)-e(n-1)]/\triangle t \quad (2)$$

where

n : time number

$\triangle t$: sampling period, [s]

$K_I = K_P / T_I$: integral gain

$K_D = K_P T_D$: derivative gain

4) SENSOR

The dynamics of sensor is given by

$$T_C d\theta/dt+\theta=\theta' \quad (3)$$

where

θ : air temperature or air relative humidity measured by the sensor, [deg] or [%]

θ': actual indoor air temperature or air relative humidity, [deg] or [%]

T_C : time constant of the sensor, [s]

The discrete form of equation (3) is as follows;

$$\theta(n)=\phi(n)\,\theta(n-1)+p(n)\,\theta'(n-1)+q(n)\,\theta'(n) \quad (4)$$

where

$\phi(n)=\exp(-\triangle t/ T_C)$

$p(n)=-\{\phi(n)+T_C[1-\phi(n)]/\triangle t\}$

$q(n)=-p(n)+[1-\phi(n)]$

5) CHILLED/HOT WATER VALVE

An equal percentage valve is used for simulation in this paper. The relationship of water flow rate and position is given by

$$G_w=G_{wmax}R^{(p-1.0)} \quad (0<p<=1.0) \quad (5)$$

$$G_w=0.0 \quad (p=0.0) \quad (6)$$

where

G_w : water flow rate, [kg/s]

$G_{w\,max}$: maximum water flow rate, [kg/s]

p : position of valve, (0.0-1.0)

$R= G_{w\,max}/ G_{w\,min}$

G_{wmin} : minimum water flow rate, [kg/s]

The hysteresis of the actuator when not equipped with a positioner will move as a function of the pilot signal, is given in Fig. 4.s

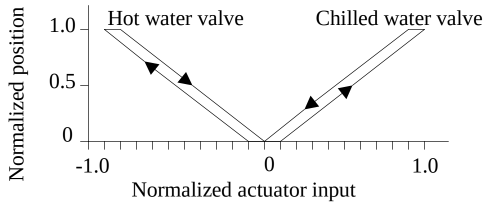

Figure 4 Hysteresis model

SIMULATION CONDITIONS

The air conditioning schedule is from 8:00 a.m. to 6:00 p.m. everyday and 5 days a week. The calculation is carried out for 3 months from June 1st to August 31st. Air temperature set point both in perimeter zone and interior zone is set at 25.0 degree. In case of simultaneous control of temperature and relative humidity, relative humidity set point of interior zone is set at 50%RH. Heat gain in conditioned spaces (occupants + lighting + power + appliances) are assumed as follows. Sensible heat gain is 1.0kW and generation of vaporizing water (latent heat gain) is 0.28 kg/h in perimeter zone. Sensible heat gain is 3.65kW and generation of vaporizing water (latent heat gain) is 1.11 kg/h in interior zone. Amount of outdoor air introduced into the space is 450 m^3/h. As conditions for PMV, 1.1 met is given for activity levels, 0.6 clo for clothing and 0.15 m/s (0.05 m/s, out of operation) for air velocity. The parameters for controller are given in Table 4. As for outdoor design conditions, Standard Weather Data of Fukuoka is adopted. Computation interval is set at 10 seconds. Automatic valve is controlled every 60 seconds to regulate water flow. As weather data, every hour data is treated by linear-interpolation. The computation is started 6 days before the beginning day June 1st. Infiltration is neglected. Actually amount of air exchange between perimeter zone and interior zone is influenced by various factor. It is very difficult to determine it as accuracy. Because perimeter zone area is approximately 1/3 of interior zone area, in calculation condition of this time, amount of air exchange between perimeter zone and interior zone is assumed to 1/3 of air handling unit supply air (i.e. 500m^3/h).

Table 4 Parameters of controller

	Perimeter zone	Interior zone	
	Air temperature controller	Air temperature controller	Air relative humidity controller
Proportional gain K_P	15.0	15.0	250.0
Integral gain K_I	0.01	0.001	0.2
Derivative gain K_D	20.0	20.0	250.0

SIMULATION RESULTS

Figure 5 shows the energy consumption in the cases of temperature-only control and simultaneous control of indoor air temperature and humidity. In the case of simultaneous control process of indoor air temperature and humidity, total energy consumption is 60% higher than in the case of temperature-only control process. The heat extraction of AHU is 40% higher than in the case of temperature-only control process. In terms of reheating energy consumption reaches 4,148.6MJ in the case of simultaneous control, comparing to 46.4MJ in the case of temperature-only control process.

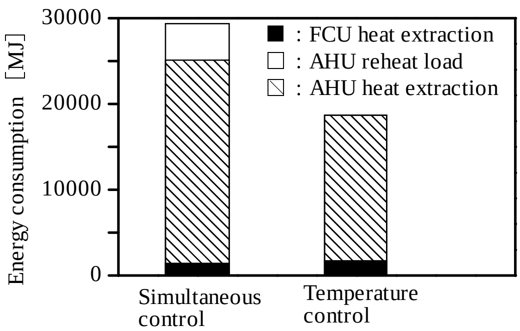

Figure 5 Energy consumption of simultaneous control process and temperature control process from June 1st to August 31st

Table 5 shows the average and standard deviations of room air temperature, relative humidity and PMV in the conditioned space during 3 months from June 1st to August 31st. As Table 5 shows, average room air temperature in the case of temperature-only control is 0.2 degree higher than in the case of simultaneous control. The energy consumption is partly effected by these room air temperature difference, but the difference of energy consumption in these two cases is mainly caused by the overcooling and reheating energy to control humidity of interior zone

The simulation results of Fig.5 show that the heat extraction of fan coil unit in the case of temperature-only control is 20% higher than in the case of simultaneous control process. The difference of

Table 5 Average value of the indoor air temperature, relative humidity and PMV

		Temperature control	Simultaneous control
Perimeter zone	Temperature [deg.] (standard deviation)	25.18 (0.42)	25.04 (0.30)
	Relative humidity [%] (standard deviation)	59.31 (5.97)	51.56 (7.96)
	PMV (standard deviation)	0.31 (0.16)	0.21 (0.14)
Interior zone	Temperature [deg.] (standard deviation)	25.17 (0.20)	24.95 (0.18)
	Relative humidity [%] (standard deviation)	60.52 (4.16)	50.16 (10.48)
	PMV (standard deviation)	0.20 (0.05)	0.07 (0.06)

room air temperature between perimeter zone and interior zone in the latter case is larger than that in the former case (Table 5). This means that the interior AHU covers a part of perimeter load by air mixing. Table 5 shows also that the simultaneous control of interior temperature and humidity reduces approximately 10% of indoor air relative humidity in perimeter zone and interior zone. As a result, PMVs of the perimeter zone and the interior zone are improved by around 0.1 points.

Figure 6 shows the weather conditions from July 9th (Mon.) to July 13th (Fri.). Figure 7-10 show the transition of room air temperature, humidity, PMV and water velocity in the coil.

Figure 7 shows the change of room air temperature and Fig.9 shows that of humidity. In temperature-only control, relative humidities of interior zone and perimeter zone both reach more than 60%RH. Especially in July 9th when outdoor air was over 90%RH humid, both of them reach 65% to 70%RH. In simultaneous control, the relative humidity of interior zone shows 50%RH or the setting point and that of perimeter zone shows under 55%RH. As the result, the PMV of interior zone is improved by 0.1 to 0.2 points and the PMV of perimeter zone is improved by 0.1 points (Fig.8, Fig.10).

In simultaneous control, room air temperature tends to swing and it takes time to reach stable condition, because of interaction between the temperature control and the humidity control (Fig.7, Fig.9).

Figure 8 shows the water velocity in the coil of AHU. In simultaneous control, chilled water velocity in the coil of AHU becomes very large to dehumidify. Besides, hot water coil is needed reheat. In order to dehumidify, air temperature should be once cooled under the dew point of room air temperature, and then reheated to a certain supply air temperature. This over-cooling and reheating cost more energy consumption to temperature-only control.

In simultaneous control, it is rather difficult to set the parameters of controller properly. Figure 7 shows that the setting time of chilled water velocity at pull-down could be considerably long in simultaneous control.

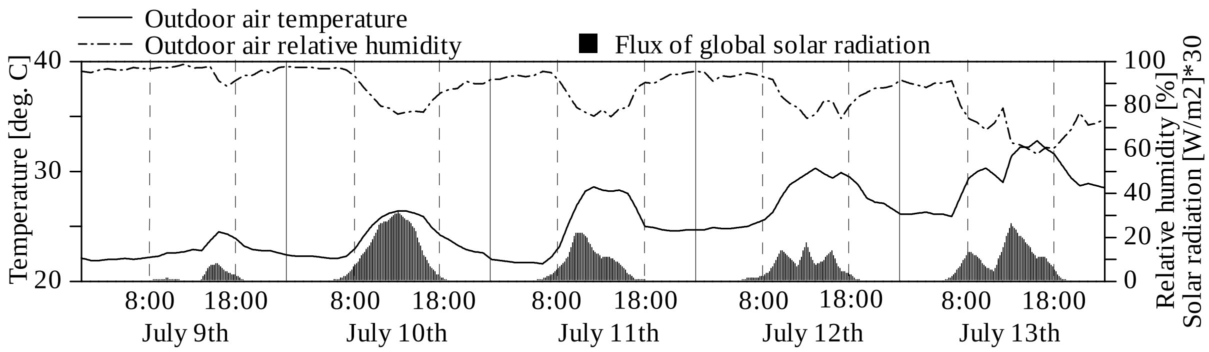

Figure 6 Standard weather data of July in Fukuoka, Japan

Figure 10 shows the difference of chilled water velocity in fan coil unit between temperature-only control and simultaneous control of interior zone. When interior air temperature and humidity are simultaneously controlled by AHU, chilled water velocity in fan coil unit of perimeter zone tends to reduce comparing to temperature-only control. In Fig.9, in simultaneous control, perimeter air temperature swings a little, but average temperature is equal to temperature-only control. On the other hand, interior air temperature of simultaneous control is 0.3 degree lower than the other (Fig.7). This is when interior air temperature and humidity are simultaneously controlled by interior zone AHU; the AHU covers also a part of perimeter load, which should be originally treated by perimeter fan coil units.

The outdoor air temperature of July 13th was so high, cooling load in perimeter zone was increased. But, because it is assumed in this simulation that air exchange between interior zone and perimeter zone is constant (500m^3/h), cooling by the air mixing between perimeter zone and interior zone reached to the limit and the automatic valves of FCUs are fully opened.

CONCLUSIONS

About dynamic simulation of air conditioning system, simulation method [8], which assumed conditioned space with one zone, was submitted. However, changes of air temperature and humidity in the room with a difference of control action in perimeter FCU and interior AHU cannot be analyzed by that method. In simulation of this time, conditioned space is divided into perimeter zone and interior zone, and each zone has air conditioning facility and the control system, which is independence. Using the simulation method presented in this paper, air temperature/humidity change of each zone and energy consumption of fan coil unit and air handling unit can be investigated. The followings are found by this simulation.

1) Total energy consumption in the case of simultaneous control is 60% higher than that of temperature-only control case. Heat extraction of AHU in the former case is 40% higher than that in the latter case. As for integrated value of the reheat energy consumption from June 1st to August 31st, it reaches 4,148.6MJ in the former case.

2) The simultaneous control of interior temperature and humidity reduces approximately 10% of indoor air relative humidity in perimeter zone and interior zone. As a result, PMVs of the perimeter zone and the interior zone are improved by around 0.1 points.

3) In simultaneous control, room air temperature tends to swing and it takes time to reach stable condition, because of interaction between the temperature control and the humidity control.

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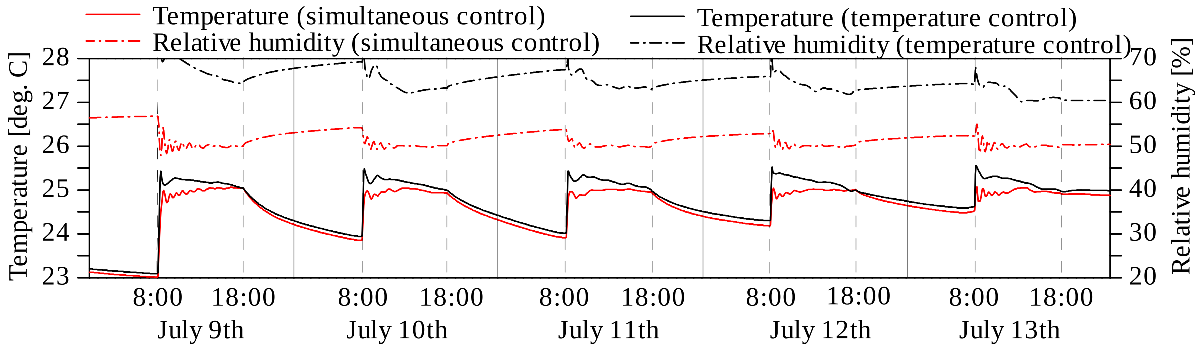

Figure 7 Responses of the interior zone air temperature and relative humidity

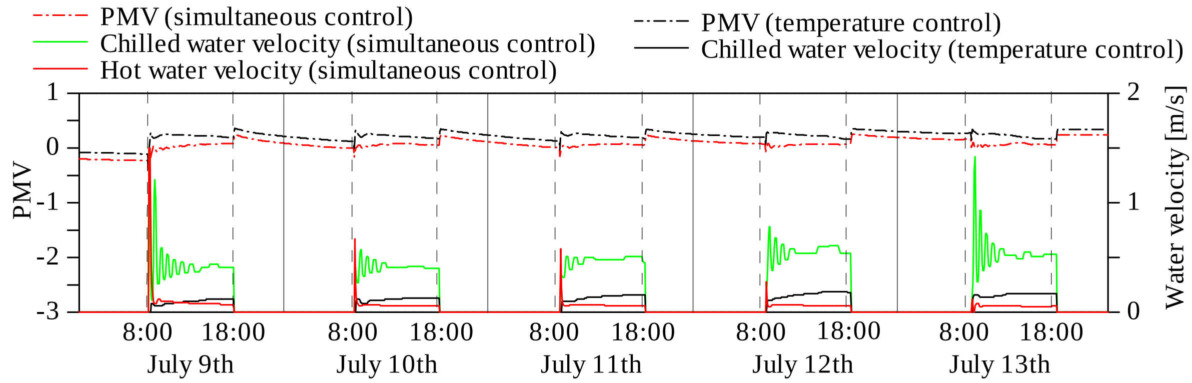

Figure 8 Responses of the interior zone PMV and water velocity in air handling unit

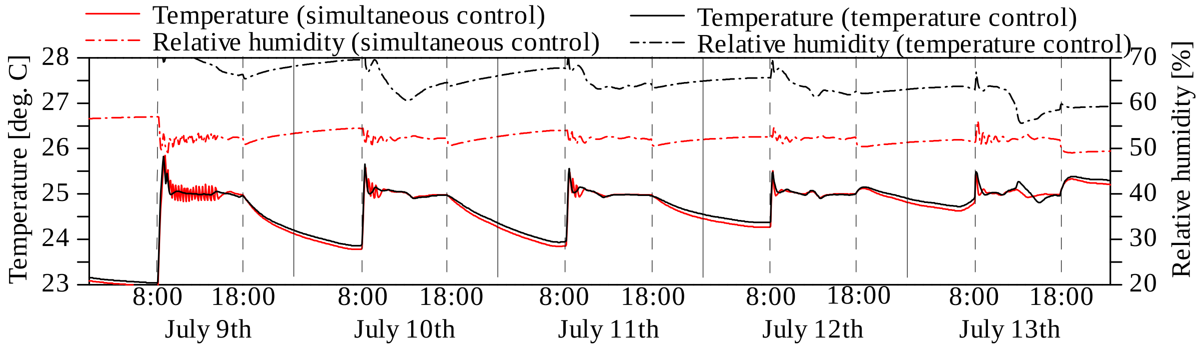

Figure 9 Responses of the perimeter zone air temperature and relative humidity

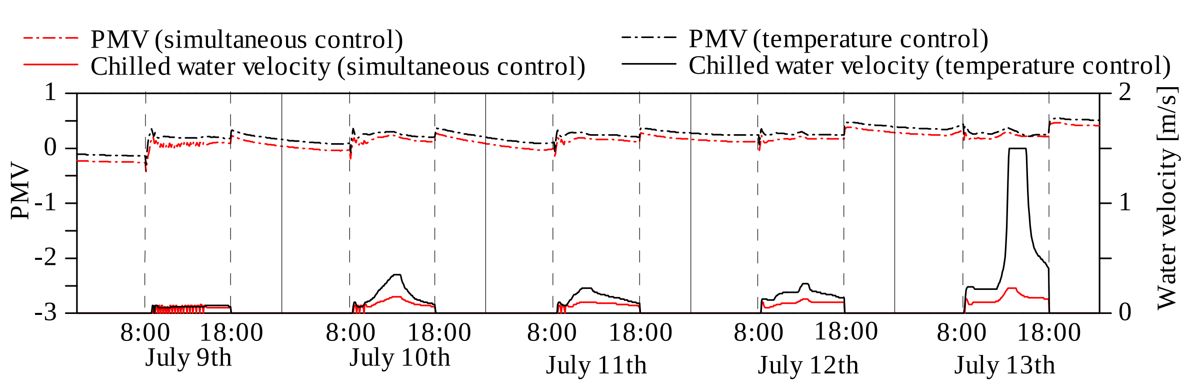

Figure 10 Responses of perimeter zone PMV and water velocity in fan coil unit

APPENDIX 1 DYNAMIC CALCULATION OF MULTI-LAYER WALL HEAT CONDUCTION BY A STATE TRANSITION METHOD [1]

When one dimension heat conduction of multi-layer wall is presumed, generally indicial response of surface heat flow for unit function excitation of air temperature (or the surface temperature) can be shown with the next expression.

$$\phi(t)=A_0+\sum_{k=1}^{\infty} A_k\exp(-\alpha_k t) \qquad (1\text{-}1)$$

where

A_0: coefficient of over-all thermal transmission

A_k: coefficient of thermal transmission (or admission) response

$-\alpha_k$: root of the characteristic expression for the thermal transmission system through walls, ($\alpha_k>0$)

When the indicial response of surface heat flow rate is actually computed, the upper limit of k is broken off by the suitable value K0, and the clause beyond K0+1 is made instantaneous heat rate.

$$\phi(t)=A_0+\sum_{k=1}^{K0} A_k\exp(-\alpha_k t)+Q\delta(t) \qquad (1\text{-}2)$$

where

$\delta(t)$: Dirac's delta function

Q: heat flow rate at $t=0^+$, and it can be shown as follows.

$$Q=\sum_{k=K0+1}^{\infty} A_k/\alpha_k \qquad (1\text{-}3)$$

Transfer function G(s) of the multi-layer wall surface heat flow for a unit function excitation can be written by next expression with multiplying Laplace transform of the expression (1-1) by s.

$$G(s)=A_0+\sum_{k=1}^{K0}[A_k s/(s+\alpha_k)]+Qs \qquad (1\text{-}4)$$

In the following, this paper assumes transfer function G(s) of arbitrary multi-layer wall with a thing of known information. Outputs of the heat conduction system on the multi-layer wall for inputs can be expressed in various expressions. Because real input and output of the multi-layer wall is discrete-time system, using z-transform, discrete data of input and output at $t=n\triangle t$ can be represented like next.

$$U(z)=u_0+u_1z^{-1}+u_2z^{-2}+\cdots \qquad (1\text{-}5)$$

$$Y(z)=y_0+y_1z^{-1}+y_2z^{-2}+\cdots \qquad (1\text{-}6)$$

where

$\triangle t$: sampling period

U(z): z-transform of input u(t)

Y(z): z-transform of output y(t)

If pulse transfer function G(z) is used, relationship of the input and the output of discrete-time system can be given by next expression.

$$Y(z)=G(z)\cdot U(z) \qquad (1\text{-}7)$$

Trapezoid hold function is used to reproduce input of discrete-time system in this paper. Laplace transform of trapezoid hold function can be expressed in expression (1-8).

$$G_h(s)=[1-\exp(-s\triangle t)]/s+h\{-\exp(-s\triangle t)/s +[1-\exp(-s\triangle t)]/(s^2\triangle t)\} \qquad (1\text{-}8)$$

where

h: gradient parameter, $hu_{n-1}= u_n-u_{n-1}$

When the expression (1-4) and the expression (1-8) are used, pulse transfer function G(z) can be written by following.

$$\begin{aligned}G(z)&=Z[G(s)G_h(s)]\\ &=A_0+\sum_{k=1}^{K0}A_k(1-z^{-1})/[1-z^{-1}\exp(-\alpha_k\triangle t)]\\ &+h\sum_{k=1}^{K0}\{A_k[1-\exp(-\alpha_k\triangle t)-\alpha_k\triangle t]/(\alpha_k\triangle t)\\ &\times z^{-1}/[1-z^{-1}\exp(-\alpha_k\triangle t)]\}+Q(1-z^{-1})/\triangle t\end{aligned} \qquad (1\text{-}9)$$

Product of element of right side Sub-Section 2,3 of the expression (1-9) and discrete data of input U(z) is defined as state variables $W_k(z)$ about k here.

$$\begin{aligned}W_k(z)=\{&A_k(1-z^{-1})/[1-z^{-1}\exp(-\alpha_k\triangle t)]\\ &+hA_k[1-\exp(-\alpha_k\triangle t)-\alpha_k\triangle t]/(\alpha_k\triangle t)\\ &\times z^{-1}/[1-z^{-1}\exp(-\alpha_k\triangle t)]\}U(z)\end{aligned} \qquad (1\text{-}10)$$

where

$$W_k(z)=w_{k,0}+w_{k,1}z^{-1}+w_{k,2}z^{-2}+\cdots \qquad (1\text{-}11)$$

$$U(z)=u_0+u_1z^{-1}+u_2z^{-2}+\cdots \qquad (1\text{-}12)$$

A state transition expression about $w_{k,n}$ is provided by substituting expression (1-11) and (1-12) into the (1-9), and comparing the coefficients of z^{-n} .

$$\begin{aligned}w_{k,n}=&w_{k,n-1}\exp(-\alpha_k\triangle t)\\ &+A_k(u_n-u_{n-1})[1-\exp(-\alpha_k\triangle t)]/(\alpha_k\triangle t)\end{aligned} \qquad (1\text{-}13)$$

where

$w_{k,0}$:$A_k u_0$

Similarly, by substituting the expression (1-9) into the expression (1-7), output expression of the multi-layer wall heat conduction system can be written by following.

$$y_n=A_0u_n+\sum_{k=1}^{K0}w_{k,n}+Q(u_n-u_{n-1})/\triangle t \qquad (1\text{-}14)$$

When indoor air temperature and humidity, sensors of temperature and humidity are adapted to the state transition concept that used trapezoid hold function, state transition expression of those can be derived.

APPENDIX 2 A DYNAMIC CALCULATION MODEL OF CHILLED WATER COIL [7]

When heat balance and mass balance are considered about water, air, tube and fin in one finned tube, the next expressions are provided.

$$\partial\theta_w/\partial t+v_w\partial\theta_w/\partial x=A\alpha_w(\theta_t-\theta_w) \qquad (2\text{-}1)$$

$$\partial\theta_a/\partial t+BG_c\partial\theta_a/\partial y=D\alpha_s(\theta_t-\theta_a) +E\alpha_s(\theta_f-\theta_a) \qquad (2\text{-}2)$$

$$\partial\theta_t/\partial t=F\alpha_s(\theta_w-\theta_t)+H\alpha_s(\theta_a-\theta_t) +K(\theta_f-\theta_t)+Vk_x(d_a-d_t) \qquad (2\text{-}3)$$

$$\partial\theta_f/\partial t=L\alpha_s(\theta_a-\theta_f)+M(\theta_t-\theta_f)+Yk_x(d_a-d_f) \qquad (2\text{-}4)$$

$$\partial d_a/\partial t+BG_c\,\partial d_a/\partial y=Z'k_x(d_t-d_a)+Y'k_x(d_f-d_a) \quad (2\text{-}5)$$

where

θ: temperature, [deg]

d: air absolute humidity, [kg/kg']

α_s: outside heat transfer coefficient, [W/(m^{2}K)]

α_w: inside heat transfer coefficient, [W/(m^{2}K)]

k_x: mass transfer coefficient, [kg/(m^{2}skg/kg')]

v_w: water velocity in tube, [m/s]

G_c: air volume, [m^3/s]

x: direction of water flow, [m]

y: direction of air flow, [m]

subscript w: water

subscript a: air

subscript t: tube

subscript f: fin

A,B,D,E,F,H,K,L,M,V,Y,Z',Y': coefficient decided by specification and material of finned tube and flow

Relationship of temperature and saturation absolute humidity is similar with next linear expression.

$$d=b_0+b_1\theta \quad (2\text{-}6)$$

Combining expressions (2-1)-(2-5) of discretization on space coordinates x, y and Laplace transforming from the time domain to the frequency domain, following is provided as 0 initial condition.

$$\begin{pmatrix} s+a_1 & 0 & -a_3 & 0 & 0 \\ 0 & s+a_4 & -a_6 & -a_7 & 0 \\ -a_9 & -a_{10} & s+a_8 & -a_{11} & -a_{12} \\ 0 & -a_{14} & -a_{15} & s+a_{13} & -a_{16} \\ 0 & 0 & -a_{19} & -a_{20} & s+a_{17} \end{pmatrix} \begin{pmatrix} \Theta_{wo} \\ \Theta_{ao} \\ \Theta_t \\ \Theta_f \\ D_{ao}' \end{pmatrix} = \begin{pmatrix} a_2 & 0 & 0 \\ 0 & a_5 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_{18} \end{pmatrix} \begin{pmatrix} \Theta_{wi} \\ \Theta_{ai} \\ D_{ai}' \end{pmatrix} \quad (2\text{-}7)$$

where

$a_1,a_2\cdots a_{20}$: coefficient decided by specification and material of finned tube and flow

s: Laplace transform variable

subscript o: outlet

subscript i: inlet

D_{ao}':$D_{ao}-b_0$

D_{ai}':$D_{ai}-b_0$

When coefficient matrix of left side in the expression (2-7) is turned with ***A*** and (2-7) is rearranged, the input and output expression in frequency domain can be written by following.

$$\begin{pmatrix} \Theta_{wo} \\ \Theta_{ao} \\ \Theta_t \\ \Theta_f \\ D_{ao}' \end{pmatrix} = \frac{1}{\det\mathbf{A}} \begin{pmatrix} a_2A_{1,1} & a_5A_{2,1} & a_{18}A_{5,1} \\ a_2A_{1,2} & a_5A_{2,2} & a_{18}A_{5,2} \\ a_2A_{1,3} & a_5A_{2,3} & a_{18}A_{5,3} \\ a_2A_{1,4} & a_5A_{2,4} & a_{18}A_{5,4} \\ a_2A_{1,5} & a_5A_{2,5} & a_{18}A_{5,5} \end{pmatrix} \begin{pmatrix} \Theta_{wi} \\ \Theta_{ai} \\ D_{ai}' \end{pmatrix} \quad (2\text{-}8)$$

where

$\det\mathbf{A}$: polynomial expression which s is a variable

$A_{i,j}$: algebraic complement of the matrix $\mathbf{A}$

When Θ_{wi}=1/s, Θ_{ai}=1/s, D_{ai}'=1/s are substituted for the expression (2-8) and using inverse Laplace transform, the indicial response of exit water temperature, exit air temperature, exit air absolute humidity, tube and fin temperature can be derived as following.

$$\phi_{ij}(t)=A_{0,ij}+\sum_{k=1}^{ks}A_{k,ij}\exp(-\alpha_k t) \quad (2\text{-}9)$$

i=w,a,d

j=w,a,t,f,d

where

$-\alpha_k$: root of characteristic expression $\det\mathbf{A}=0$, ($\alpha_k>0$)

k_s: number of the roots

$A_{0,ij}$, $A_{k,ij}$: coefficient of indicial response decided by Heaviside expansion.

Referring to the expression (1-13) and (1-14) of APPENDIX 1, state transition expression and output expression for finned tube can be given.

$$w_{k,n}=w_{k,n-1}\exp(-\alpha_k\Delta t)+A_k(u_n-u_{n-1})[1-\exp(-\alpha_k\Delta t)]/(\alpha_k\Delta t) \quad (2\text{-}10)$$

$$y_n=A_0u_n+\sum_{k=1}^{ks}w_{k,n} \quad (2\text{-}11)$$

where

$w_{k,0}$:A_ku_0

Expression (2-12) is provided when substituting the expression (2-11) into the expression (2-10).

$$y_n=G_1u_n-G_2u_{n-1}+G_{3,n-1} \quad (2\text{-}12)$$

where

$$G_1=A_0+\sum_{k=1}^{ks}A_k[1-\exp(-\alpha_k\Delta t)]/(\alpha_k\Delta t)$$

$$G_2=\sum_{k=1}^{ks}A_k[1-\exp(-\alpha_k\Delta t)]/(\alpha_k\Delta t)$$

$$G_{3,n-1}=\sum_{k=1}^{ks}w_{k,n-1}\exp(-\alpha_k\Delta t)$$

G_1,G_2,$G_{3,n-1}$ are different by kinds of the output. G_1,G_2 are not related with inputs, but change by wet or dry condition of finned tube. $G_{3,n-1}$ depends on wet or dry condition of finned tube and kinds of the input. The outputs are determined by the sum of all inputs when analyzes input and output relationship of one finned-tube with the expression (2-12).

$$\begin{pmatrix} \theta_{wo,n} \\ \theta_{ao,n} \\ \theta_{t,n} \\ \theta_{f,n} \\ d_{ao,n}' \end{pmatrix} = \begin{pmatrix} G_{1,ww} & G_{1,aw} & G_{1,dw} \\ G_{1,wa} & G_{1,aa} & G_{1,da} \\ G_{1,wt} & G_{1,at} & G_{1,dt} \\ G_{1,wf} & G_{1,af} & G_{1,df} \\ G_{1,wd} & G_{1,ad} & G_{1,dd} \end{pmatrix} \begin{pmatrix} \theta_{wi,n} \\ \theta_{ai,n} \\ d_{ai,n}' \end{pmatrix} - \begin{pmatrix} G_{2ww} & G_{2,aw} & G_{2,dw} \\ G_{2,wa} & G_{2,aa} & G_{2,da} \\ G_{2,wt} & G_{2,at} & G_{2,dt} \\ G_{2,wf} & G_{2,af} & G_{2,df} \\ G_{2,wd} & G_{2,ad} & G_{2,dd} \end{pmatrix} \begin{pmatrix} \theta_{wi,n-1} \\ \theta_{ai,n-1} \\ d_{ai,n-1}' \end{pmatrix} + \begin{pmatrix} G_{3,n-1,ww}+G_{3,n-1,aw}+G_{3,n-1,dw} \\ G_{3,n-1,wa}+G_{3,n-1,aa}+G_{3,n-1,da} \\ G_{3,n-1,wt}+G_{3,n-1,at}+G_{3,n-1,dt} \\ G_{3,n-1,wf}+G_{3,n-1,af}+G_{3,n-1,df} \\ G_{3,n-1,wd}+G_{3,n-1,ad}+G_{3,n-1,dd} \end{pmatrix} \quad (2\text{-}13)$$

where

$G_{1,wa}$: G_1 of the expression (2-12)

subscript wa: exit air temperature response when inlet water temperature changes. The others resemble with this.

To calculate the outputs of the whole chilled water coil which consists of the finned tubes by the expression (2-13), interaction of the inputs and the outputs among the finned tubes have to be considered.