A SYSTEMS APPROACH TO THE OPTIMAL OPERATION OF HVAC PROCESSES IN BUILDINGS

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ABSTRACT

A systems approach is developed for predictive operation of HVAC processes in buildings, which is described with an objective function, a state-space model, and a number of constraints. The state-space model is transformed from z-transfer functions that are identified in real-time. The system constraints are described mathematically so that they can be easily applied. Dynamic programming techniques are combined with on-line simulation to enhance efficiency in searching for the optimal operation strategies. Simulation studies show that the techniques are computationally efficient. Results also indicate that the predictive control of floor heating systems may lead to significant savings in operating cost.

INTRODUCTION

A considerable amount of thermal mass in a radiant floor heating system can be utilised for dynamic operation. Predictive operation of electric heating systems can reduce peak power demands. Moreover, the average temperature of building thermal mass may be lowered through night set-point setback. Hence, more solar radiation energy transmitted through windows may be absorbed during the following day. Furthermore, the capacity of heating systems may also be reduced by means of predictive control of thermal energy storage. This makes it possible to reduce heating equipment size, which translates into savings in the initial cost. Consequently, the part load efficiency can be enhanced if a gas or oil boiler is used in a heating system.

There are two main techniques available in the theory of optimal control, such as the maximum principle as formulated by Pontryagin (1962) and dynamic programming presented by Bellman (1957). The computational efficiency and flexibility are key issues in choice of the technique for on-line optimisation of operation strategies. The maximum principle has been widely adopted in simulation studies on the dynamic operation of HVAC systems (Fan et al and Townsend et al). The technique is efficient for solving the problem without constraints. However, the efficiency will rapidly decrease with increasing number of constraints. The issue becomes more complicated when there are several operation stages in which different models may be employed. An HVAC system in operation is usually associated with a considerable number of constraints. In dealing with constraints, dynamic programming seems to be more flexible and efficient. The main disadvantage of this technique is that it may become inefficient or even infeasible if the dimension of system state variables is very large, or the number of possible system operating states is considerably high. The reason is that the optimal values of objective functions and stage decisions have to be calculated and stored in a computer for each state. This weakness could be remedied or partly overcome by reducing the number of discrete grids of states to be searched.

A framework of the optimal predictive control system was developed in the previous work (Chen and Athienitis 1996) for the dynamic operation of building thermal systems. It integrates a weather predictor, a setpoint optimiser, a system identifier, and a generalised predictive controller with feedforward control scheme to achieve high building thermal performance. The weather predictor and the generalised predictive controller have been presented in that article (Chen and Athienitis 1996).

The focus of this study is on development of the real-time setpoint optimiser, which is one of the essential parts of the optimal predictive control system. An objective function subject to a state-space model of the HVAC process and a number of operating constraints are established in the next section. An algorithm for multistage decision of the building thermal system is then presented. In the final section, the techniques will be applied to a floor heating system to show the potential for reduced energy consumption and operating cost.

MODEL FOR OPTIMAL OPERATION

A building thermal system model needs to be identified before applying any optimising techniques in real-time. The operative temperature is considered as the controlled variable. The thermal process may be modelled by the following z-transfer function:

$$A(z^{-1})T_e(t) = B(z^{-1})u(t-n_{td}) + \mathbf{H}(z^{-1})^T \mathbf{TS}(t) + e_o \quad (1)$$

where $T_e(t)$ is the globe temperature, which is approximately equal to the operative temperature; u(t) is a control input; **TS**(t) is a vector representing the measurable but independent sources, such as solar radiation and ambient temperature; e_o denotes average predict error; t is time; n_{td} is the discrete dead time; $A(z^{-1})$ and $B(z^{-1})$ are polynomials and $\mathbf{H}(z^{-1})$ is a polynomial vector in the backward operator.

$T_e(k)$ can be determined when the past operative temperatures $T_e(k-j)$ (j>0), the heat supply u(k-1) and weather conditions are known. Hence, $T_e(k)$ is chosen as state variables. Thus:

$$x_i(k) = T_e(k-(n-i)) \qquad (i = 1, \cdots, n-1) \tag{2}$$

It follows that:

$$x_i(k+1) = x_{i+1}(k)$$

To simplify notation, it is assumed that n_{td} is equal to 1. Substituting Equation (2) into (1) and considering only one term in the thermal energy supply, results in a discrete-time state equation in the vector-matrix form:

$$\mathbf{X}(k+1) = \mathbf{M}\,\mathbf{X}(k) + \mathbf{m_1}\,(b_1\,u(k\text{-}1) + \mathbf{H}(z^{-1})\mathbf{TS}(k) + e_o) \tag{3}$$

where the coefficient matrix **M** and the vector m_1 are given by:

$$\mathbf{M} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix}$$

$$\mathbf{m_1} = [\ 0 \quad 0 \quad \cdots \quad 1\ ]^T$$

and the vector of state variables is expressed by:

$$\mathbf{X}(k) = [x_1(k) \quad x_2(k) \quad \cdots \quad x_{n-1}(k)]^T$$

The objective function for predictive operation of the building thermal processes may be established as follows:

$$J^* = \min_{u(k)} \sum_{k=0}^{N-1} C_s(k)\, u(k)\, \Delta t \tag{4}$$

where superscript * indicates optimum; N is the number of time intervals over the period of interest; C_s represents the utility rate structure ($/kWH) and Δt the discrete time interval (Hour); u is supply heating power (kW), which is also called the decision variable here. C_s is set to 1 if the utility rate is constant or if energy consumption rather than operating cost is considered in applications.

The objective function is subject to the state-space model Equation (3), and the initial and final conditions as follows:

$$\mathbf{X} = \mathbf{X}_o, \quad \mathbf{X}_{N1} \le \mathbf{X}(N) \le \mathbf{X}_{N2} \tag{5}$$

where $\mathbf{X}_0$ is the initial state of the thermal system; $\mathbf{X}_{N1}$ and $\mathbf{X}_{N2}$ represent the acceptable range of the system state (i.e. indoor operative temperature) at the end of the period. Note that the global heating cost over the period of N stages is equal to the sum of separable criteria $C_s(k)u(k)\Delta t$ at each stage. A decision u(k) to be made will minimise the global operating cost. It implicitly depends on the current system states, because the objective function must be subject to the state-space model of the thermal process. This dependence can be expressed by a stage cost function:

$$C_s(k-1)\,u(k-1)\,\Delta t = L(\mathbf{X}(k), u(k\text{-}1), k) \tag{6}$$

There are some other constraints when operating the building thermal system. The supply heat must be less than the system capacity u_{max}:

$$0 \le u(k) \le u_{\max} \tag{7}$$

The indoor thermal comfort must be within the acceptable range recommended by ASHRAE Standard 55-1992, which is based on a 10% dissatisfaction criterion. The following comfort constraints must be satisfied for people in typical winter closing during, and light and primarily sedentary activity. The boundaries of the comfortable operative temperature may be expressed by:

$$\begin{gathered} T_e(k) \ge 19.82 + 0.016x(60 - RH(k)) \\ 30\% \le RH(k) \le 60\% \\[1em] T_e(k) \le 23.45 + 0.023x(60 - RH(k)) \\ 23\% \le RH(k) \le 60\% \end{gathered} \tag{8}$$

where RH is the relative humidity. The limits of the relative humidity may be described by:

$$\begin{gathered} (30 - 1.75\,x\,(T_e(k) - 20.3))\% \le RH(k) \le 60\% \\ 20.3 \le T_e(k) \le 24.3 \end{gathered} \tag{9}$$

and the mean air speed should be subject to:

$$\bar{v}_{air} \le 0.15\ m/s \tag{10}$$

Otherwise the indoor temperature has to be raised to offset increased air speed. The air speed should be limited by a well designed air distribution system. The surface temperature of the floor must be kept within:

$$18\,°C \le T_{sf} \le 29\,°C \tag{11}$$

The maximum rate of the operative temperature change should be subject to:

$$\frac{\Delta T_e(k)}{\Delta t} \leq 0.5\,°C/h \qquad (12)$$

During the occupancy time, the peak-to-peak amplitude $\Delta T_{max,ocp}$ of the temperature fluctuation is also limited by:

$$\Delta T_{\max,ocp} \leq 3.5°C \qquad (13)$$

MULTISTAGE DECISION OF THE THERMAL PROCESS

The problem can be significantly simplified by applying Bellman's principle of optimality (Bellman 1965). The global energy cost in Equation (4) is thus restructured in the recurrence form:

$$J^*[\mathbf{X}(k), k] = \min_{u(k-1)} \{L(\mathbf{X}(k), \mathbf{u}(k-1), k) + J^*[\mathbf{X}(k+1), k+1]\} \qquad (14)$$

where $J^*[\mathbf{X}(k), k]$ indicates the minimum energy cost obtained at stage k using a sequence of optimal decisions u^*_k ($u^*_k=\{u^*(k-1), u^*(k), \ldots, u^*(N-1)\}$. It can be observed from the above equation that the past and current decisions do not directly influence the future decisions. They affect the future decision indirectly through the future system states $\mathbf{X}(k+1)$. The current optimal decision $u^*(k-1)$ and minimum energy cost $J^*[\mathbf{X}(k), k]$ at stage k can be determined by minimisation of the sum of the current stage cost function $L(\mathbf{X}(k), u(k-1), k]$ and the future optimal cost function $J^*[\mathbf{X}(k+1), k+1]$, determined at stage k+1. There are two search procedures in application of dynamic programming. The first is a backward procedure (k=N-1, N-2, …, 0) in which the minimum cost function is recurrently computed from the final stage N to the initial stage. Thus, the optimal cost function $J^*[\mathbf{X}(k), k]$ at stage k is recursively calculated using the minimum cost function $J^*[\mathbf{X}(k+1), k+1]$ obtained at stage k+1. The second procedure is a forward process in which the optimal system states (i.e. set-points) are computed from the initial stage to the final stage. The initial or past system states and the optimal decisions found in the previous procedure are recurrently substituted into the z-transfer functions (1) to determine the optimal set-points and heat supply curve.

The required final system condition (5) is treated by:

$$J^*[\mathbf{X}(N), N] = \begin{cases} 0 & \mathbf{X}(N) \in [\mathbf{X}_{N1}, \mathbf{X}_{N2}] \\ \infty & \mathbf{X}(N) \notin [\mathbf{X}_{N1}, \mathbf{X}_{N2}] \end{cases} \qquad (15)$$

which indicates that the objective function is set to zero when the operative temperature at the final stage is in the desired range; otherwise, it is penalised. The infinity is replaced by a large penalty number in a computer program.

In order to find the numerical solution, the operation range of the operative temperature must be discrete. A system state is the combination of a few consecutive operative temperatures over the same number of successive time intervals. All the possible system states were searched in a proper order so that the identification of system states refers to the ordinal number of states, rather than the operative temperatures. This will considerably reduce the search time and the demand for storage space in a computer, since the ordinal number is an integer and one number replaces a number of temperatures. Equation (14) shows that the calculation of the optimal cost function $J^*[\mathbf{X}(k), k]$ at stage k needs to use the value of $J^*[\mathbf{X}(k+1), k+1]$ at stage k+1. Moreover, the optimal decision $u[\mathbf{X}(k), k]$ for each state at each stage is also needed to find the optimal set-point in the forward procedure. Consequently, it is necessary to store all the optimal cost functions and heat supply decisions at the previous stage using a computer.

The increase of the dimension of system state variables and the operation range of system states will result in a significant increase of demand for computer storage and computation time. This is the main weakness of dynamic programming techniques. Any approach that can reduce the demand on computer resources should be incorporated with the application to enhance the efficiency of dynamic programming techniques.

It is well known that a constraint can reduce the operation range of thermal systems. Therefore, as many practical constraints as possible should be utilised when adopting dynamic programming in searching for the optimal set-points.

The acceptable operation range described with Equations (8) and (9) may not be feasible sometimes since the future states of the building thermal system depend on the past and current states, the weather conditions in the near future and the maximum HVAC system capacity. Hence, the feasible operation boundaries may first be determined with on-line simulation. It is easy to find the high and low limits of the operative temperature if night set-point setback is not considered. The calculation of the feasible boundaries of the future system states starts at the current state using both full and zero capacity until the feasible limits intersect with the acceptable operation boundaries. A trial and error approach is used when night set-point setback is adopted. The heat supply strategy for the night setback is estimated by the following equations:

$$n_{sb} = n_{q0} + n_{q1}$$
$$T_{e,l0} = T_{e0} - n_{q0}\Delta T_{e0} \quad (16)$$
$$T_{e,l1} = T_{e1} - n_{q1}\Delta T_{e1}$$

where n_{sb} is the number of time intervals over the night setback; n_{q0} and n_{q1} represent the number of time intervals with zero and full system capacities, respectively; ΔT_{e0} and ΔT_{e1} indicate average temperature decrease and increase, when the heating system is off and fully on, respectively. T_{e0} is the operative temperature when the night setback starts; T_{e1} is the required minimum comfort temperature when a room is occupied; $T_{e,l0}$ and $T_{e,l1}$ are the lowest temperatures during the night setback, which are calculated from the temperature-down process when the heating system is shut off, and temperature-up process when the heating system is fully on. The lowest feasible setback profile can be determined by adjusting n_{q0} and n_{q1} until $T_{e,l0}$ and $T_{e,l1}$ are approximately equal.

DETERMINATION OF OPTIMAL OPERATION STRATEGIES

The model and the algorithm described in the last two sections have been implemented into a computer program and applied to a passive solar floor heating room. The heating process models identified in real-time are described elsewhere (Chen 1997) and used in the simulation study. The temperature increment between states was taken as 0.25 °C. A system state is the combination of four consecutive operative temperatures over four successive time intervals. Computer calculations show that the techniques are computationally efficient and flexible. It takes about 10.7 to 14.3 seconds on a computer with Pentium 166 MHz processor to find one optimal operation strategy over 24 hours. Applications in this section are aimed at two targets: the reduction of operating cost and the utilisation of solar energy.

Reduction of Operating Cost

The electrical heating load during the utility high-demand time of day may be shifted to the time when the utility rate is low. In order to do this, the variation of the utility rate has to be known. The rate generally varies with the overall electricity demand. The utility rate structure (Appendix A) generated for Albuquerque, NM, given by Winn and Winn (1985), was used in this study.

Several optimal set-points and heat supply power profiles are presented in Figures 1 through 4. They were obtained on a day with daily total incident solar radiation of 6.77 MJ/Day/m^2 and the typical normal ambient temperature with an average of -5.8 °C at Montreal in December (Chen 1997). Four different cases were considered for two operation strategies and two objective functions. No night setback was considered for both Figures 1 and 2 while night setback was considered for Figures 3 and 4. Minimum energy consumption was used as an objective index for Figures 1 and 3 while minimum energy cost was employed for Figures 2 and 4. Operating (energy) cost and energy consumption for these four cases are summarised in Table 1 in which M_c represents the daily operating (energy) cost and E_c indicates the daily energy consumption.

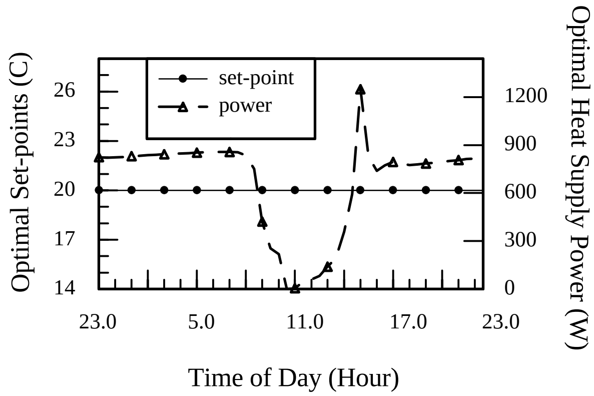


Figure 1 Optimal operation strategies without night setback and without taking utility rate into account

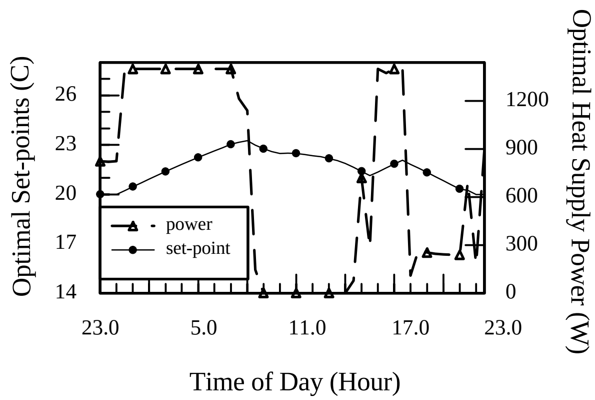


Figure 2 Optimal operation strategies taking utility rate into account and without night setback

Table 1 Operating Cost and Energy Consumption with the Daily Solar Radiation of 6.77 MJ/Day/m^2 and the Mean Ambient Temperature of -5.8°C

Desired Operation	Minimum Energy Consumption		Minimum Operating Cost	
	M_c Dollar/Day	E_c MJ/Day	M_c Dollar/Day	E_c MJ/Day
No Night Setback	1.71	58.1	1.34	62.3
Night Setback	1.38	50.2	1.27	60.1

Dynamic operation strategies were also optimised with different weather conditions. A moderate ambient temperature of 2.2 °C and a low daily solar incident radiation of 1.75 MJ/Day were considered. Summaries of operating costs and energy consumption without and with night setback are given in Tables 2 and 3, respectively. Comparing the results in the three tables, we may observe the following:

(1) When minimising energy consumption, cost is not minimised. Comparing the daily cost (M_c) values for minimum operating cost to those for minimum energy consumption in Table 2, it can be observed that when night setback is not desired, decreases of 22% to 27% in operating cost may be achieved by predictive control of the floor heating system, and utilisation of the varying utility rate.

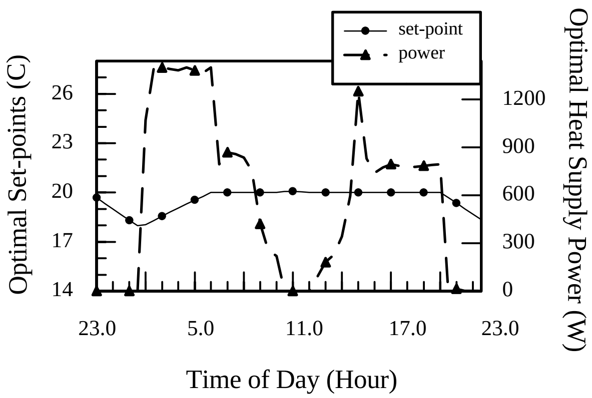


Figure 3 Optimal operation strategies with night setback and without taking utility rate into account

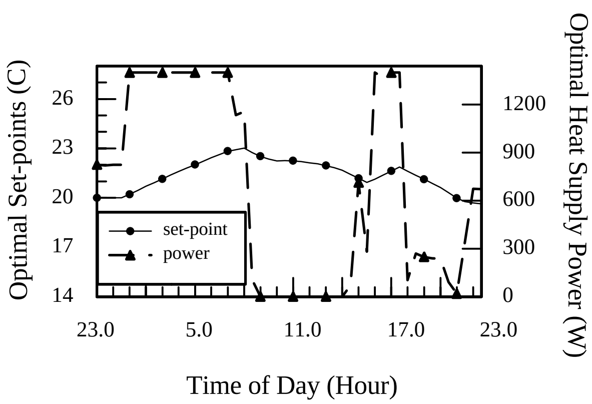


Figure 4 Optimal operation strategies taking utility rate into account with night setback

(2) If night setback is desirable, savings in operating cost significantly decreases because the energy savings due to night setback with minimisation of energy consumption are much higher than the energy savings with minimisation of operating cost. This phenomenon can be observed in Table 1, and by comparison of Tables 2 and 3. When average ambient temperature is -5.8 °C, for example, savings in operating cost with night setback (in Table 3) are 11% since the daily operating (energy) cost is $1.59 with minimisation of operating cost and $1.78 with minimisation of energy consumption. In comparison, the savings without night setback under the same conditions are 22% (in Table 2). However, the influence of night setback on the savings decreases considerably with increase of ambient temperature. When average ambient temperature is 2.2 °C, operating cost is reduced by 24% with night setback and by 27% without night setback.

(3) The ratio of heating loads to amount of thermal mass in a building is an important factor that affects the cost-savings. This ratio for the test-room is lower as compared with that for some floor heating systems in a thermally massive building. It is hence expected that the savings in higher mass system should be higher than the results obtained in this study.

Table 2 Operating Cost and Energy Consumption without Night Setback (the daily Solar Radiation of 1.75 MJ/Day/m^2)

Average Ambient Temperature °C	Minimum Energy Consumption		Minimum Operating Cost	
	M_c Dollar/Day	E_c MJ/Day	M_c Dollar/Day	E_c MJ/Day
2.2	1.35	42.8	0.99	45.7
-5.8	2.12	66.2	1.66	69.2

Table 3 Operating Cost and Energy Consumption with Night Setback (the daily Solar Radiation of 1.75 MJ/Day/m^2)

Average Ambient Temperature °C	Minimum Energy Consumption		Minimum Operating Cost	
	M_c Dollar/Day	E_c MJ/Day	M_c Dollar/Day	E_c MJ/Day
2.2	1.13	38.0	0.86	44.3
-5.8	1.78	58.4	1.59	67.5

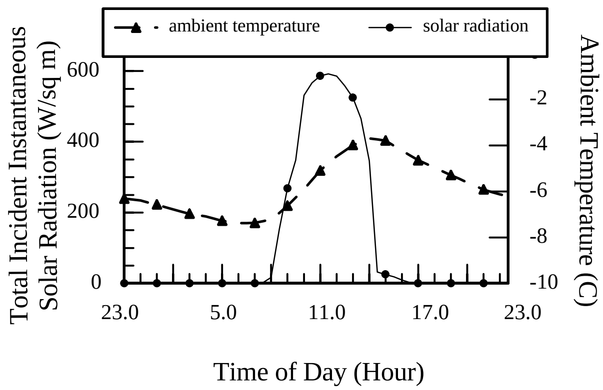
The results also indicate that the successful predictive control of heating systems considerably depends on local weather conditions, desired operation strategies (with or without night setback) and building heating system design. A heating system should be designed with systematic analysis of the dynamic heating operation under local weather conditions. Other possible options should also be evaluated. For instance, active heat storage may need to be added into a heating system when night setback is considered. This allows the design of a heating system that is suitable for the desired operation strategy and to maximise the savings in operating cost.

The temperature setpoint given in Figures 1-4 is the optimal operation strategy based on the past thermal system status and the predictive outdoor conditions in the next 24 hours. The setpoint can be updated with the new measured data at a certain period, depending on the speed of microprocessor or computer used. A proper controller is selected or developed to follow the optimal operation strategy closely. A generalised predictive controller, for instance, may be adopted to compensate for the time delay caused by the thermal mass of floor heating systems, as well as the dynamic setpoint. Although there is always a deviation between the real temperature and the setpoint in practice, amount of energy conservation should be close to the maximum as long as the room temperature is controlled around the optimal setpoint.

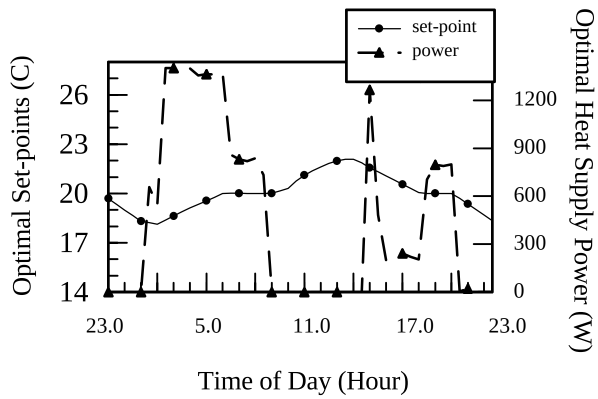
Utilization of Solar Energy

Dynamic programming techniques were also applied for the utilisation of solar energy. A sunny day (9.96 MJ/Day) with the typical normal ambient temperature in December was considered as shown in Figure 5(a). Optimal set-points and heat supply power profile are presented in Figure 5(b). The operative temperature is set back during the period between 8 pm and 6 am to precool the thermal mass. Consequently, it can absorb more solar radiation to prevent the room from overheating. It can be observed that the heating system is fully on at around 2 am to raise the operative temperature to the minimum comfort temperature at 6 am when the room starts to be occupied. The room should be kept with the minimum comfort temperature before sunny hours. Some heat may have to be supplied after sunny hours if allowed changes in the operative temperature must be less than or equal to 0.5 °C/h as required by ASHRAE Standard 55-1992. Analysis of the results shows that the floor thermal mass cannot be fully utilised for storage of solar energy since the floor has to be heated before heating the room air. Therefore, a small air heating system may need to be added to fully utilise the floor mass to store solar energy.

The techniques developed in this work may also be applied to the other predictive operation strategies. For example, the peak heating load may be reduced by storing heat in the building envelope mass a few hours ahead. The increase of indoor temperature for the peak load shifting will consume more heating energy. This strategy, however, only operates for a few hours in a whole year when the outdoor temperature is extremely low. Therefore, the increase of energy consumption due to the peak load shifting is negligibly small. Taking Montreal weather as an example, the time when the peak shifting operation is needed is less than two percent of total heating hours in one year if the capacity of a heating system is reduced by 10%. On the other hand, a heating system with smaller capacity requires less equipment cost, and has an improved part load efficiency when a gas or oil boiler is used. An example simulation run with BESA for an apartment building located in Montreal shows that the energy consumption is reduced by about 1.5% when the size of a gas boiler is reduced by 10% (Chen and Athienitis 1993).



(a)



(b)

Figure 5 Optimal operation strategies with night setback and without taking utility rate into account

CONCLUSION

A systems approach has been developed for multiple-stage optimal operation of building and HVAC systems. It was applied to a passive solar floor heating system with a number of practical system and thermal comfort constraints. Computer simulations demonstrate that the acceptable operation range to be searched for can be reduced to the feasible operation range through on-line simulation. This will greatly enhance the efficiency of dynamic programming techniques.

The simulation results also indicate that the predictive operation of passive solar floor heating systems may lead to significant operating energy savings, between 10% and 27% in this study, mainly depending on weather conditions, operation requirements (with or without night setback) and building thermal system design. Operating energy savings through the predictive operation of the heating system may be further improved by the proper design of thermal mass and heating subsystems.

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NOMENCLATURE

$A(z^{-1})$ = polynomial
$B(z^{-1})$ = polynomial
C_s = utility rate structure, $/kWh
e_o = average predict error
E_c = daily energy consumption, MJ/day
$\mathbf{H}(z^{-1})$ = a polynomial vector
J^* = objective function, $ or MJ/day
k = discrete time
M_c = daily operating (energy) cost, $/day
n_{td} = discrete dead time
n_{sb} = number of time intervals over night setback;
n_{q0} = number of time intervals with system fully off
n_{q0} = number of time intervals with system fully on
N = number of time intervals with period of interest
RH = relative humidity, %
t = time, sec
T_e = global temperature, °C
T_{sf} = floor surface temperature, °C
TS = a vector of predictable independent sources
u = a control input (auxiliary heat), kWh
v_{air} = indoor air speed, m/s
x = state of thermal systems
X = a vector of system states
z^{-1} = backward operator

APPENDIX A

Utility Rate Structure

Figure 6 shows the utility rate structure generated for Albuquerque, NM, which is given by Winn and Winn (1985).

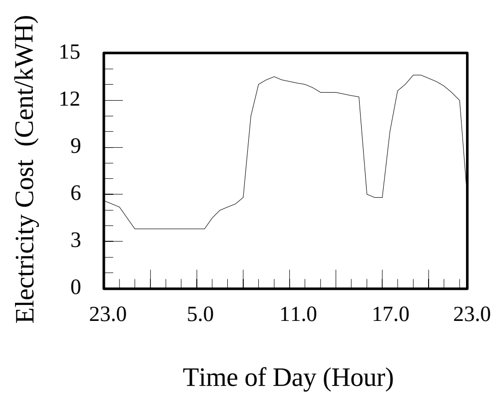

Figure 6 Utility rate structure