SKY LUMINANCE DISTRIBUTUION MODEL FOR SIMULATION OF DAYLIT ENVIRONMENT

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ABSTRACT
It is important to apply the natural light effectively for the low energy consuming daylighting design. A reliable standard sky is absolutely required that copies after the real sky luminance distribution for the simulation and evaluation of the daylit environment. Any applicable standard sky, however, has never found that shows accurately the actual state of daylight. A research work was carried out by the authors in order to establish a suitable sky model that can represent all the sky luminance distribution from the clear sky to the overcast sky continuously. The model proposed here was named “All Sky Model”. The detailed descriptions on the research are explained in this paper as well as All Sky Model.

INTRODUCTION
An excellent standard of the sky luminance distribution is necessary for daylighting design. CIE Standard Clear Sky and Overcast Sky (1994A, 1996) show the two extreme sky conditions. Most of the real skies lie between the two CIE Standard Skies. They show various aspects of the sky conditions and the diverse distribution of sky luminance. The goal of this research work is to standardize all sky conditions by formulizing their luminance distribution. A numerical equation is introduced in this paper by which the absolute values of the luminance and luminance distribution of all kinds of skies from the clear sky to the overcast sky can be estimated. The sky proposed in this paper as an advanced standard sky is named “All Sky Model”.

SKY LUMINANCE MODELS RECOMMENDED AND PROPOSED
Moon and Spencer (1942) surveyed and arranged the previous research works, and proposed luminance distribution of the overcast sky as the standard. This overcast sky model was simplified a little, and has been recommended as the CIE Standard Overcast Sky (1955). The CIE Standard Overcast Sky is similar to the considerably dark sky covered with thick clouds.

Kittler (1967) proposed luminance distribution of the clear sky model as a standard. This clear sky model has been adapted as the CIE Standard Clear Sky (1973). The CIE Standard Clear Sky shows very close luminance distribution of the completely clear up sky.

There is no recommended zenith luminance of the overcast and clear skies by CIE, though both of the CIE Standard Skies are shown in the relative values to the zenith luminance. There are some proposals (1994B) about the zenith luminance of the overcast and clear skies by several research workers.

Both CIE Standard Skies show the extreme sky conditions of the cloudy condition and the clear up condition. The frequency of occurrence of the luminance distribution close to both CIE Standard Skies is considerably small. Most of the real skies lie between the two CIE Standard Skies.

Nakamura (1985) proposed the Intermediate Sky. An equation concerning the zenith luminance of the Intermediate Sky was proposed at the same time. The Intermediate Sky is a kind of the average sky models of each solar altitude of all the skies except similar sky to the two CIE Standard Skies.

Littlefair (1981) proposed a sky luminance distribution model of each solar altitude called BRE AVERAGE SKY based on the measurement data by Wegner (1975).

Kittler (1985) assumed that the sky condition fluctuates homogeneously from the clear sky to the overcast sky, and proposed the HOMOGENEOUS SKY to show the absolute value of the sky luminance distribution.

Perraudeau (1988) classified all the skies into five categories, that is, the overcast sky, the intermediate overcast sky, the intermediate sky, the intermediate clear sky, and the clear sky. And he proposed equations by which the sky luminance distributions were expressed.

Perez (1993) proposed the ALL WEATHER MODEL as the function of Sky Clearness and Sky Brightness by which all the skies were classified into eight categories. The relative diffusion indicatrix of this model is a little different from the CIE Standard Clear Sky.

Recently, Kittler, Darula and Perez (1997) proposed a new sky luminance distribution model, which classified all skies into fifteen categories.

There are many numerical models to show sky luminance distribution, however, all of these models do not continuously explain all sky conditions which fluctuate from the clear sky to the overcast sky.

RAW DATA AND THEIR ACQUISITION

The raw data for this research work were gathered from March 1992 to September 1993 at the IDMP measurement station of Takenaka Corp. Tokyo in Japan (35°40'03" N, 139°49'3" E). Those of the global illuminance, zenith luminance and so forth were applied that were obtained every 1-minute through a whole day. The raw data of the sky luminance distribution were gained every 15-minute at the 145 measuring points allover the sky vault by a sky scanner (EKO: Sky Scanner MS-300) developed by Nakamura (1991).

ARRANGEMENT OF THE SKY LUMINANCE DISTRIBUTIONS

Two kinds of the coefficient of correlation of every measured sky luminance distribution were calculated by comparing the obtained value at 145 measuring points and the calculated values of the CIE Standard Skies, respectively. The difference of the exponents of two corresponding coefficients of correlation were defined and named "ordering index" by the authors (1997A). It was applied to put the sky luminance distribution data in order. All the data of the sky luminance distributions were divided into 15 groups by every 5-degree of their solar altitude. The data of each group were line up linearly from the clear sky to the overcast sky by the ordering index. The relative ranking of a datum in a group to the total number of the data in the group indicates its cumulative frequency of occurrence. The data belongs to a group were classified into 20 sets according to the cumulative frequency of occurrence. The total number of sets amounts 300. The average relative sky luminance distribution of each set was finally arranged as the preparation of regression analysis stated below.

PARAMETER FOR THE REGRESSION ANALYSIS

In order to find an appropriate parameter to explain the sky luminance distribution, all kinds of the data measured at the IDMP station in Tokyo and the values by various combinations of some of them were inspected again and again (1997B).

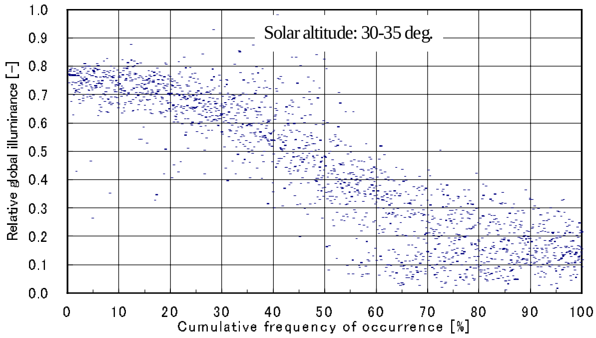

Fig.1 Relation between cumulative frequency of occurrence and relative global illuminance

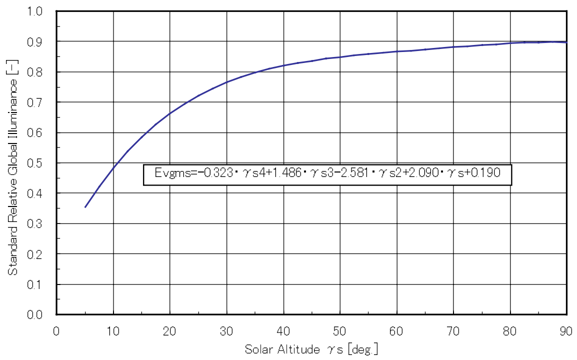

Fig.2 Standard relative global illuminance

A parameter named "normalized global illuminance" was finally adopted as calculated by the following procedures.

The relative global illuminance was taken into account at the first step. It is the value of the global illuminance multiplied by the relative optical air mass and divided by the extraterrestrial normal values of illuminance.

$$Evgm = m \cdot Evg / Evo \ [\text{-}] \qquad (1)$$

$Evgm$: Relative global illuminance [-], Evg : Global illuminance [lx], Evo : Extraterrestrial normal illuminance [lx], m : Relative optical air mass [-].

Fig.1, as an example, shows the relation between the relative global illuminance and the cumulative frequency of occurrence of the sky luminance distribution. A good relevance is found between them.

The standard relative global illuminance was introduced as the second step because the relative global illuminance was considered to be normalized. It is the relative global illuminance of the CIE Standard Clear Sky when its luminous atmospheric transmittance is 0.75. The standard relative global illuminance can be estimated by the following regression equation (2).

$$Evgms(\gamma_s) = -0.323 \cdot \gamma_s^{\ 4} + 1.486 \cdot \gamma_s^{\ 3} - 2.581 \cdot \gamma_s^{\ 2} + 2.090 \cdot \gamma_s + 0.190 \qquad (2)$$

$Evgms(\gamma_s)$: Standard relative global illuminance [-],

γ_s : Solar altitude [rad.].

Fig.2 shows the standard relative global illuminance. The normalized global illuminance defined finally as the value of the relative global illuminance divided by the standard relative global illuminance of the same solar altitude. The normalized global illuminance can be estimated by the following equation (3).

$$Nevg = Evgm / Evgms(\gamma_s) \quad (3)$$

$Nevg$: Normalized global illuminance [-].

Regardless of the solar altitude, the normalized global illuminance is approximately 1 for the clear sky and is approximately 0.2 for the overcast sky.

THE BASIC DATA FOR THE REGRESSION ANALYSIS

All of the average sky luminance distributions of 300 sets were characterized by every 5-degree of the solar altitude and every 5 percent of the cumulative frequency of occurrence. They were applied as the basic data for 300 regression equations. An equation was deduced based upon the 300 regression equations. The absolute values of the luminance and luminance distribution of all kinds of skies can be estimated by the equation.

THE BASIC EQUATION FOR SKY LUMINANCE DISTRIBUTION

The sky luminance distribution of the CIE Standard Clear Sky consists of two equations, i.e., $f(\zeta)$ and $\phi(\gamma)$. The structure of the new equation was decided following the CIE Standard Clear Sky as follows:

$$\frac{L_{va}(\gamma_s, \gamma, \zeta)}{L_{vz}} = \frac{\phi(\gamma) \cdot f(\zeta)}{\phi(\pi/2) \cdot f(\pi/2 - \gamma_s)} \text{ [-]} \quad (4)$$

$L_{va}(\gamma_s, \gamma, \zeta)$: Luminance of a sky element [cd/m^2], L_{vz} : Zenith luminance [cd/m^2], γ : Altitude of a sky element [rad.], ζ : Angle between the sun and a sky element [rad.].

In order to cover all average sky luminance distributions, i.e., including the overcast sky and so forth, the forms of the equations $f(\zeta)$ and $\phi(\gamma)$ were decided after repeated trials as follows:

$$f(\zeta) = 1 + b \cdot (\exp(c \cdot \zeta) - \exp(c \cdot \pi/2)) + d \cdot \cos^2 \zeta \quad (5)$$

$$\phi(\gamma) = 1 + a \cdot (1 - \sin^{0.6} \gamma) \quad (6)$$

a, b, c and d: coefficients.

THE REGRESSION ANALYSIS FOR THE COEFFICIENTS

Two kinds of luminance values of every data were excluded from the regression analysis. One was the value gained at the lowest measuring point of 6 degree. It was influenced obstructively too much to be adapted. The other was that got at the sky element within 20 degree from the sun. It often showed the value of sky luminance effected by the sun. Every set has available about 100 values were prepared for the regression analysis.

The regression analyses were repeated 300 times to get the coefficients *a*, *b*, *c* and *d* of each set. The values of each coefficient were inspected carefully and those of the coefficient *b* were considered to have the most regular relation to the normalized global illuminance. The coefficients *b* were fixed as a regression equation of the function of the normalized global illuminance as follows:

$$b = 23.4 \cdot (1.6 \cdot Nevg)^{5.9} \exp(-0.17 \cdot Nevg) \cdot (1.1 - Nevg)^{1.5} \quad (7)$$

The same procedure was carried out after the equation (7) was substitute to the equation (4) and (5), and got the regression equation of coefficient *d* as follows:

$$d = 2.06 \cdot Nevg^5 - 6.40 \cdot Nevg^4 + 6.02 \cdot Nevg^3 - 1.31 \cdot Nevg^2 + 0.08 \cdot Nevg \quad (8)$$

The regression equation of coefficient *c* and *a* were decided in the almost same way as follows:

$$c = 62.16 \cdot Nevg^6 - 257.62 \cdot Nevg^5 + 405.67 \cdot Nevg^4 - 296.6 \cdot Nevg^3 + 99.30 \cdot Nevg^2 - 16.34 \cdot Nevg + 0.43 \quad (9)$$

$$a = 9.93 \cdot Nevg^3 - 10.68 \cdot Nevg^2 + 7.09 \cdot Nevg - 2.11 \quad (10)$$

If *Nevg*>1.0 then *Nevg*=1.0 in equations (7) - (10).
If *Nevg*<0.24 then *Nevg*=0.24 in equation (10).

COMPARISON OF RELATIVE SKY LUMINANCE DISTRIBUTIONS BETWEEN MEASURED AND CALCULATED SKIES

Fig.3 shows the relative sky luminance distributions of averaged value measured and estimated value for solar altitude 32.5°. In Fig.3, (1) shows the clear sky, (20) shows the overcast sky and the skies among the clear sky and the overcast sky are shown sequentially besides.

And Rms is a coefficient of correlation between the measured sky and the calculated sky. The measured values and calculated values are corresponding to each sky condition very well.

Fig.4 shows the coefficients of correlation of relative sky luminance distributions between the averaged value and calculated values. The coefficients of correlation are 0.95 or more in most skies.

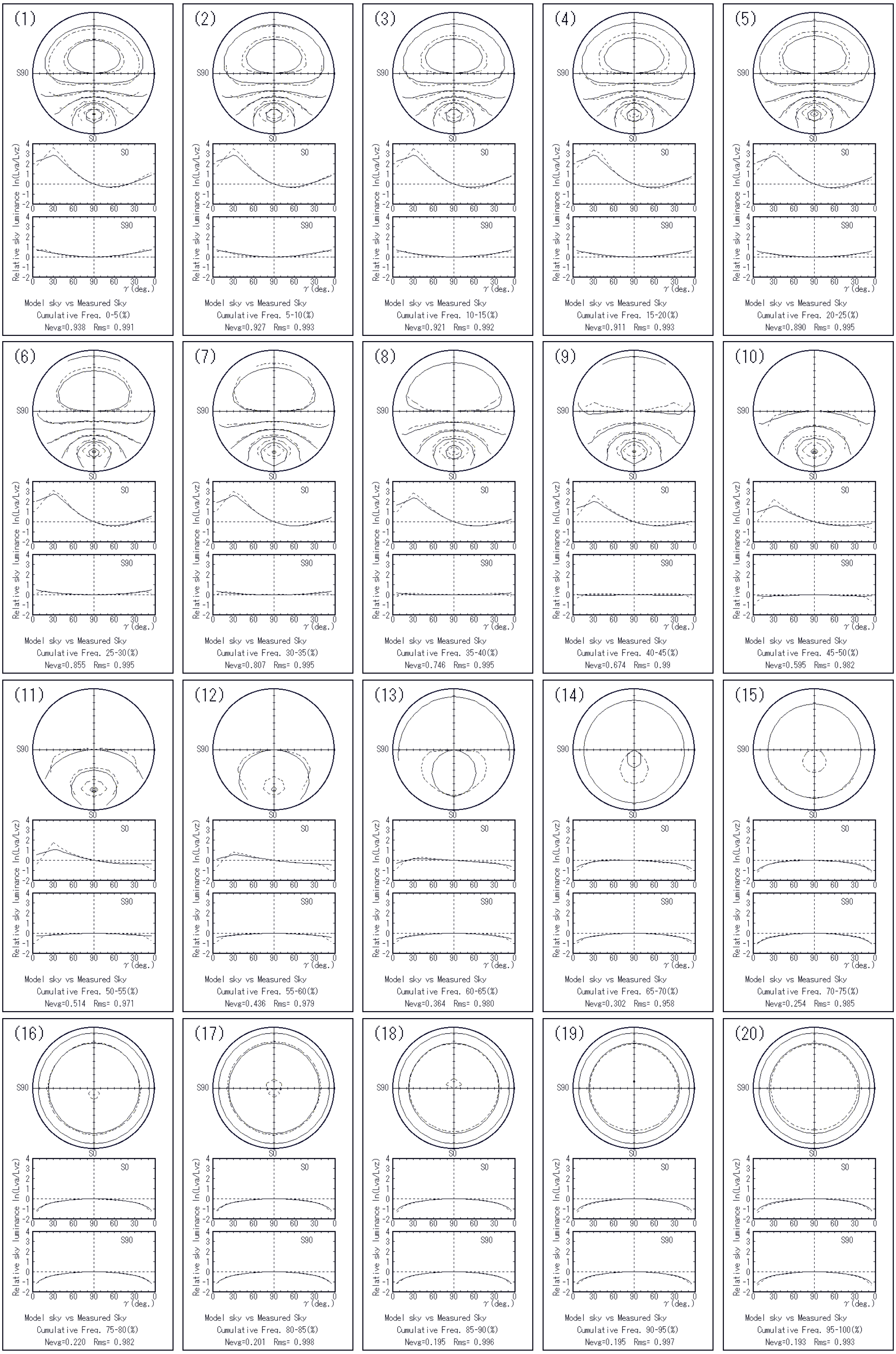

Fig.3 Comparison of relative sky luminances between measured and calculated skies (Solar altitude: 30-35 deg)

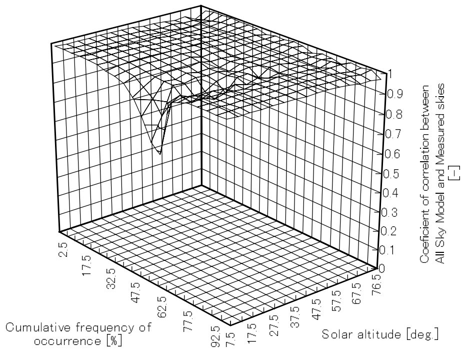

Fig.4 Coefficients of correlation between measured sky and ALL SKY MODEL

THE REGRESSION ANALYSIS OF THE ZENITH LUMINANCE

For the design of daylit environment, the absolute value of the sky luminance distribution is indispensable. Therefore, the zenith luminance, which can show all the sky conditions, was examined. The equation (4) for all the sky luminance distribution of all sky conditions can be applicable as the coefficients of the equation (5) and (6) are given. The values brought by equation (4), however, are relative and not absolute, though the absolute ones are often required for the advanced daylighting design. The absolute values can be gained with the relative ones and the zenith luminance. As any suitable expression of the zenith luminance for the equation (4) is not found, it is necessary to develop the excellent equation for the zenith luminance that is corresponding to the equation (4), i.e., the normalized global illuminance.

The regression equation for the zenith luminance was acquired as the function of the solar altitude and the normalized global illuminance by the almost same procedure above stated as follows:

$$L_{vz} = \exp(A \cdot Nevg^5 + B \cdot Nevg^4 + C \cdot Nevg^3 + D \cdot Nevg^2 + E \cdot Nevg + F) \ [\text{cd/m}^2] \quad (11)$$

$$A = 18.373 \cdot \gamma_s + 9.955 \quad (12)$$

$$B = -52.013 \cdot \gamma_s - 37.766 \quad (13)$$

$$C = 46.572 \cdot \gamma_s + 59.352 \quad (14)$$

$$D = 1.691 \cdot \gamma_s^2 - 16.498 \cdot \gamma_s - 48.670 \quad (15)$$

$$E = 1.124 \cdot \gamma_s + 19.738 \quad (16)$$

$$F = 1.170 \cdot \ln(\gamma_s) + 6.369 \quad (17)$$

Fig.5 shows the zenith luminance for the solar altitude corresponding to the normalized global illuminance.

The sky luminance distribution of all sky conditions in terms of absolute values proposed here by the authors was named “All Sky Model”. Its luminance values were calculated by the equation (4).

Fig.6 shows the examples of the luminance values by the All Sky Model correspond to normalized global illuminances. The errors of the calculated values by the equation (4) from the basic data were checked and found negligible

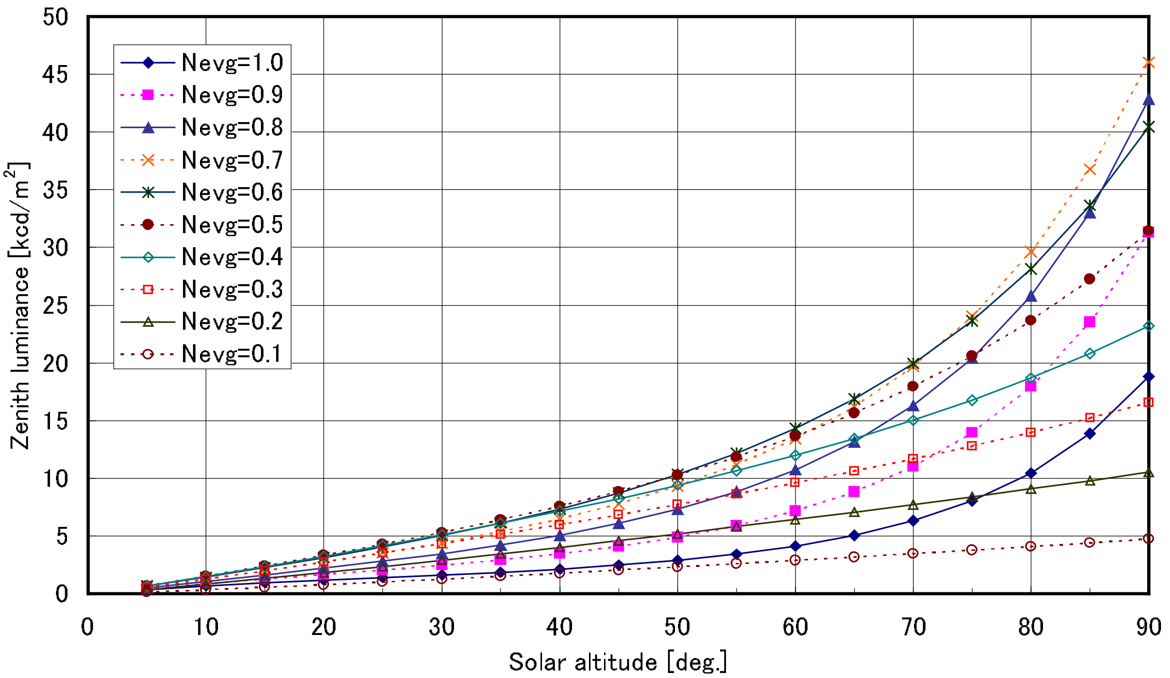

Fig. 5 Zenith luminance at various sky conditions

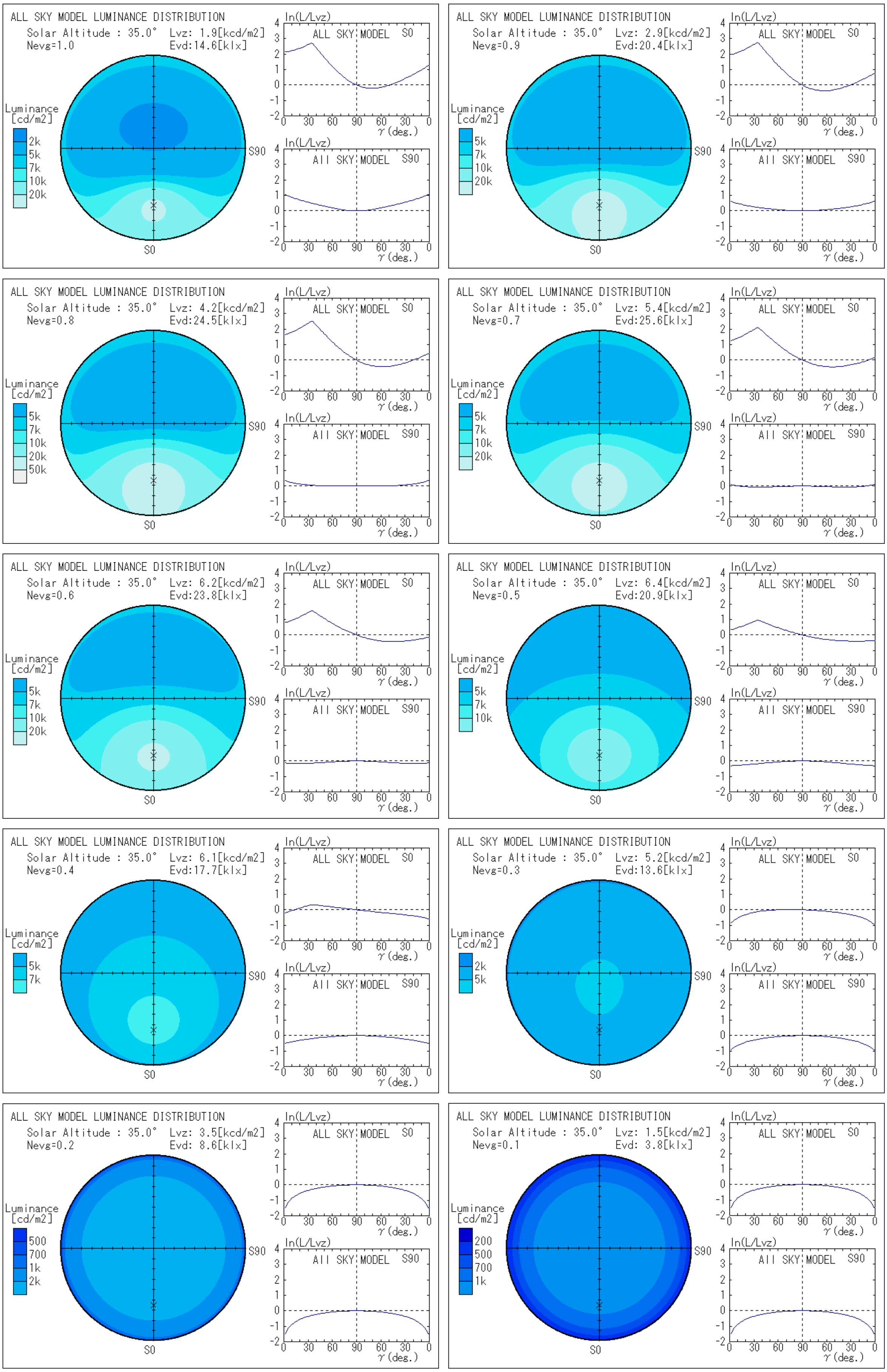

Fig.6 Absolute value of sky luminance distribution from clear sky to overcast sky (Solar Altitude: 35deg.)

CONCLUSIONS

When only the global illuminance is obtained, the All Sky Model can be composed, i.e., it is possible to estimate the sky luminance and sky luminance distribution in absolute value of all sky conditions.

The global illuminance is one of the most basic daylight data gained by ordinary daylight measurement and get at all the IDMP stations in the world. It can be also estimated with appropriate luminous efficacy from the irradiance that is always gained by meteorological observatory.

All Sky Model can be composed by the global illuminance with a personal computer and applied usefully to the advanced simulation and design of daylit environment.

The fruit of this research work convinced the authors of its great contribution to the progress of the simulation and design techniques of daylit environment.

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NOMENCLATURE

Evg : Global illuminance [lx]

Evo : Extraterrestrial normal illuminance [lx]

$Evgm$: Relative global illuminance [-]

$Evgms(\gamma_s)$: Standard relative global illuminance [-]

$Nevg$: Normalized global illuminance [-]

$L_{va}(\gamma_s, \gamma, \zeta)$: Luminance of a sky element [cd/m^2],

L_{vz} : Zenith luminance [cd/m^2]

γ_s : Solar altitude [rad.]

γ : Altitude of a sky element [rad.]

ζ : Angle between the sun and a sky element [rad.]

m : Relative optical air mass [-]