Optimal Operation Control of HVAC Systems

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ABSTRACT

The operation of technical building devices demands a lot of heating, cooling and electric energy. This also causes monetary costs and pollutant emission as well. Optimization investigations are very useful to make the running of plants more efficient. This paper shows two optimization methods. Some practical applications clearly demonstrate the profits of an optimized operation of technical building installation.

GOAL

For the management of temperature, humidity and air quality specified values are demanded for a determined time of the day (period of use). These parameters have a given tolerance.

The control of HVAC is to choose such that the loss is at its minimum. This is a task for optimal control. The solution is founded on variational principles. This work tests the Kuhn-Tucker-theorem and the discrete maximum principle of Pontrjagin. The procedures were practically tested using examples.

DESCRIPTION OF TASK

The run optimization for a HVAC is a matter of variational calculation, with

- system equations
- quality functional
- restrictions
- boundary conditions

The systems equations achieve the result of the heat and mass balances of the HVAC and of the building. The system variable is a function of the control function **u** and the disturbance function.

$$\mathbf{x}=f(\mathbf{u}, \mathbf{z}) \qquad (1)$$

Here are

- $\mathbf{x}$ – system variables
- $\mathbf{u}$ – control function
- $\mathbf{z}$ – disturbance function

The unknown time-dependent control functional is the result of the optimization.

For the quality function it is given

$$K_{op} = \min(f(\mathbf{u}, \mathbf{x}_0)) \qquad (2)$$

The function f is a continuous linear or non-linear function. $\mathbf{x}_o$ describes the initial value of the system. The quality functional describes the minimum of energy for heating or cooling

$$K = \int_0^T \dot{Q}(t)\,dt \overset{!}{=} \mathrm{Min} \qquad (3)$$

or the costs

$$K = \int_0^T f\left(\dot{Q}, p(t)\right) dt \overset{!}{=} \mathrm{Min} \qquad (4)$$

The specific costs can be time-dependent (cost for electricity energy).

The control function and the system function may be restricted. The control function is restricted by the calculation of the HVAC (of the boiler and the heating surface). Room temperature and humidity display restrictions that result from building physics, hygiene and comfort.

For the system function boundary conditions are given for the period of use. Fig. 1 shows the boundary conditions and restrictions for temperature and humidity.

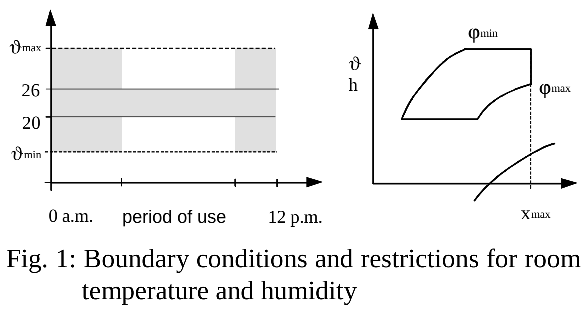

Fig. 1: Boundary conditions and restrictions for room temperature and humidity

In winter it is demanded to hold the temperature within the range

$$20°C \le \vartheta_i \le 26°C$$

and humidity within the range

$$30\,\% \le \varphi \le 65\,\%$$

during the period of use.

METHODOLOGY

Constrained optimization

A function, that depends on the initial state of the system $\mathbf{x}_o$, the time-scheduled open loop control $\mathbf{u}$ and the disturbances $\mathbf{z}$, serves as a quality criterion (see Eq. (2) to (4)).

The function f refered to in Eq. 2 is any linear or nonlinear, continual function which has to be formulated in correspondence with the respective optimization problem. It is possible to subject the finding of the optimum K_{opt} to nonlinear constraints.

$$g(\mathbf{u}) \le 0 \qquad (5)$$

These constraints g cover both boundary conditions and restrictions of **u**. The time-scheduled function **u** which minimizes the quality criterion that is subject to the constraints is calculated using Lagrange's principle. Thus the optimization problem

$$\min\left\{ f(\mathbf{u}) : g_i(\mathbf{u}) <= 0, i = 1,2 \ldots m \right\} \qquad (6)$$

has a Lagrangian function, which is defined as

$$L(\mathbf{u}, \boldsymbol{\lambda}) = f(\mathbf{u}) + \boldsymbol{\lambda}^T \mathbf{g} \qquad (7)$$

The Lagrange multiplier function

$$\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_m)^T$$

indicates how the optimum solution will change as a function of the constraints. If the equality constraints $\mathbf{g}(\mathbf{u}) = \mathbf{0}$ hold true, f and L are identical.

The algorithm is based on the solution of the Kuhn-Tucker equations. The Kuhn-Tucker theorem in R^n is stated as: $f(\mathbf{u})$, $g_1(\mathbf{u}),...g_m(\mathbf{u})$ are convex in $\mathbf{R^n}$; $\mathbf{u}^0$ is the solution of the optimization problem, if the local Kuhn-Tucker conditions will be satisfied ([1], [2]):

$$\mathbf{g}(\mathbf{u}^0) \le \mathbf{0} \qquad (8)$$

$$\boldsymbol{\lambda}^{0^T} \mathbf{g}(\mathbf{u}^0) = \mathbf{0} \qquad (9)$$

$$\nabla L(\mathbf{u}^0, \lambda^0) = \nabla f(\mathbf{u}^0) + \boldsymbol{\lambda}^{0^T} \nabla \mathbf{g}(\mathbf{u}^0) = 0 \qquad (10)$$

The solution of the Kuhn-Tucker equations can be found using gradient methods. For this purpose Newton methods are suited well.

The Kuhn-Tucker theorem is valid for convex functions. A function $f(\mathbf{u})$ is convex, if its Hessian is positively semidefinite for all $\mathbf{u} \in U$. The Hessian of the function $f(\mathbf{u})$ is defined from the second partial differentials (Eq. 11). It is positively semidefinite, if the value

$$Q = \mathbf{x}^T \mathbf{H}_f \mathbf{x}$$

becomes non-negative regardless of the choice of the real values **x**. A peculiarity of convex optimization is the fact, that a local minimum of a convex function in a convex set is a global minimum at the same time. That is why the use of local methods for finding the global minimum is possible.

$$H_f = \nabla^2 f(\mathbf{u}) = \begin{pmatrix} \frac{\partial^2 f(\mathbf{u})}{\partial u_1^2} & \frac{\partial^2 f(\mathbf{u})}{\partial u_1 \partial u_2} & \cdots & \frac{\partial^2 f(\mathbf{u})}{\partial u_1 \partial u_n} \\ \frac{\partial^2 f(\mathbf{u})}{\partial u_2 \partial u_1} & \frac{\partial^2 f(\mathbf{u})}{\partial u_2^2} & \cdots & \frac{\partial^2 f(\mathbf{u})}{\partial u_2 \partial u_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f(\mathbf{u})}{\partial u_n \partial u_1} & \frac{\partial^2 f(\mathbf{u})}{\partial u_n \partial u_2} & \cdots & \frac{\partial^2 f(\mathbf{u})}{\partial u_n^2} \end{pmatrix} \qquad (11)$$

The local quadratic approximation of the Lagrangian function L (Eq. 7) is

$$q_k(\mathbf{d}) = \nabla f(\mathbf{u}_k)^T \mathbf{d} + \frac{1}{2} \mathbf{d}^T \nabla^2 L(\mathbf{u}_k, \lambda_k) \mathbf{d} \qquad (12)$$

with the step direction **d** and the iteration index k.

Now the general optimization problem (Eq. 6) can be written as

$$\begin{pmatrix} \mathbf{min}\left(\frac{1}{2} \mathbf{d}^T \mathbf{H}_k \mathbf{d} + \nabla f(\mathbf{u}_k)^T d \right) \\ \nabla \mathbf{g}(\mathbf{u}_k)^T d + \mathbf{g}(\mathbf{u}_k) \le 0 \end{pmatrix} \qquad (13)$$

$\mathbf{H}_k$ is a theoretical strictly positive definite approximation of the Hessian of the Lagrangian function (7)

$$\mathbf{H}_k \approx \nabla^2 L(\mathbf{u}, \lambda) = \nabla^2 f(\mathbf{u}) + \sum_{i=1}^{m} \lambda_i \nabla^2 g_i(\mathbf{u})$$

which is calculated using Quasi-Newton methods. The iterative updating of the Hessian can be done by

$$\mathbf{H}_{k+1} = \mathbf{H}_k - \frac{\mathbf{H}_k \mathbf{s}_k (\mathbf{H}_k \mathbf{s}_k)^T}{\mathbf{s}_k^T \mathbf{H}_k \mathbf{s}_k} + \frac{\mathbf{y}_k \mathbf{y}_k^T}{\mathbf{y}_k^T \mathbf{s}_k} + + \Phi\left[\mathbf{s}_k^T \mathbf{H}_k \mathbf{s}_k\right] \mathbf{v}_k \mathbf{v}_k^T \qquad (14)$$

with

$$\mathbf{s}_k = \mathbf{u}_{k+1} - \mathbf{u}_k \qquad (15)$$

$$\mathbf{y}_k = \nabla \mathbf{L}(\mathbf{u}_{k+1}, \lambda_\mathbf{k}) - \nabla \mathbf{L}(\mathbf{u}_k, \lambda_\mathbf{k})$$

$$\mathbf{v}_k = \left[\frac{\mathbf{y}_k}{\mathbf{y}_k^T \mathbf{s}_k} - \frac{\mathbf{H}_k \mathbf{s}_k}{\mathbf{s}_k^T \mathbf{H}_k \mathbf{s}_k} \right]$$

$$\Phi_k \in [\mathbf{0,1}]$$

Setting $\Phi = 0$ it is the method of Broyden-Fletcher-Goldfarb-Shanno (BFGS). With $\Phi = 1$ it is the method of Davidon-Fletcher-Powell (DFP).

The solution of Eq. 13 can be found with methods of the Quadratic Programming([1], [2]). There the step direction **d** will be calculated to allow the calculation of a new set of open-loop control variables.

$$\mathbf{u}_{k+1} = \mathbf{u}_k + \alpha_k \mathbf{d}$$

With an updated $\mathbf{H}_k$ an iteration with the step size α_k along direction **d** has to be done until the minimum ($\mathbf{u}^0$, λ^0) is reached. The Lagrange multipliers are to be calculated by the use of the optimization conditions (Eq. 10)

$$\nabla \mathbf{g}\left(\mathbf{u}^0\right)\lambda^0 = \nabla \mathbf{f}\left(\mathbf{u}^0\right)$$

The constraints of the optimization result from the desired values for the controlled variables or from the limitation of process variables depending on an open-loop control variable which is not allowed to go beyond a limit.

Real processes are characterized by restricted manipulated variables u_i. Otherwise it would be possible to drive to any point in an infinitely short time interval. Of course these restrictions of the manipulated variables affect the optimum looked for. The constraints (Eq. 5) have to get beyond the upper and lower bounds $\mathbf{u}_{min}$ und $\mathbf{u}_{max}$:

$$\mathbf{u}_{min} - \mathbf{u} - \mathbf{d} \le 0$$
$$\mathbf{u} - \mathbf{u}_{max} + \mathbf{d} \le 0$$

The discrete maximum principle of Pontrjagin

This investigation tests the discrete maximum principle of Pontrjagin as well. The method of KATZ [3] is modified for a HVAC. For the system a set of equations is given.

$$\frac{dx_i}{dt} = \sum_{\nu=1}^{M} a_{\nu i} x_\nu + \sum_{\rho=1}^{P} b_{\rho i} u_\rho + \sum_{j=1}^{R} c_{ji} z_j \qquad (16)$$
$$(i = 1 \ldots M)$$

The system equations can be expressed in a discrete form as

$$x_i^{n+1} = x_i^n + \Delta t \sum_{\nu=1}^{M} a_{\nu i} x_\nu^n + \ldots$$
$$+ \Delta t \sum_{\rho=1}^{P} b_{\rho i} u_\rho + \Delta t \sum_{j=1}^{R} c_{ji} z_j \qquad (17)$$
$$(i = 1 \ldots M)$$

with

n	–	time step (t = n Δt)
Δt	–	time base
t	–	time
M	–	number of system variables
P	–	number of control variables
R	–	number of disturbance variables

The quality function is calculated as follows

$$K = \int_0^T f dt \qquad (18)$$

$$\frac{dK}{dt} \sim \frac{K^{n+1} - K^n}{\Delta t} = f^{\,n+1} \qquad (19)$$

or

$$K^{n+1} = K^n + \Delta t f^{\,n+1} \qquad (20)$$

The system equation M + 1 may be calculated using Eq. (21)

$$x_M^{n+1} = x_{M+1}^n + \Delta t f^{\,n+1} \qquad (21)$$

The Hamilton function is calculated as follows

$$H^{n+1} = \sum_{i=1}^{j+1} z_i^{n+1} x_i^{n+1} \overset{!}{=} \mathrm{Max} \qquad (22)$$

The covariant vector $\mathbf{z}_j^n$ is

$$z_j^n = \frac{\partial H^{n+1}}{\partial x_j^n} \left(n = 1, 2, \ldots N\right) \qquad (23)$$

with the process being devided into N time steps. The system of equations is given by

$$z_1^n = z_1^{n+1}\alpha_{11} + z_2^{n+1}\alpha_{21} + z_3^{n+1}\alpha_{31} + \ldots$$

$$z_2^n = z_1^{n+1}\alpha_{12} + z_2^{n+1}\alpha_{22} + z_3^{n+1}\alpha_{32} + \ldots$$

$$\vdots$$

$$z^n_{j+1} = -1 \qquad (24)$$

OPTIMIZATION AND ANALYSIS

Electric heating

For a room the manipulated variable is the supplied heat flux $\dot{Q}$.

The time-scheduled heat flux profile has to be determined to minimize either the energy consumption itself or the cost of it. As boundary conditions (see Fig. 1) the desired value of the air temperature has to be kept at 22°C during the period of use. The minimum temperature should be at 16°C. The specific energy costs are both constant and time-dependent. The maximum heat capacity is limited as a function of the nominal heat requirement

$$\dot{Q} = a\,\dot{Q}_N$$

With *a*=1 and standard conditions being assumed a continuous operation is necessary. Intermittent heating is achieved and additional losses (airchange) are compensated under standard conditions when *a* is set greater than 1.

Optimization of heat consumption

The optimum operation of an electric heater that is installed both in an old and a new house has to be calculated. The optimization is aiming at the following:

Manipulated variable $u = \dot{Q}$

Temperature	Variant a	Variant b
$\vartheta_i > 16^\circ C$	0 a.m. – 12 p.m.	
$\vartheta_i = 22^\circ C$	7 a.m. – 9 p.m.	7 a.m. – 8 a.m. 4 p.m. – 9 p.m.

Tab. 1 Boundary conditions

Limitations: $\dot{Q} \leq 940.2$ W (old building standard)
$\dot{Q} \leq 493.5$ W (new building standard)

Quality function:

– minimal heat consumption per day

$$K = \int_0^{24h} \dot{Q}(t)\,dt \overset{!}{=} Min$$

Figures 2 and 3 display the profiles of the daily heat flux and the corresponding air temperature in a new building. The profiles of the old building will not be displayed.

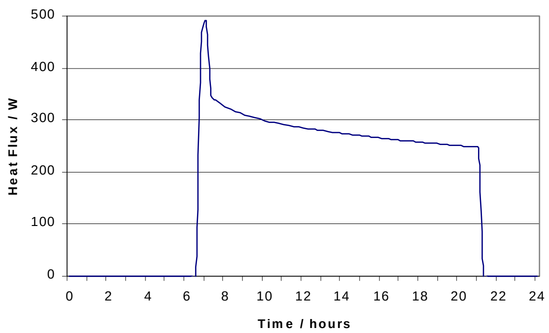

Fig. 2 Heat flux profile for electric heating

The optimization calculations result in a pre-heating procedure with maximum power. During the period of use only a controlled heating output is realized.
The energy savings in comparison with continuous heating are in a range of 3.2 to 15.6 % (Tab. 2). They are bigger at variant b) and for the old houses.

Building type	Variant a	Variant b
New	-3.2 %	-8.4 %
Old	-6.9 %	-15.6 %

Tab. 2 Relative savings of heating energy consumption

The pre-heating time is also a result of the optimization.

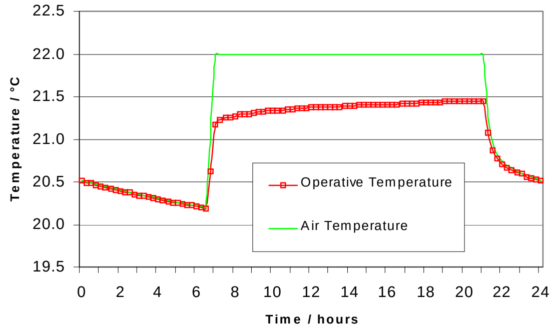

Fig. 3 Temperature profile for electric heating

Optimization of the energy costs

Instead of an optimization of the heat consumption a minimum of the total energy costs per day is to be searched out.

$$K = \int_0^{24h} p(t)\dot{Q}(t)\,dt \overset{!}{=} Min$$

The specific costs p(t) change twice a day (Tab. 3).

Time	Specific costs p(t)
10 p.m. – 6 a.m.	0.10 DM/kWh
6 a.m. – 10 p.m.	0.20 DM/kWh

Tab. 3 Specific costs of heating energy

The optimum open loop control differs substantially from the optimization of the heat consumption. In this case the initial conditions are very important. The starting point of the optimization were stationary conditions. Fig. 4 shows the heat flux and Fig.5 demonstartes the air temperature profiles with an optimization time interval of one day.

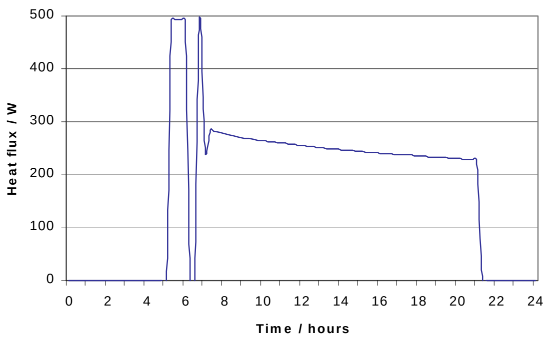

Fig. 4 Heat flux profile for electric heating; optimization of costs; variable specific costs

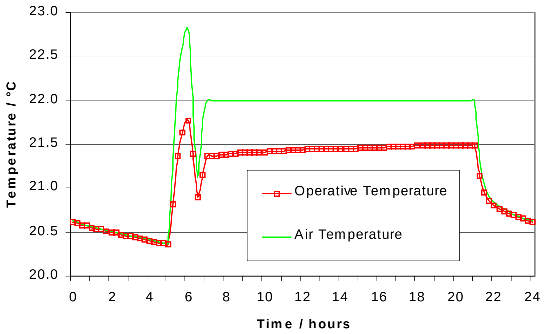

Fig. 5 Air temperature profile for electric heating; optimization of costs; variable specific costs

Figure 6 shows the development of relative cost compared with the costs caused by continuous heating. It starts with savings of about 44 % on the first day and ends with additional costs of 10 % under quasi-stationary conditions on the 16th day.

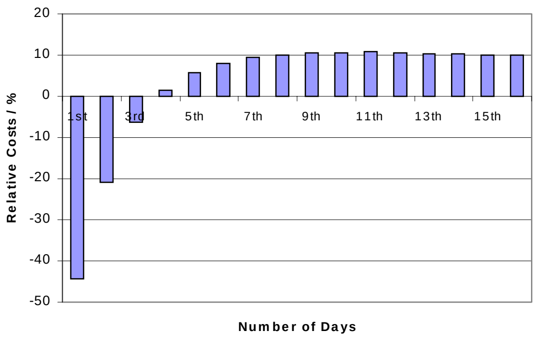

Fig. 6: Development of relative costs

Why are the costs of the calculated optimum higher than the original cost of the continuous heating ?
The reason is the thermal storage of the building envelope. The first three days profit from the heat gains of the building structure. Both the heat requirements and the costs are decreased and the calculated function of the manipulated variable is really proved to be optimum. But from the fourth day onward the optimization reverses. If we now make a comparison with continuous operation there is an additional heat consumption in combination with higher costs. Nevertheless each single one-day-interval shows an optimum solution. The possible return to a continuous operation would cause higher costs now because of the discharged thermal storage of the envelope. The approach to take this effect into consideration leads to an enlarged optimization interval. Fig. 7 displays the heat flux calculated for a 3-day-interval. The heat consumption and the costs are different for each day. The excessive heating on the first two days reduces the heating requirement on the third day. Because of the lower energy costs in the night time in this period the heat input is maximum.

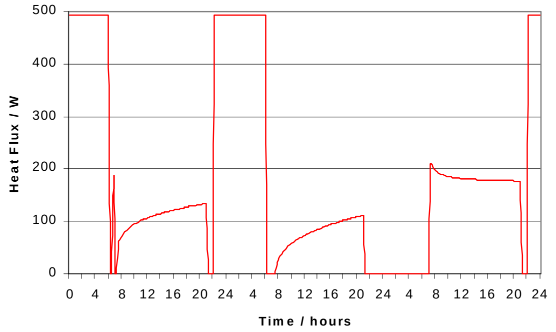

Fig. 7: Heat flux profile for electric heating; optimization of costs; variable specific costs; 3-day-interval

During the time of higher costs only the actually heating has to be provided. The resulting temperature profile is displayed in Figure 8.

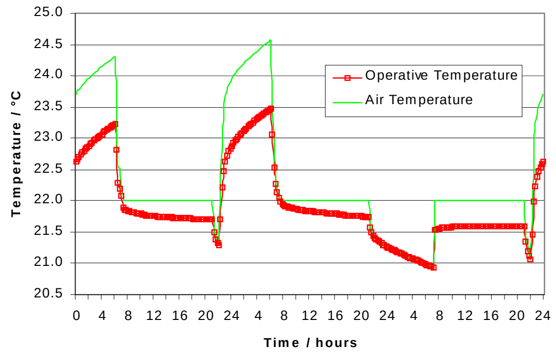

Fig. 8: Temperature profile for electric heating; optimization of costs; variable specific costs; 3-day-interval

As we can the temperatures are higher in the first two nights. The air temperature never falls below the set point temperature. The relations between the heating energy consumption and the costs of the intermitted and the continuous operation are displayed in Fig. 9.

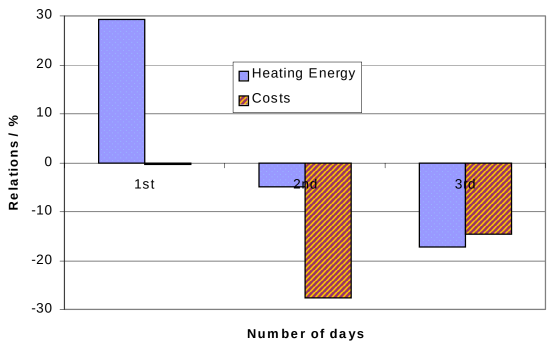

Fig. 9: Relations of Heating Energy and Development of costs; 3-day-interval

For a check of the results the optimization interval was extended to seven days. The optimized heat flux function and the resulting temperature profile are shown in Figures 10 and 11.

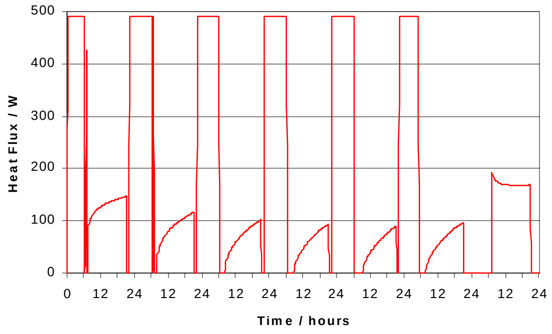

Fig. 10: Heat flux profile for electric heating; optimization of costs; variable specific costs; 7-day-interval

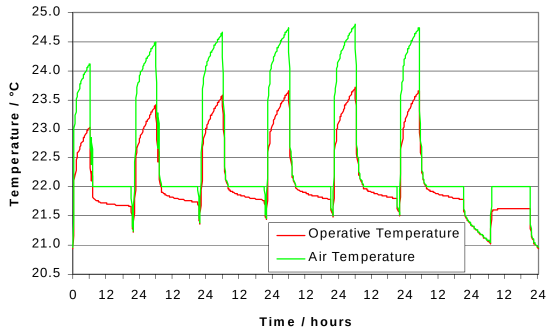

Fig. 11: Temperature profile for electric heating; optimization of costs; variable specific costs; 7-day-interval

The statements made for the 3-day-interval could be applied to the enlarged interval too. On six days the heating is maximum during the period of low specific energy costs and the last day does not require any additional heating in the night time. The relative heating energy consumption and the costs respectivley are displayed in Figure 12.

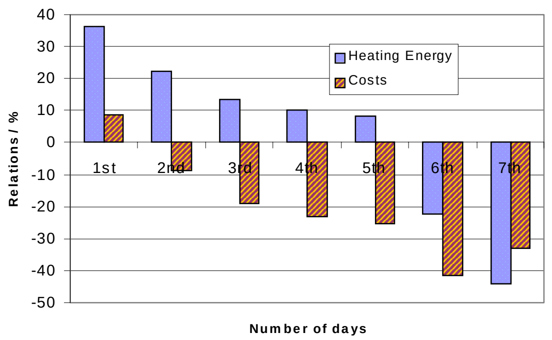

Fig. 12: Relations of heating energy and development of costs; 7-day-interval

As we can see from Tab. 4 the increasing savings of costs are a result of an additional heating output during the whole interval.

Interval	Costs	Heating energy
1 day	+10.0 %	-2.5 %
3 days	-14.1 %	+2.3 %
4 days	-18.1 %	+2.9 %
7 days	-20.3 %	+3.4 %
35 days	-27.1 %	+6.8 %

Tab. 4 Relative savings of the costs and of the heating energy consumption; old building

Water heating

The heating of the room is done by a radiator. The boundary conditions are the same as described in the latter section. The open loop control variable is the inlet temperature limited to

$$\vartheta_V = \vartheta_{V\,max}.$$

In a heating plant without any storage capabilities the maximum heating power $\dot{Q}_{max}$ has to be taken into consideration. Then the inlet temperature is stated as

$$\vartheta_V \le \frac{\dot{Q}_{max}}{\dot{m}c} + \vartheta_R$$

This is a function which depends on the return temperature. The heating surface is overdimensioned by 15 %.

Figures 13 and 14 show the optimized heat flux and temperature profiles.

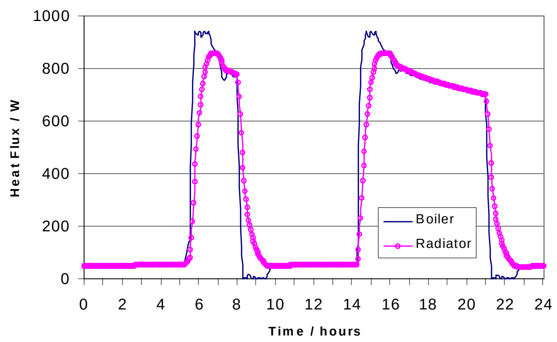

Fig 13: Heat flux profile for water heating; limited heating rate

The energy savings in comparison with continuous heating are in a range of 3.0 to 13.8 % (Tab. 5).

Building type	Variant a	Variant b
New	-3.0 %	-7.5 %
Old	-5.9 %	-13.8 %

Tab. 5 Relative savings of heating energy consumption; limited energy rate

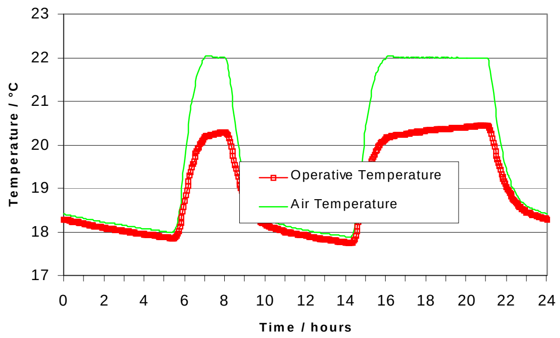

Fig. 14: Temperature profile for water heating; limited heating rate

Operation of an air-conditioning system

The decision upon the operation of an airconditioning system (Fig. 15) should be such, that the operating costs will be kept at a minimum.

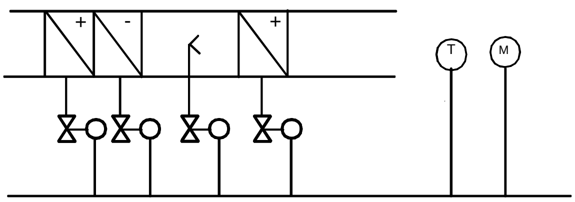

Fig. 15: Air conditioning system

The radiation for the room, the outside air conditions temperature and humidity are shown in Fig.16.

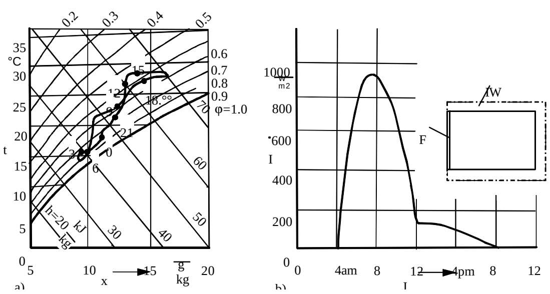

Fig. 16: Outside air conditions

Referring to [4] the room and the system are discribed by a set of differential equations of first order as

$$\frac{d\mathbf{x}}{dt} = f(\mathbf{x}, \mathbf{y}, \mathbf{z})$$

The boundary conditions are

ϑ_i=20 °C
x_i=7.4·10^{-3} kg/kg

in the time t = 8a.m. to 6 p.m.

The inlet air conditions are stated as follows

$$\vartheta_z = 14 \ldots 28^\circ C$$
$$x_z = 0.007 \ldots 0.01\,kg/kg$$
$$\dot{m}_L = 0 \wedge 0.139\,kg/s$$

The optimal operation of the system is shown in Figure 15. There are two control alternatives: cooling with outside air or cooling with the air-conditioning system. The system runs from 6.15 a.m. to 7.00 a.m. and from 7.15 a.m. to 7.30 a.m. (the time base for optimization is 0.25 h) and the air-conditioning system works from 5.30 am to 6.00 am and from 7.30 am with a temperature of the inlet air of 14°C and a humidity of 10 · 10^{-3} kg / kg. In this case the cost reduction for the optimal control is 49 % in comparision with 24 h run. The reduction of ventilator costs are of interest.

In this optimization case costs for

- ventilator
- preheater
- reheater
- humidifier
- chiller

are calculated

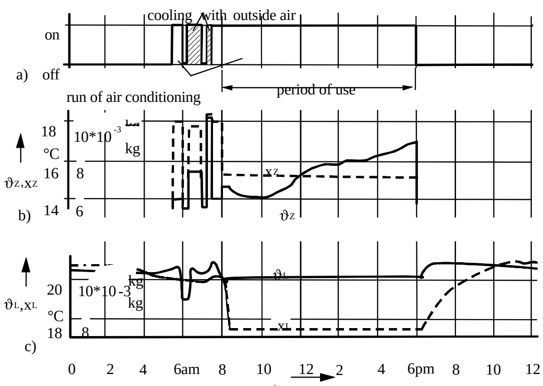

Fig. 15: Optimal control of the air conditioning system

CONCLUSIONS

This paper shows the capabilities of optimization used to calculate an open loop control function minimizing either energy consumption or energy costs. Some results did not meet the expectations made on them (i.e. optimization of the energy costs with variable specific costs). Nevertheless this is just the beginning of the optimization calculations. There is still a large number of problems to be solved in the field of the operation of technical building devices.

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NOMENCLATURE

x	absolute humidity
t	time
$p(t)$	costs
$u(t)$	time-scheduled open loop control variable
$\dot{m}$	mass flow
Q	heat flux
λ	Langrangian multiplier
ϑ	temperature