DYNAMIC OPTIMIZATION TECHNIQUE FOR CONTROL OF HVAC SYSTEM UTILIZING BUILDING THERMAL STORAGE

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ABSTRACT

This paper proposes a dynamic optimization technique for building heating and cooling systems. The proposed algorithm returns trajectories for space temperature setpoints throughout a specified period that will minimize objective functions such as running cost or peak energy consumption. It can be applied to buildings for which the thermal mass allows various choices of temperature setpoints. The algorithm also specifies an optimal on-off schedule for HVAC equipment. The discussion includes modeling of the building's thermal characteristics, optimization techniques, and example studies.

INTRODUCTION

It is well known that a building's thermal mass influences thermal conditions within the space. Thermal mass is generally considered to be negative in the case of intermittent air conditioning, since the heat load tends to increase due to heat storage load. However, taking an HVAC system with heat storage tanks as an analogy, there would appear to be a possibility of storing heat in the building structure during times of non-occupancy, thus reducing equipment capacity requirements or saving running costs by utilizing cheap night-rate electricity.

The merits of making use of building thermal mass in this way, and control theories associated with the concept, have been discussed in previous papers (for example, [1]-[4]). Braun[1] brought an optimization technique to bear on this question and discussed the conditions for, and the magnitude of, cost savings arising from the use of dynamic building control. But it cannot deliver an optimal on-off schedule such as turning on and off repeatedly. Jung et al.[4] described an optimization algorithm for systems which store heat in the plenum chamber above the ceiling. However, their algorithm requires the objective function to be expressed as quadratic, and it cannot minimize running cost or peak electrical demand directly.

The purpose of this paper is to describe an optimization technique which delivers an optimal on-off schedule and temperature setpoints for directly optimizing objective functions such as running cost or peak electrical demand.

Nagai and Matsuo[2] have already given a formulation for the algorithm used, but in this paper, the 'parsimony' expression of the state representation is introduced — allowing optimization over a longer period such as several months to a year. The proposed algorithm is applied to a system in which heat is stored in the contents of a room, not a plenum chamber, and assumes perfect forecasts of weather conditions and internal heat.

OPTIMIZATION THEORY

The optimization theory applied in this paper is dynamic programming (DP) [5]. Consider a dynamic system of which the state transition equation is

$$f(\mathbf{x}(k+1), \mathbf{x}(k), \mathbf{u}(k), k) = 0, \quad \mathbf{x}(0) = \mathbf{x}_0 \tag{1}$$

for $k = 0,1,..., N-1$

Then assume that the optimization problem is expressed as

$$\text{Minimize } J(\mathbf{x}(0),...,\mathbf{x}(N), \mathbf{u}(0),...,\mathbf{u}(N-1)) \tag{2}$$

Subject to

$$C_i(\mathbf{x}(k), \mathbf{u}(k), k) \geq 0, \quad i = 1,2,\ldots,m, \quad k = 0,1,\ldots,N-1 \tag{3}$$

$$C_i(\mathbf{x}(N), N) \geq 0, \quad i = 1,2,\ldots,m \tag{4}$$

DP is one of the available algorithms that can solve optimization problems such as the above. The main characteristics of DP are the following:

1) Any expression of f in (1) is acceptable if $\mathbf{x}(k+1)$ is

calculated for given $\mathbf{x}(k)$, $\mathbf{u}(k)$, and k. Continuity of $\mathbf{x}(k+1)$ is not required. Thus, discontinuity arising from different operational mode such as 'on' and 'off' can be dealt with.

2) Various expressions of J in equation (2) are acceptable. For example, the expressions

$$J = \sum_{k=0}^{N-1} g_1(\mathbf{x}(k), \mathbf{u}(k), k)) \tag{5}$$

or

$$J = \max_{k=0}^{N-1} (g_2(\mathbf{x}(k), \mathbf{u}(k), k)) \tag{6}$$

can be considered. No restrictions apply to the expression of g_1 and g_2. Thus, optimization problems such as minimizing cumulative energy or peak energy can be solved using equation (5) or (6).

3) If $\mathbf{x}(k)$ and/or $\mathbf{u}(k)$ are continuous, they should be scattered over discrete values.

4) As the order of the vector $\mathbf{x}(k)$ or $\mathbf{u}(k)$ increases, the complexity of the calculation increases explosively.

ROOM MODEL

A room model is obtained by considering the heat balance equations associated with the room. These equations are assumed to be expressed as follows:

$$E_s(k) = L_s(k) + H_s(z^{-1}) \cdot \theta_R(k) \tag{7}$$

$$E_l(k) = L_l(k) + H_l(z^{-1}) \cdot x_R(k) \tag{8}$$

Where $H_s(z^{-1})$ and $H_l(z^{-1})$ are transfer functions of discrete-time system, and $H_s(z^{-1}) \cdot \theta_R(k)$ and $H_l(z^{-1}) \cdot x_R(k)$ represent the heat fluxes caused by the histories up to the present of θ_R and x_R, respectively. Here, the orders of $H_s(z^{-1})$ and $H_l(z^{-1})$ should be made as low as possible so as to avoid the extreme computational requirements known as the "curse of dimensionality" in DP.

Matsuo[6] shows that the transfer function of sensible heat absorbed by a multi-layer wall can be well approximated by the second order. Thus, we assume here that $H_s(z^{-1})$ can also be represented at the second order, if there was more than one wall with a significant effect on heat absorption and their transfer functions were considerably different, a higher order might be needed to obtain an accurate approximation. Since the main aim of this paper is to demonstrate the basic methodology of the algorithm, we select the lower order situation. For the same reason, the dynamic effects of latent heat absorption are neglected. Higher orders may be needed according to the heat capacity of the room contents.

To summarize the above argument, $H_s(z^{-1})$ and $H_l(z^{-1})$ can be written as

$$H_s(z^{-1}) = p_0 + \frac{p_1 z^{-1}}{1 - r_1 z^{-1}} + \frac{p_2 z^{-1}}{1 - r_2 z^{-1}} \tag{9}$$

$$H_l(z^{-1}) = p_{0,l} \tag{10}$$

On the other hand, sensible (latent) heat extraction is assumed to be expressed in terms of the difference between supply air temperature (humidity) and return air temperature (humidity) of an air-conditioner. Namely,

$$E_s(k) = C_a \rho_a V_a (\theta_R(k) - \theta_s(k)) \tag{11}$$

$$E_l(k) = r_w \rho_a V_a (x_R(k) - x_s(k)) \tag{12}$$

Substituting equations (9) through (12) into equations (7) and (8) and rearranging yields the following:

$$(C_a \rho_a V_a - p_0)\theta_R(k) - C_a \rho_a V_a \theta_s(k) = L_s(k) + A_1(k) + A_2(k) \tag{13}$$

$$(r_w \rho_a V_a - p_{0,l}) x_R(k) - r_w \rho_a V_a x_s(k) = L_l(k) \tag{14}$$

$$A_1(k+1) = r_1 A_1(k) + p_1 \theta_R(k) \tag{15}$$

$$A_2(k+1) = r_2 A_2(k) + p_2 \theta_R(k) \tag{16}$$

At time step 'k', the unknown quantities are $\theta_s(k)$, $\theta_R(k)$, $x_s(k)$ and $x_R(k)$. These variables are determined by coupling equations (13) and (14) with other equations used in the HVAC model.

SYSTEM MODEL

The HVAC system should be modeled as follows:

$$h_i(\theta_s(k), \theta_R(k), x_s(k), x_R(k), \mathbf{u}(k), k) = 0, \quad i = 1,2 \tag{17}$$

where $\mathbf{u}$ is the vector of the control input to the HVAC system. The unknowns, $\theta_s(k)$, $\theta_R(k)$, $x_s(k)$ and $x_R(k)$ are calculated from simultaneous equations (13), (14) and (17). We assume that the dynamics of the HVAC equipment are relatively minor compared with those of the building structure itself, and can thus be neglected. By adopting this assumption, the vector of state variables $\mathbf{x}(k)=(x_1(k), x_2(k))^T$ for the whole system, including the building structure, can be defined as

$$x_1(k) = A_1(k) \quad (18)$$

$$x_2(k) = A_2(k) \quad (19)$$

Where $A_1(k)$ and $A_2(k)$ are expressed by equations (15) and (16).

Using equations (13) to (19), the state transition equation (1) is obtained for a given $\mathbf{x}(k)$ and $\mathbf{u}(k)$. Incidentally, other state vectors $\mathbf{x}(k)$ can also be defined. For example, $x_2(k)=A_1(k)+A_2(k)$ can be defined as a state variable instead of equation (19). In this case, $x_2(k)$ represents the sensible heat storage load.

EXAMPLE

System

The system adopted to demonstrate the algorithm is schematically illustrated in Fig.1.

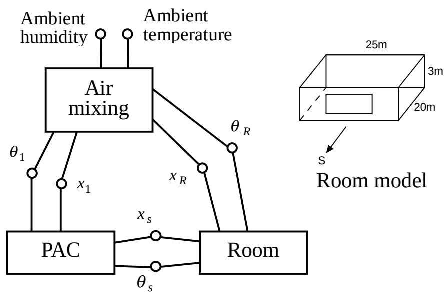

Fig.1 System schematic

The system consists of three modules:

Room module: Expressed by equations (13), (14), (15) and (16). The parameters in these equations ($L_s(k)$, $L_l(k)$, p_0, p_1, p_2, r_1, r_2 and $p_{0,l}$) are determined using the building heat transfer simulation program 'HASP/ACLD/8501'. Temperature of the adjacent spaces opposite the floor and the ceiling is assumed to be the same as that of the space considered here.

Air mixing module: Mixes outdoor air with return air from the room only when the room is occupied. The amount of outdoor air in the mix is constant over time.

Package air-conditioner (PAC): Heats or cools the inlet air according to the given loading ratio. The moisture characteristic of the inlet air, its volume, and the loading ratio determine the state of the heat-exchange coil: wet or dry. The wet coil model introduced here assumes that the by-pass factor is constant. The heating or cooling capacity varies according to the characteristics of the inlet and outdoor air, as shown in Fig.2 (where only the cooling situation is illustrated). This module provides the electricity consumption associated with the compressor, air supply fan, and condensing fan. The consumption of the compressor varies with the loading ratio and the air characteristics, as with the capacity. The electricity consumption of the supply fan and the condensing fan is considered constant. Fig.3 shows the partial loading characteristic of the PAC studied here.

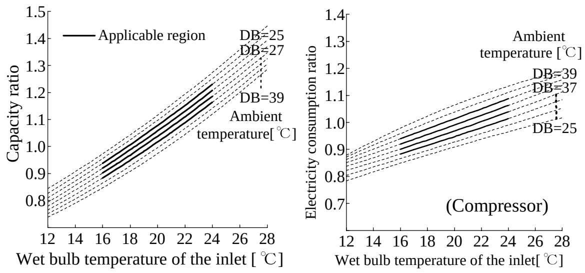

Fig.2 Capacity and electricity consumption characteristics of the PAC (cooling)

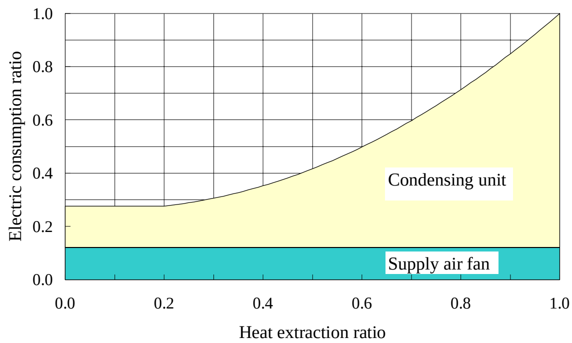

Fig.3 Partial loading characteristic of the PAC (ambient dry-bulb temperature: 35℃, inlet wet-bulb temperature: 19℃)

The volume of air passing through the supply fan is assumed to be constant (CAV). More detailed data are listed in Table 1. The PAC module equations are combined with the air mixing module equations and expressed in the form of equation (17). (Though different equations are used according to whether the coil is wet or dry.)

Table 1 Calculation conditions

Room	
Floor area	500m^2
Exterior walls	RC of 150mm with Plasterboard of 12mm
Floor and ceiling	RC of 150mm
Windows	single-glazed with an overall thermal conductance of 6.2W/m^2℃
Windows area	25m^2 (1/3 of the south exterior wall)
Infiltration	0.1 times of the room volume
Internal heat	25W/m^2 (lighting and the other electric equipment), 0.15 persons/m^2 (persons)
Occupied hours	from 9 a.m. to 18 p.m. on weekday
Thermal conditions	24～26℃ for occupied hours, 18～31℃ for non-occupied hours
Package air-conditioner	
Nominal capacity	75.6kW (ambient dry-bulb temperature: 35℃, inlet wet-bulb temperature: 19℃)
Energy consumption	34.2kW (compressor under full load), 6.3kW (supply air and condensing fan)
Air volume	12,960 m^3/h (supply air), 2,000 m^3/h (outdoor air)
By-pass factor	0.2
Others	
Electricity rates	0.133 $/kWh (from 8 a.m. to 21 p.m.), 0.033 $/kWh (the other hours)
Weather conditions	standard weather data of Osaka,Japan[7]

Input and State Variables

There is a single input variable: the loading rate of the PAC. So, input variable **u** is a scalar here. Its value ranges from -1 to 1, where the value 1 (-1) gives the maximum heat extraction (supply) of the coil. There are two state variables as stated before, and in this case study they are $x_1=A_1$ and $x_2=A_1+A_2$. The ranges of these state variables are derived from the permitted range of room temperatures using equations (15) and (16).

Discretization

Both input variable and state variables should be scattered discretely when applying DP. With regard to u, a value of zero should be included because this represents the state in which the compressor and condensing fan are turned off. In this case study, discretized values are obtained by dividing the admissible range into 30 as regards u and x_1, and 20 as regards x_1.

A step interval is set to one hour. Thus, states of operation including temperature setpoints continue at least one hour.

Objective Functions

Three optimization cases are examined here:

CASE-1:minimize cumulative electricity consumption

CASE-2: minimize peak electricity consumption

CASE-3: minimize cumulative running cost

The results will be compared with conventional operation (CASE-0): room temperature set to 26 °C when the room is occupied period, with the apparatus off otherwise.

RESULTS

Summer Peak Period

Fig.4 shows the ambient temperature and the solar radiation incident on the south-oriented vertical wall from 8/4 to 8/10. From Monday to Thursday, the peak daytime temperature is almost constant, and solar radiation has a tendency to decline slightly day by day.

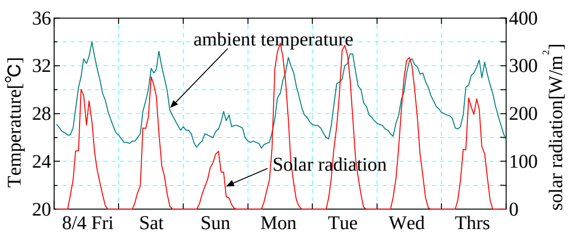

Fig.4 Ambient temperature and solar radiation on the south vertical surface

Fig.5 shows the room temperature trajectories selected to minimize each objective function. Fig.6 and 7 show equipment heat extraction and electricity

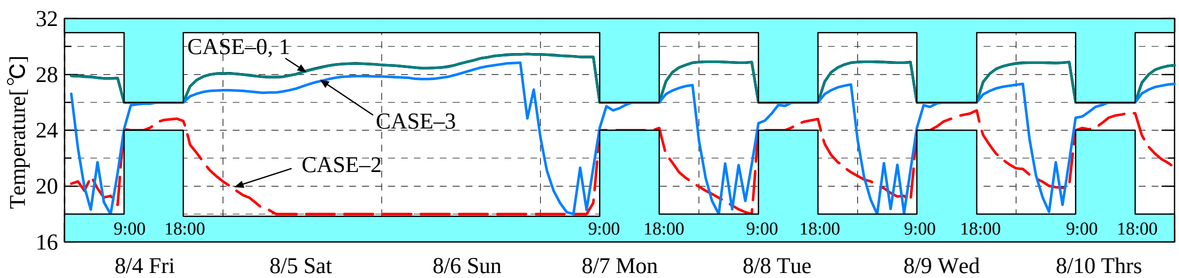

Fig.5 Room temperature minimizing each objective function (shadowed areas are infeasible regions)

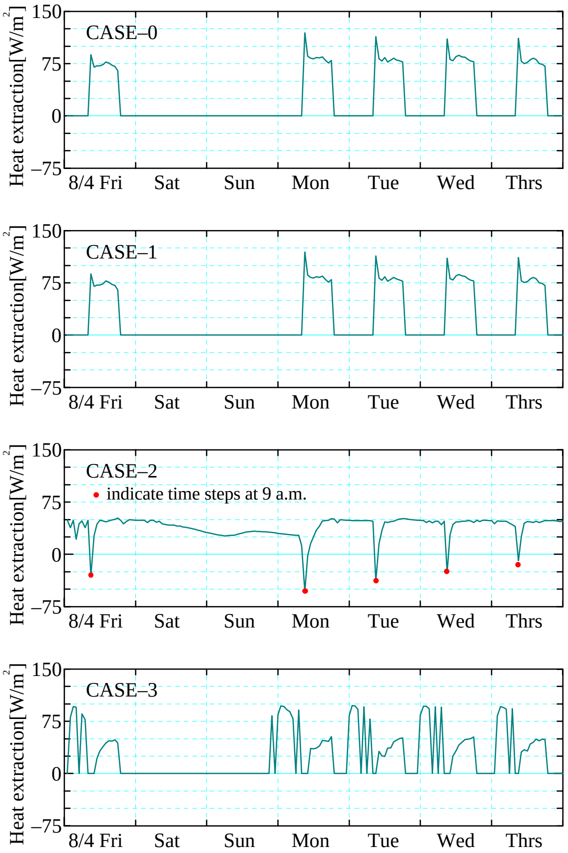

Fig.6 Heat extraction load (August)

consumption, respectively.

CASE-1 (minimize cumulative electricity consumption) is almost the same as CASE-0 (conventional operation) in all figures.

In CASE-2 (minimize peak electricity consumption), the room temperature is held to the lower limit (18°C) throughout unoccupied days, and raised sharply to the lower comfort limit (24°C) at 9 am on Monday morning by heating the space. Fig.6 illustrates how the equipment heats the space at 9 am, except on unoccupied days, even though it is summer peak season. The amount of heat supplied at 9 am gradually declines day by day. This strategy leads to flat electricity consumption, as illustrated in Fig.7, and the peak demand is considerably lower than with the other cases.

In CASE-3 (minimize cumulative running cost), the equipment turns on and off repeatedly (termed "of-off operation" in this paper) after reaching the lower temperature limit during weekday nights. One of the reasons for this on-off operation rather than continuous operation may be the higher loading rate that it achieves.

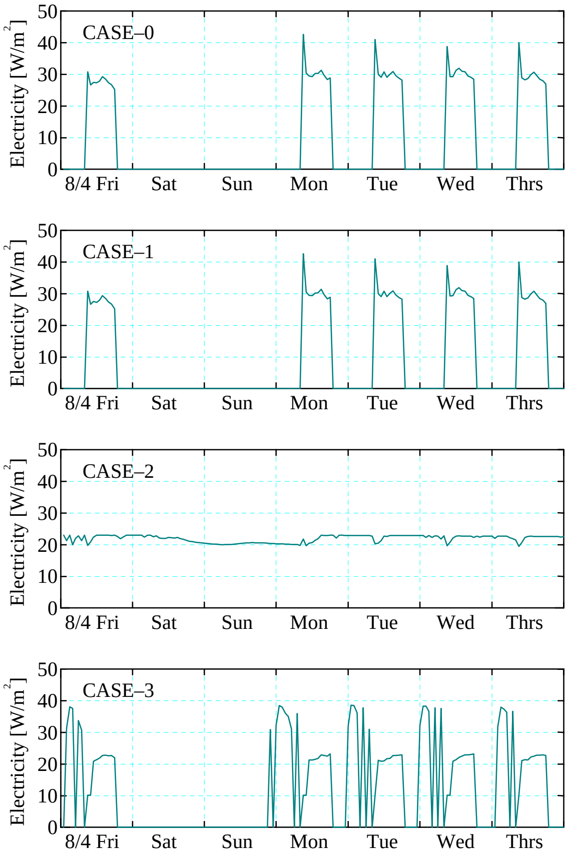

Fig.7 Electricity consumption (August)

Off-Peak Period

During the off-peak period, the strategy for minimizing cumulative energy consumption (CASE-1) is not the same as during conventional operation, as shown in Figs.8 and 9. According to Fig.9, on-off operation during the occupied period in CASE-1 leads to a higher loading rate (during on periods) compared to conventional operation. As illustrated in Fig.3, the loading rate influences energy efficiency, especially when the loading rate is low. This is one of the reason why on-off operation is suggested.

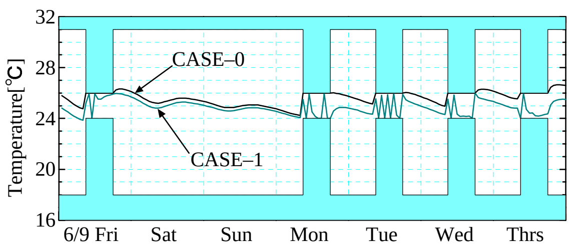

Fig.8 Room temperature (June, shadowed areas are infeasible regions)

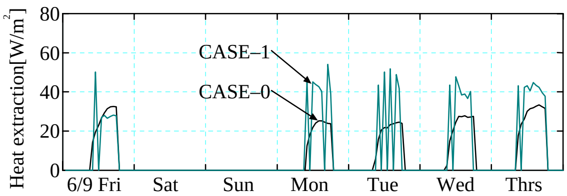

Fig.9 Heat extraction load (June)

Analysis by Calculation Period

Table 2 shows the results of calculations for the period from June 1 to September 30. The cumulative electricity consumption (CASE-1) is not much less than that obtained with conventional operation (CASE-0). This means that conventional operation is a near-optimal strategy for minimizing cumulative electricity consumption in this case study. The peak electricity consumption (CASE-2) is drastically reduced from conventional operation, but cumulative electricity consumption and cost are much higher. Operational factors such as the use of heating immediately after cooling are partially responsible for these energy and cost increases. Lower value of system COP made possible by the lower average loading rate also has an influence.

The peak loading rate in CASE-2 is 0.36, so the selection of a PAC with lower capacity may improve energy consumption and cost. The cumulative energy cost is reduced by about 10% from conventional operation by optimization (CASE-3).

Table 2 Summary of the optimization result (the shadowed figures are optimal values for each criterion)

	Criteria	CASE-0	CASE-1	CASE-2	CASE-3
(1)	Cumulative Heat Extraction [Wh/m2]	43,176	44,762	95,621	51,995
(2)	Peak Heat Extraction [W/m2]	133.6	119.1	53.9	110.6
(3)	Cumulative Electricity [Wh/m2]	20,422	20,054	60,927	27,130
(4)	Peak Electricity [W/m2]	50.3	42.7	23.2	44.9
(5)	Cumulative Energy Cost [$]	1,361	1,337	2,789	1,225
(6)	System COP = (1)/(3)	2.11	2.23	1.57	1.92
(7)	Peak Loading Rate = (2)/(PAC Capacity)	0.88	0.79	0.36	0.73
(8)	Average Loading Rate (operational)	0.33	0.36	0.22	0.37
(9)	Operation Hour [h]	870	813	2,897	927

CONCLUSIONS

A dynamic optimization technique for building heating and cooling systems is proposed. This new method is able to indicate optimal operation in terms of objective functions such as energy consumption and peak electrical consumption. As an illustration of the technique, a system consisting of a room equipped with a package-type air-conditioner is examined. The results indicate that operation regimes differ considerably according to the criteria selected.

Generally, optimization techniques can be expected to occasionally give unexpected results, as seen in this paper. And, it does give a hint as to how the operation of HVAC systems might be improved, even if this algorithm cannot be introduced into real control devices. So far as dynamic building control is concerned, more case studies are needed to establish a general operating strategy. In particular, parameters such as building structure, building location, the partial loading characteristics of the HVAC system, and the energy rate structure can be expected to influence the results.

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NOMENCLATURE

$\mathbf{x}$: state variable (vector)
$\mathbf{x}_0$: state variable at initial step (vector)
$\mathbf{u}$: input variable (vector)
k : time step
N : number of total time steps
f, J, C_i, g_1, g_2, h_i : function
m : number of constraints per step
E_s : sensible space heat extraction
E_l : latent space heat extraction
L_s : sensible space cooling load
L_l : latent space cooling load
H_s, H_l : transfer function
z^{-1} : delay operator
θ_R : room temperature
x_R : room absolute humidity
p_0, p_1, p_2, r_1, r_2, $p_{0,l}$: time constant
C_a : specific heat of air
r_w : evaporative heat of wator
ρ_a : density of air
V_a : supply air volume
θ_s : supply air temperature
x_s : supply air absolute humidity
^{T} : transposition
θ_1 : inlet air temperature of PAC
x_1 : inlet air absolute humidity of PAC