A NEW ALGORITHM FOR THE SIMULATION OF SOUND PROPAGATION IN SPATIAL ENCLOSURES

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ABSTRACT

Traditional algorithms for the simulation of sound propagation in spatial enclosures include the image-source method and the ray-tracing method. These algorithms are computation intensive and take a long time to generate results. In this paper, a new algorithm is proposed for the simulation of sound propagation in spatial enclosures that is relatively fast. This algorithm is based on modeling the surfaces of a spatial enclosure as sound radiators. The purpose of the algorithm is to generate an energy response graph at a certain listening location based on a sound source. Acoustical parameters such as Clarity (C50, C80) can be calculated from this graph. These parameters can then be used to assess the acoustical quality of the space.

INTRODUCTION

The author has developed a computer-aided design system for the preliminary spatial design of auditoriums. This system has been presented at national and international conferences (Mahalingam, 1992, 1998). This design system uses an algorithmic process to derive the spatial form of an auditorium from acoustical, functional and programmatic parameters. The acoustical environments generated by the design system need to be assessed. A simple and efficient model of sound propagation in a spatial enclosure was needed to complete the design system. The model presented in this paper is an attempt to accomplish this goal, and extend the capabilities of the design system developed by the author.

Traditional methods for modeling sound propagation in spatial enclosures have included the image-source method, the ray-tracing method, and a hybrid of these two methods. These methods have been computation intensive. Different researchers have tackled the problem of developing a computationally suitable algorithm using different approaches. Stephenson compared the image-source method with a sound particle simulation method (Stephenson, 1990). Vorlander developed a hybrid approach combining the image-source method and the ray-tracing method (Vorlander, 1989). This hybrid method is used in acoustical simulation software such as EASE (Electro-Acoustical Simulator for Engineers). Vian and Van Maerke developed a method using cones instead of rays in their simulation model (Vian & Van Maerke, 1986). Based on the experience of these researchers, a strategy to tackle the computational overhead of these methods is to eliminate the exponential growth of calculations. The radiosity method of modeling the distribution of light provides us with a new concept to model the propagation of sound. This model is based on a diffuse propagation of sound in a spatial enclosure. This model is primarily used to generate the energy response graph at a listener location for a particular sound source. Various acoustical parameters that are used to assess the quality of an acoustical environment can be derived from the energy response graph (Bradley, 1990). This allows the propagation model to act as a precursor for the acoustical evaluation of a spatial enclosure.

SIMULATION CONCEPT

The new model for the propagation of sound in a spatial enclosure is conceptually simple. The propagation of sound in a spatial enclosure is modeled as the radiation of sound from a source to all the planes that enclose the space, and a re-radiation from each plane to all other planes of the spatial enclosure until the intensity of sound drops to a preset limit. This radiation of sound is calculated till a significant portion of the sound is dissipated, or until the reverberation time is reached. Traditionally this has been assumed to be a drop of 60 dB (decibels) in sound intensity level, or a reduction in intensity to 1/1000000 of the original intensity. This propagation model is based on the finality of events. The sound leaving the source will reach all the planes of the spatial enclosure unless it dies out. The sound energy arriving at a plane will be absorbed, transmitted, or reflected. This reflected sound will then reach all other planes unless it dies out. The process of absorption, transmission and reflection will be repeated at the receiving planes. This whole cycle will be repeated until the sound is completely dissipated.

REPRESENTATION OF THE SPATIAL ENCLOSURE

The spatial enclosure or room is represented as a collection of planes that together enclose the spatial volume (see Fig. 1). Each plane is stored as a collection of points. The parameters (a, b, c, d) of the normalized plane equation ($ax + by + cz + d = 0$) are computed from these points. The area of each plane and its centroid are computed. Each plane is also assigned an absorption coefficient and transmission coefficient based on the material of the plane. The distance of the plane from the source, and the distance and angle between each plane and every other plane, are also computed in the propagation calculations. This is made easy because the plane is defined in parametric form.

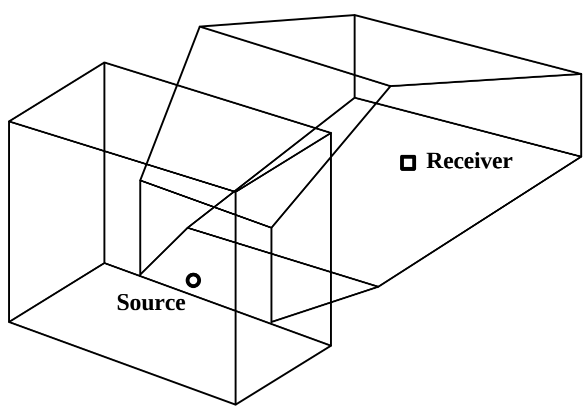

Figure 1. Spatial model of an auditorum showing the enclosing planes. This spatial model is generated by the design system developed by the author.

The centroid is approximated as the center of the bounding volume of the plane. The coordinates of the centroid (x_c, y_c, z_c) are given by the relation:

$x_c = (x_{max} - x_{min}) / 2$

$y_c = (y_{max} - y_{min}) / 2$

$z_c = (z_{max} - z_{min}) / 2$

The perpendicular distance (d) from a point (x_j, y_j, z_j) to a normalized plane ($ax + by + cz + d = 0$) is given by the relation:

$d = |ax_j + by_j + cz_j + d|$ (Bowyer and Woodwark, 1983, pp.107)

The angle (q) between two normalized planes represented by ($a_1x + b_1y + c_1z + d_1 = 0$) and ($a_2x + b_2y + c_2z + d_2 = 0$) is given by the relation:

$q = \cos^{-1} (a_1a_2 + b_1b_2 + c_1c_2)$ (Bowyer and Woodwark, 1983, pp. 110)

The distance (d) between two normalized planes whose centroids are represented by (x_{c1}, y_{c1}, z_{c1}) and (x_{c2}, y_{c2}, z_{c2}) is given by the relation:

$d^2 = ((x_{c1} - x_{c2})^2 + (y_{c1} - y_{c2})^2 + (z_{c1} - z_{c2})^2)$ (Bowyer and Woodwark, 1983, pp. 96)

The square root of d^2 gives the distance between the planes.

REPRESENTATION OF THE SOUND ENERGY

The source sound is represented as intensity and the unit used is watts/m^2. The sound intensity of the source is measured at unit distance (1m) from the sound source. This enables the easy calculation of sound intensity loss due to the Inverse-Square Law. The source sound is assumed to be an impulse of a very short duration, which is enough to generate the energy response graph (see Fig. 2).

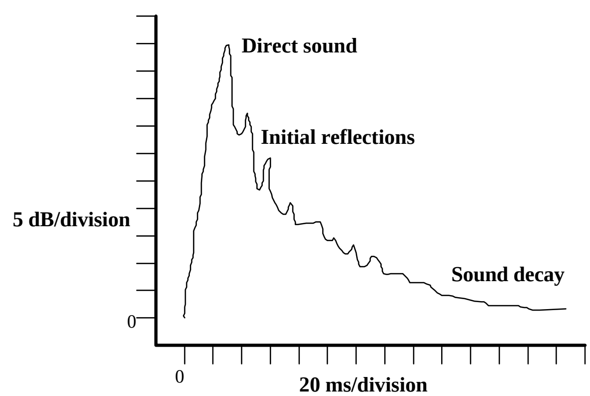

Figure 2. An energy response graph. The intervals shown in the graph are 20 ms (milliseconds). The data in the model is calculated at 1 millisecond intervals.

REPRESENTATION OF THE SOUND PROPAGATION IN THE ENCLOSURE

A sound of a given intensity is assumed to radiate spherically from the sound source. This sound reaches all the planes of the enclosure. The sound energy reaching a plane is modified by many factors. First, the sound energy is reduced in intensity having traveled the distance to the receiving plane, based on the Inverse-Square Law. The sound energy is also modified by the angle the receiving plane makes with the source plane based on the cosine of the angle between them. After the sound is modified based on these factors, the absorption of sound energy at the receiving plane is computed based on the absorption coefficient of the material of the plane. Next, the transmission of sound energy through the receiving plane is also computed based on the transmission coefficient of the material of the plane. The time of arrival of the sound at the receiving plane is recorded in milliseconds. After the sound intensity level at each receiving plane is calculated based on the various factors, the sound intensity is assumed as a new source, and the intensity is radiated to all other planes. This re-radiation is computed from each plane to all other planes. This re-radiation is computed until the sound intensity reaches 1/1000000 of the original intensity for each plane.

As the sound radiation is calculated from a plane to all other planes, a component of the sound intensity is calculated at a listener location. The intensity of the sound received at the listener location and the time of arrival of the sound is recorded for each radiation from plane to planes. After the sound has completed its radiation and is dissipated, the intensity of sound at each time interval is then compiled to create an energy response graph for the listener location. The intensity variation per millisecond (the time interval of choice for acousticians) is plotted as a graph. This intensity can also be transformed and expressed in decibels (dB).

COMPUTATIONAL IMPLEMENTATION

The different components that make up the sound propagation model are implemented as computational objects using the principles of object-oriented computing, as defined in the programming language Smalltalk. These objects include the planes that define the enclosure, and the propagation process model.

Computational objects have two components, data and operations. For example, the computational object that represents the planes of the enclosure is defined as:

Class: AcousticalPlane

Data (instance variables)

a, b, c, d (normalized parameters of the plane)

points (collection of vertices of the plane)

absorptionCoefficient

transmissionCoefficient

area

centroid

currentIntensity (sound intensity at plane)

arrivalTime (time of arrival of sound at plane)

Operations (methods)

distanceFrom: aPoint (calculates distance from the plane to a point)

angleBetween: aPlane (calculates the angle between one plane and another)

computeArea (calculates area of the plane)

computeCentroid (calculates the approximate centroid of the plane)

setCurrentIntensity: anIntensity (sets the current sound intensity at the plane based on incident sound intensity, absorption and transmission at plane)

setArrivalTime: aTime (sets arrival time of sound)

The computational object that represents the propagation model is defined as:

Class: SimulationModel

Data (instance variables)

soundSourceIntensity (intensity of sound source)

sourceLocation (location of source)

listenerLocation (location of listener)

planes (collection of planes used in propagation model)

energyResponseIntensities (a Dictionary (special data structure in Smalltalk) of energy response calculations at millisecond intervals)

Operations (methods)

radiateFromSource (computes radiation from source to listener location and all planes)

radiateFromPlaneToPlane (computes the radiation from one plane to all planes)

propagationProcess (the propagation process algorithm)

COMPUTATIONAL OVERHEAD

This algorithm models the diffuse propagation of sound. Because surface to surface radiation is used rather than rays, the computation time is significantly less in this algorithm. Also, the number of radiations does not increase exponentially but is based on a constant cyclical iteration (see Fig. 3).

For purposes of estimating the magnitude of calculations involved in the propagation model, a single radiation of sound from the source to a receiving plane, or from one plane to another, is treated as one calculation. The computation of the sound intensity component at the listener location for each radiation is also treated as one calculation.

The first propagation of sound from the source to all the planes of the spatial enclosure results in calculations equal to the number of planes (n). This also results in 1 calculation of sound intensity received directly at the listener location. Therefore, the total number of calculations at the end of the first cycle of propagation is (n + 1). The re-radiation from each plane to all other planes results in (n - 1) calculations. This process is repeated for all planes. Therefore for one cycle of re-radiation from all planes results in $(n * (n - 1)) = (n^2 – n)$ calculations. This also results in (n) calculations of the sound intensity received at the listener location from each plane. The total number of calculations for one cycle of re-radiation will be $(n^2 – n) + n = n^2$ calculations. This cycle is repeated (m) times until the sound intensity drops to 1/1000000 of the original intensity at each plane. Therefore, the total number of calculations in the model will be:

$(n + 1) + m(n^2) = mn^2 + n + 1$.

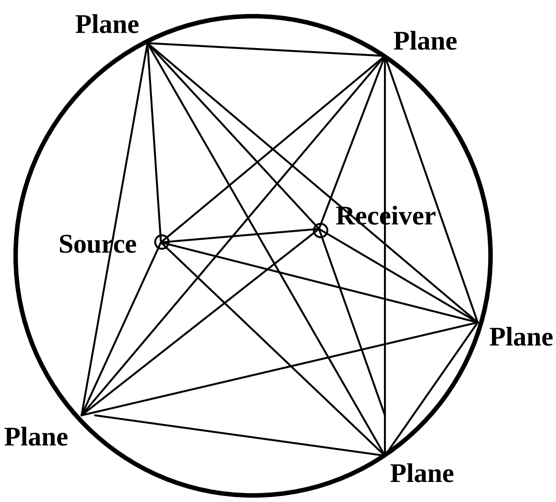

Figure 3. Diagram of finite cyclic net of calculations

This equation clearly does not reflect an exponential growth in calculations. For n = 30 and m = 25, the total number of calculations will be 22,531.

The resolution of the energy response graph is determined by the number of sound intensity calculations at the listener location. In this model, the number of calculations will be:

$m(n) + 1 = mn + 1$

Since some of the intensities calculated can occur at the same time, and therefore, have to be added, the number of discrete data points calculated for the energy response graph will be a maximum of:

$mn + 1$

This will be the resolution of the graph. If n = 30 and m = 25, a maximum of 751 data points will be generated for the graph. If the propagation lasts a total of 3 seconds (3000 milliseconds), the ratio of data points generated to total possible data points will be 1:4. This indicates a coarse resolution for the energy response graph.

CONCLUSIONS

This propogation model provides a relatively fast method to calculate an energy response graph at a listener location for a sound source. The model has some drawbacks however. It does not take into account the interference of the sound, which makes it less accurate. Also, in situations where the planes of the spatial enclosure are highly absorbant, the resolution of the energy response graph is reduced, because the number of re-radiation calculation cycles (m) is reduced. This model also does not take into account the non-spherical nature of the spatial enclosure. The spherical propagation is not deformed by the form of the spatial enclosure. This aspect can be incorporated in a more sophisticated propagation model. A more sophisticated version of this propagation model is currently being developed by the author, which will address these issues.

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NOMENCLATURE

Bounding Volume: It is the minimal rectangular volume that encloses a plane.

C50: A measure of clarity that is the ratio of the total sound energy received at a listener location in the first 50 milliseconds to the total energy received afterward.

C80: A measure of clarity that is the ratio of the total sound energy received at a listener location in the first 80 milliseconds to the total energy received afterward.

Decibel (dB): This is a measure of sound intensity level given by the relation: $dB = 10 \log (I_1/I_0)$ where I_1 is the given sound intensity and I_0 is a reference intensity.

Dictionary: A special class in Smalltalk which represents a data structure where values of any type can be stored and referenced by keys, which also can be of any type. In this model, the keys used are millisecond intervals, and the values are intensities.

Resolution: A measure of the number of data points or calculations. A fine resolution has more data points or calculations than a coarse resolution.

Smalltalk: A completely object oriented programming language developed at Xerox Palo Alto Research Center. This language comes with its own development environment in a product called Visual Works.