A SIMPLE BUILDING MODELLING PROCEDURE FOR MATLAB/SIMULINK

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ABSTRACT

A computationally-efficient building thermal model is developed for short-timescale investigations applicable to control system design. A lumped-capacity treatment of the building elements is used which can be summarised by a simple analogue electric circuit. The method is procedurally-transparent and leads to a state-space description of a building space which is implemented using MATLAB/SIMULINK. The model is tested using experimental data from a building with high thermal capacity. The accuracy of second-order descriptions of the high thermal capacity building elements compared with first-order descriptions is also examined. Good agreement with experimental data over short simulation periods is obtained using first-order element descriptions.

INTRODUCTION

This work details the development of a building model for coupling to a convective heat emitter model for the primary purpose of investigating control system design. The building model was required to adequately reflect the interaction of the emitter and the space (and include the effects of the external climate). Thus the model would investigate typically shorter periods of time than normally associated with many of the building thermal models available today. A lumped capacitance model was investigated allowing interaction with the structure and external climate without dominating the modelling process. The model treats radiative and convective heat inputs in the same way which is considered acceptable since the associated emitter model (not reported here) is fully convective.

SIMPLIFIED BUILDING THERMAL MODELS

Lorenz and Masy (1982) describe a simple lumped capacity model which has since been frequently used. This is a two time constant model, one time constant associated with the air mass and the other associated with the internal and external structural mass. The two time constants describe the traditionally accepted form of building cooling profile, a relatively rapid fall in room temperature associated with the air capacity followed by a slower fall in room temperature associated with the structural mass.

Levermore (1992) develops a heating system optimiser model that considers the building as a first order description. In this model all the building fabric initial temperature are assumed to be variant. The various construction elements are assumed to warm up and cool down to their relative steady-state temperatures. The internal structure is considered to be at the same temperature as the internal air and any internal resistance to heat flow is ignored. Levermore (1988) states that a typical two time constant model has a short time constant associated with the air, windows and doors and a dominant time constant associated with the fabric. The dominant time constant was considered to be the controlling factor. This assumption effectively ignores the short time constant and is considered adequate for temperature prediction for optimiser controls but is clearly limited when investigating high frequency problems in which the heating system dynamics are required to play a part.

McLaughlin *et al* (1981) in discussing cooling response curves state that buildings exhibit two or three time constants. In contrast to Levermore and Lorenz and Masy the shorter time constant is claimed to be due to the cooling of the air and the heat emitter and not the air capacity alone.

Tindale (1993) refined the basic Lorenz and Masy model to address some of the reported inadequacies. One of these inadequacies was the poor modelling of buildings with very high thermal capacity. As many modern buildings have these characteristics, or are “thermally heavyweight” as defined by the CIBSE response factor method (CIBSE, 1986), it was considered important that the building model proposed in the present work adequately modelled thermally heavyweight spaces.

LUMPED CAPACITANCE MODEL

The Lorenz and Masy model achieves its simple two time constant form by including a proportion of the internal structure (floors, ceilings and partitions) in the single lumped external structural capacity. This simplification had benefits at the time of development, but the advent of cheap computing power has diluted the case for very simplified models. Hence the model used in this work is based on the description first proposed by Lorenz & Masy, subsequently applied by Crabb *et al* (1987) and, later, by Tindale (1993). A similar approach has also been taken by Bérnard *et al* (1985). The difference here is that simplifications regarding the treatment of internal construction elements which were adopted by the previous workers were not assumed in the present work. Our approach is similar to that used by Achterbosch *et al* (1992) though their work was based on housing whereas the present work is relates to commercial-scale buildings with high thermal capacity.

The procedure described in the following considers a room model consisting of external wall elements and internal floor, ceiling and partition walls. This is the same as the room from which experimental data (described later) have been obtained. However, the method is sufficiently transparent for alternative room formats to be easily considered. An analogue circuit form of interpretation is shown in Figure 1.

Whilst the thermal resistances (hence thermal transmittances) and thermal capacitances can be calculated trivially, the Lorenz and Masy (1982) prescriptions for the single capacity equivalent of a multi-layer construction have been used to calculate the split between inner and outer region thermal resistances for the external walls – the "accessibility factor".

Based upon Figure 1, energy balances about each internal temperature node give rise the to following.

External wall 1:

$$\frac{\mathrm{d}T_{\mathrm{w}_1}}{\mathrm{d}t}=\frac{A_{\mathrm{w}_1}}{C_{\mathrm{w}_1}}\left[Ui_{\mathrm{w}_1}(T_{\mathrm{ai}}-T_{\mathrm{w}_1})+Uo_{\mathrm{w}_1}(T_{\mathrm{ao}}-T_{\mathrm{w}_1})\right] \quad (1)$$

External wall 2:

$$\frac{\mathrm{d}T_{\mathrm{w}_2}}{\mathrm{d}t}=\frac{A_{\mathrm{w}_2}}{C_{\mathrm{w}_2}}\left[Ui_{\mathrm{w}_2}(T_{\mathrm{ai}}-T_{\mathrm{w}_2})+Uo_{\mathrm{w}_2}(T_{\mathrm{ao}}-T_{\mathrm{w}_2})\right] \quad (2)$$

Floor:

$$\frac{\mathrm{d}T_{\mathrm{f}}}{\mathrm{d}t}=\frac{A_{\mathrm{f}}}{C_{\mathrm{f}}}\left[\frac{pQ_{\mathrm{s}}}{A_{\mathrm{f}}}+U_{\mathrm{f}}(T_{\mathrm{ai}}-T_{\mathrm{f}})\right] \quad (3)$$

Ceiling:

$$\frac{\mathrm{d}T_{\mathrm{c}}}{\mathrm{d}t}=\frac{A_{\mathrm{c}}}{C_{\mathrm{c}}}\left[U_{\mathrm{c}}(T_{\mathrm{ai}}-T_{\mathrm{c}})\right] \quad (4)$$

Partitions:

$$\frac{\mathrm{d}T_{\mathrm{ip}}}{\mathrm{d}t}=\frac{A_{\mathrm{ip}}}{C_{\mathrm{ip}}}\left[\frac{(1-p)Q_{\mathrm{s}}}{A_{\mathrm{ip}}}+U_{\mathrm{ip}}(T_{\mathrm{ai}}-T_{\mathrm{ip}})\right] \quad (5)$$

Air:

$$\frac{\mathrm{d}T_{\mathrm{ai}}}{\mathrm{d}t}=\frac{1}{C_{\mathrm{a}}}\left[\begin{array}{l}Q_{\mathrm{p}}+Q_{\mathrm{e}}+\\(A_{\mathrm{g}}U_{\mathrm{g}}+U_{\mathrm{v}})(T_{\mathrm{ao}}-T_{\mathrm{ai}})+\\A_{\mathrm{w}_1}Ui_{\mathrm{w}_1}(T_{\mathrm{w}_1}-T_{\mathrm{ai}})+\\A_{\mathrm{w}_2}Ui_{\mathrm{w}_2}(T_{\mathrm{w}_2}-T_{\mathrm{ai}})+\\A_{\mathrm{f}}U_{\mathrm{f}}(T_{\mathrm{f}}-T_{\mathrm{ai}})+\\A_{\mathrm{c}}U_{\mathrm{c}}(T_{\mathrm{c}}-T_{\mathrm{ai}})+\\A_{\mathrm{ip}}U_{\mathrm{ip}}(T_{\mathrm{ip}}-T_{\mathrm{ai}})\end{array}\right] \quad (6)$$

Equations (1)-(6) can be readily stacked using the state-space notation:

$$\dot{\mathbf{T}} = \mathbf{AT}+\mathbf{Bi}$$

in which $\dot{\mathbf{T}}$ is a vector of derivatives; **A,B** are matrices of coefficients; **T** is a vector of states and **i** a vector of inputs (Equation (7)).

$$\begin{bmatrix}\dot{T}_{w_1}\\ \dot{T}_{w_2}\\ \dot{T}_{f}\\ \dot{T}_{c}\\ \dot{T}_{ip}\\ \dot{T}_{ai}\end{bmatrix}=\begin{bmatrix}\frac{-A_{w_1}}{C_{w_1}}\left[Ui_{w_1}+Uo_{w_1}\right] & 0 & 0 & 0 & 0 & \frac{A_{w_1}Ui_{w_1}}{C_{w_1}}\\ 0 & \frac{-A_{w_2}}{C_{w_2}}\left[Ui_{w_2}+Uo_{w_2}\right] & 0 & 0 & 0 & \frac{A_{w_2}Ui_{w_2}}{C_{w_2}}\\ 0 & 0 & \frac{-A_{f}U_{f}}{C_{f}} & 0 & 0 & \frac{A_{f}U_{f}}{C_{f}}\\ 0 & 0 & 0 & \frac{-A_{c}U_{c}}{C_{c}} & 0 & \frac{A_{c}U_{c}}{C_{c}}\\ 0 & 0 & 0 & 0 & \frac{-A_{ip}U_{ip}}{C_{ip}} & \frac{A_{ip}U_{ip}}{C_{ip}}\\ \frac{A_{w_1}Ui_{w_1}}{C_a} & \frac{A_{w_2}Ui_{w_2}}{C_a} & \frac{A_{f}U_{f}}{C_a} & \frac{A_{c}U_{c}}{C_a} & \frac{A_{ip}U_{ip}}{C_a} & \frac{-1}{C_a}\begin{bmatrix}A_gU_g+U_v+\\ A_{w_1}Ui_{w_2}+A_{w_2}Ui_{w_2}+\\ A_fU_f+A_cU_c+A_{ip}U_{ip}\end{bmatrix}\end{bmatrix}\times\begin{bmatrix}T_{w_1}\\ T_{w_2}\\ T_{f}\\ T_{c}\\ T_{ip}\\ T_{ai}\end{bmatrix}+\ldots$$

$$\ldots\begin{bmatrix}0 & 0 & 0 & \frac{A_{w_1}Uo_{w_1}}{C_{w_1}}\\ 0 & 0 & 0 & \frac{A_{w_2}Uo_{w_2}}{C_{w_2}}\\ 0 & 0 & \frac{p}{C_f} & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & \frac{(1-p)}{C_{ip}} & 0\\ \frac{1}{C_a} & \frac{1}{C_a} & 0 & \frac{(A_gU_g+U_v)}{C_a}\end{bmatrix}\times\begin{bmatrix}Q_p\\ Q_e\\ Q_s\\ T_{ao}\end{bmatrix}\qquad(7)$$

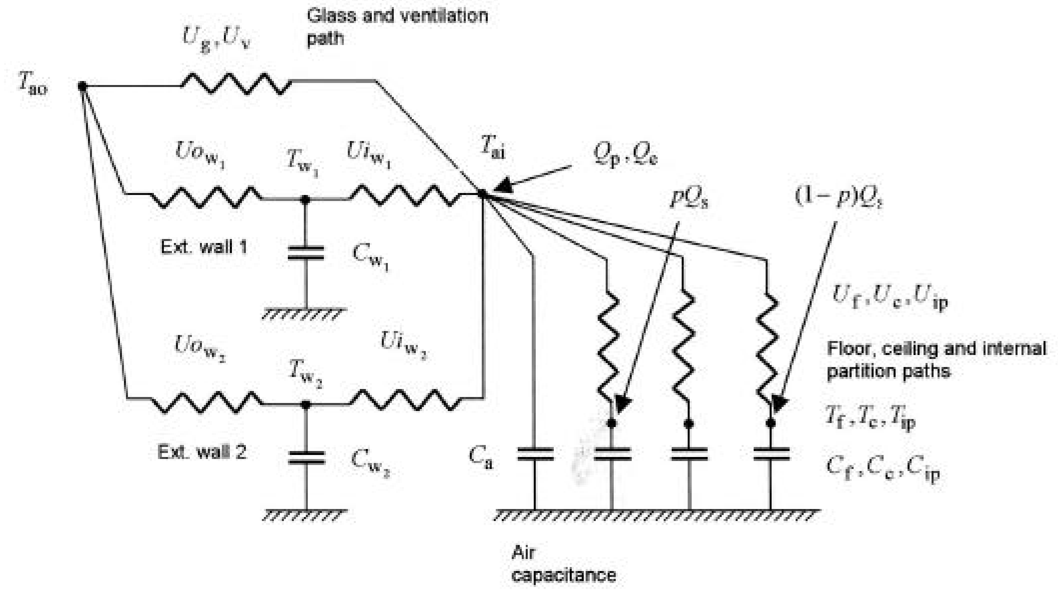

Figure 1: Lumped Capacitance Model

MODEL IMPLEMENTATION

The model is implemented using the Simulink structure of Figure 2. Inputs are heat supplied by the plant, Q_p, internal "casual" heat gains, Q_e, solar radiation through glazing, Q_s, and external air temperature, T_{ao}. A state-space block from the Simulink "linear" library represents the building model. The derivatives were integrated using a 4th -order Runge-Kutta scheme and an integration interval of 0.1hr was found to work well.

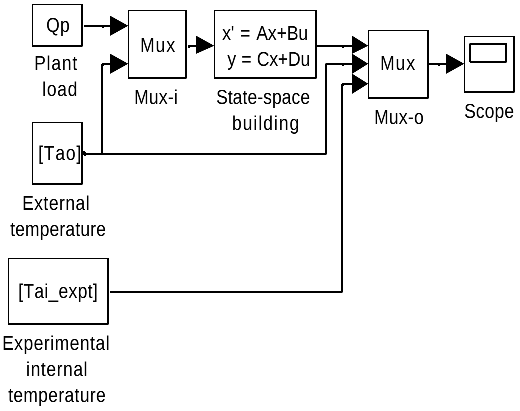

Figure 2: Simulink Model Structure

Sample model data were taken from a north-facing corner room in a University of Northumbria campus building - the Northumberland Building. Besides the room's major northern exposure (with substantial glazing), there is a west facing wall with minor well-shaded glazing. The building itself is the subject of extensive demonstration research into photovoltaic cladding and, consequently, is extensively monitored. Further details can be found in Hudson & Underwood (1996).

Model parameters were calculated using standard property data for the construction materials involved (CIBSE, 1986). The air change rate used in the ventilation transmittance was based on an earlier oil-tracer test which indicated an average air change rate of 0.5hr^{-1} under typical mild weather conditions for the room considered.

INITIALISATION

Model implementation was initially hampered by the choice of initial conditions. Clearly, the initial temperature for the room air can be set routinely but initial temperatures for structural elements, especially internal structural elements, required further consideration. These considerations are not important for long term simulations since the inevitable initial errors have limited impact on later results. However, for short-term dynamics for which this model was primarily designed the impact of inappropriately-set initial conditions would have an unacceptable influence on the accuracy of results.

The initial conditions for internal structural elements were especially problematical. The room from which the experimental results were obtained was heated by a fully convective system which would normally imply internal structure temperatures lower than the room air temperature. However, this was not found from field monitoring. Over a ten-day period the air and mean radiant temperatures were recorded in the test room. The mean radiant temperature was measured using a globe thermometer, which was situated at one of the occupants working positions, approximately at the centre of the room. Air velocity results were obtained and the low mean values observed (<0.1ms^{-1}) indicated that the globe temperature would give an accurate indication of mean radiant temperature. The measurements were taken between March – April 1998 and are shown in Figure 3.

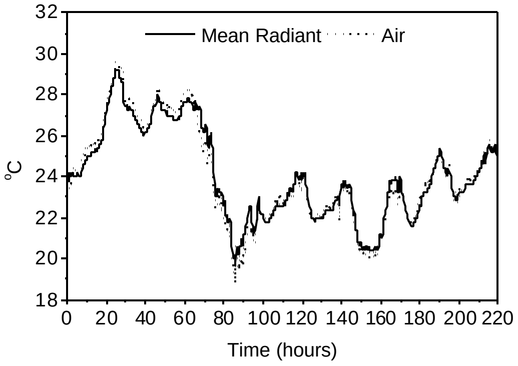

Figure 3: Air and Mean Radiant Temperatures

The variation between mean radiant and air temperature is small under cyclic conditions of room heating and cooling. These results indicate that setting the initial temperature of the internal structure at the room air temperature at the start of the cooling period is not unreasonable. This is the same assumption as Levermore uses in his whole building single time constant model (Levermore (1992)). Note that this comparison was made in a north facing room under typically overcast conditions. This conclusion may well be invalid for rooms experiencing significant solar heat gain.

Over the short timescale considered, the initialisation of the structural elements' temperatures was found to be less significant. Fixing this temperature at the mean of the internal air and external air temperatures was found to work very well in relation to model and experimental results. This would clearly be the correct choice for an homogenous slab at (initial) steady-state conditions. The error in this assumption would be dependent on the thermal-capacitance-mix of materials present. For most external building elements which are dominated by high thermal capacitance materials, the error is likely to be minor.

VALIDATION

The model of Figure 1 was initialised as described above and applied to the north facing test room. The room itself was chosen so that basic model validation could be investigated by neglecting solar radiation and monitoring data were chosen from periods in which the room was not in use (i.e. $Q_s = Q_e = 0$) . Monitoring data were collected in January 1997, a predominantly overcast period at the test site.

Three model validation trials were carried out:

- Cool-down (room temperature response to plant off).
- Preheating (room temperature response to plant on).
- Cool-down/heating/cool-down (plant fully on for a short period between periods of cool-down)

The plant capacity, Q_p, in the test room was known to be 3.6kW. No account was taken of the plant dynamics (in all trials, the room heating was either fully on or fully off). Results are given in Figures 4, 5 and 6 respectively.

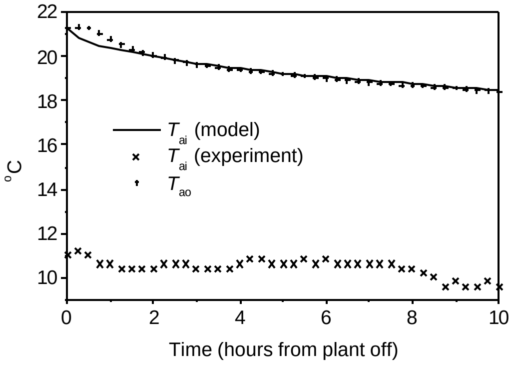

Figure 4: Cool-down Response

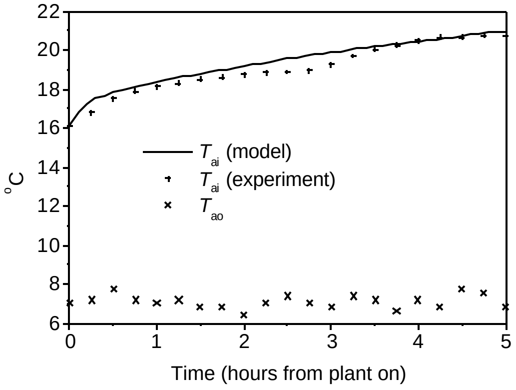

Figure 5: Preheating Response

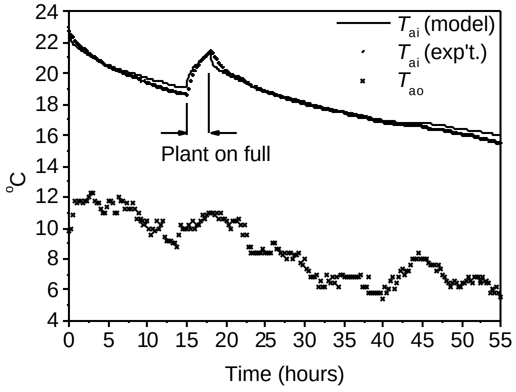

Figure 6: Cool-down/Heating/Cool-down Response

The model gives good agreement with experimental data for the short-term dynamics considered and for both heat-up and cool-down conditions. Towards the end of the longer of the three trials (Figure 6), the model and experimental results are seen to begin to diverge slightly (>45hr). It is likely that this divergence would continue as a result of the compromised accuracy of the lumped capacity method. Evidently, the lumped capacity method works well for short-term dynamics of special interest to control loop design, but accuracy for long term investigations (certainly involving high thermal capacity buildings) is uncertain.

SECOND-ORDER ELEMENTS

The high thermal capacity paths consisting of the two external walls and ceiling (Figure 1) were re-modelled using second-order lumped capacities by splitting the internal resistances in two and sharing the capacitances equally. This was done to establish whether the first-order descriptions for these elements was in any way dubious in terms of accuracy. Results, which are based on the same heating transient for the preheating case above, are given in Figure 7. The results show that there is no appreciable advantage in using the higher order description for short-term transient analysis.

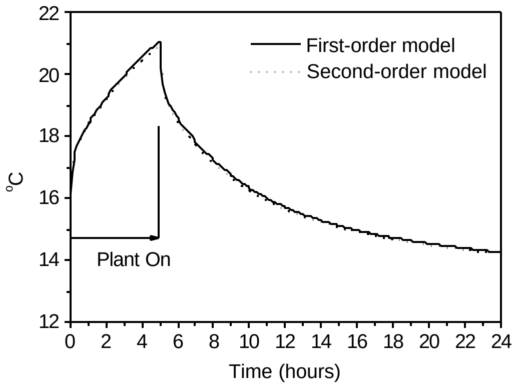

Figure 7: First and Second-order Model Comparison

CONCLUSIONS

A simple thermal model has been developed specifically for the analysis of short-term dynamics in buildings of high (or low) thermal capacity and has been implemented as a state-space block using MATLAB/Simulink. The model is procedurally-transparent, easy to interpret and can be initialised with reference to internal and external air temperatures. The model has been applied to a building exhibiting high thermal capacity and results compare very favourably with test data taken from the building for relatively short time periods and under both heating and cooling conditions. A comparison between second-order and first-order representations of the higher thermal capacity building elements present has revealed that there is no advantage to be gained by using the higher-order description as far as short-term dynamics are concerned.

Further work needs to be done on heating system modelling, the treatment of short-wave and long-wave radiation and the validity of the model (if necessary with higher-order elements) for longer term simulations.

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NOMENCLATURE

T	temperature (^{o}C)
Q	heat flux (W)
p	fraction of solar radiation entering floor
A	surface area (m^{-2})
U	thermal transmittance ($Wm^{-2}K^{-1}$)
Ui	inner thermal transmittance ($Wm^{-2}K^{-1}$)
Uo	outer thermal transmittance ($Wm^{-2}K^{-1}$)
U_v	ventilation thermal transmittance (WK^{-1})

Subscripts:

w_1	external wall 1
w_2	external wall 2
f	floor
c	ceiling
ip	internal partition
ai	air – internal
ao	air – external
s	solar
g	glazing
e	casual sources (internal heat gains)
p	plant