IMPROVEMENT OF THE FLUID NETWORK'S MODULE CONSTRUCTION FOR HVACSIM+

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ABSTRACT

This paper describes improvements to the calculation of duct/pipe system using HVACSIM+. The improvements described resolve problems relating to fluid backflow calculation, and improvements are suggested to achieve a convergence of duct/pipe network calculation.

TRNSYS and HVACSIM+ are used separately to set the upstream (input) and the downstream (output) in the joint-ports of each duct/pipe component, and calculate the state in the downstream (or upstream) according to the state (pressure, flowrate, temperature and humidity) of the upstream (or downstream) in the module. Neither TRNSYS nor HVACSIM+ could resolve backflow deriving from the flow information. This problem was solved by improving the TYPE design.

TRNSYS and HVACSIM+ processed the variable of each category in parallel. That is, they did not differentiate between the relative importance of variables in each category. In four variables that affect the calculation of duct/pipe network (pressure, flowrate, temperature, and humidity), both pressure and flowrate are coupling variables. Due to the improvement, pressure/ flowrate computation was achieved in one module.

INTRODUCTION

Dynamic simulation of air-conditioning systems is coming of age. In recent years, the successes of such packages as TRNSYS and HVACSIM+ have created interest in research into air-conditioning systems. The two packages are general-purpose simulation programs that treat components of the air-conditioning system as individual modules (TYPEs) which are combined as options to construct an air-conditioning system. The connections of each module are accomplished by inputs and outputs (I/O) that handle flow information of mass and energy, and others. Simulation program comprising modules that functioned together eased the task of constructing a simulation, and contributed to the diffusion of simulation in air-conditioning industry.

A method of I/O design, however, directly restricts the range of simulation. Particularly, module design with TRNSYS and HVACSIM+ could not accurately resolve fluid backflow, when calculating ducts and piping systems. ASHRAE 825-RP improved the convergence of duct/pipe system calculation for TRNSYS and HVACSIM+, but fell short of changing the fundamental design of modules to cope with fluid backflow. Fluid backflow calculation is indispensable to simulation of thermal storage tanks and induction ducts, as well as by-pass pipe in the piping system or by-pass duct in the duct system.

There is also the coupling problem in the fluid network calculation. Calculation of coupled variables with other non-coupled variables would enlarge the variable vector of convergence calculation, and aggravate the convergence of simulation. This paper discusses improvements to these issues.

BACKFLOW PROBLEM

Fig. 1 shows an example of a typical air-conditioning system. This system comprises fluid networks of air and water. HVACSIM+ completely divided the duct and pipe networks of air-conditioning systems into three modules--flow split, duct/pipe, and flow merge--based on a physical model, to make this program fit for general purposes. Fig. 2 partly illustrates relationships between flow inputs and outputs in the module of the duct system, when constructing the system.

When the pressure of supply-air fan and return-air fan falls to unbalance in the system (Fig.1), the direction of airflow in the exhaust-air duct and the

outdoor-air duct may be reversed during simulation. The old TYPE of HVACSIM+ was designed unidirectionally, so the simulation result would be in error when backflow was generated. Correspondence to backflow is crucial to duct/pipe networks having by-pass pipes.
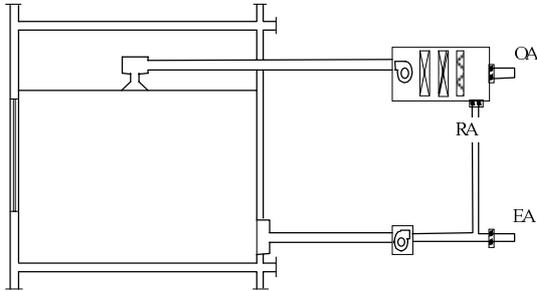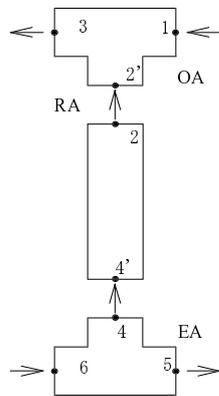

Fig. 1 Typical air-conditioning system

Fig. 2 Connection of old TYPE

As shown in Fig. 2, TRNSYS and HVACSIM+ separately set the upstream (input) and the downstream (output) in the joint-port of each duct/pipe component, and calculate the state in the downstream (or upstream) according to the state (pressure/flowrate, temperature and humidity) of the upstream (or downstream) in the module. When backflow is generated, the entrance set as the upstream initially becomes the exit of the downstream. Neither TRNSYS nor HVACSIM+ could respond to this change since they did not consider the backflow in the flow information. Because the output of the module is calculated using the state of the appointed joint-port, the result of any computation would be in error.

Taking the old type TYPE 4: MERGE (term 'old type' is used in this paper to describe the TYPE of HVACSIM+ Ver. 7) as an example for a pipe network. Thus assume that the fluid flows into inlet 1

and 2 and out from outlet 3 (Fig.3). Calculation models are described by equations (1) through (4).

$$w_3=w_1+w_2 \qquad (1)$$
$$T_3=(w_1T_1+w_2T_2)/w_3 \qquad (2)$$
$$P_1=P_3+K\cdot[\text{sign}(w_1)w_1^2+\text{sign}(w_3)w_3^2]/2 \quad (3)$$
$$P_2=P_3+K\cdot[\text{sign}(w_2)w_2^2+\text{sign}(w_3)w_3^2]/2 \quad (4)$$

Where,

w_1 – flowrate at inlet 1

w_2 – flowrate at inlet 2

w_3 – flowrate at outlet 3

T_1 – temperature at inlet 1

T_2 – temperature at inlet 2

T_3 – temperature at outlet 3

P_1 – pressure at inlet 1

P_2 – pressure at inlet 2

P_3 – pressure at outlet 3

K – flow resistance coefficient

The old TYPE cannot obtain thermal information from the components connected to outlet 3 even when backflow is generated, thus calculating the temperature at outlet 3 from equation (2). This causes error in the simulation result.
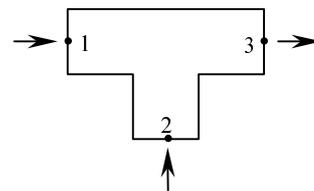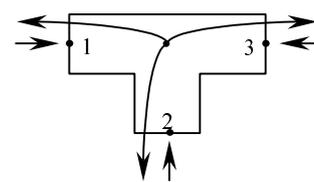

Fig. 3 Inputs/output of old TYPE

Fig. 4 Inputs/output of new TYPE

This is the background of introducing a new method of constructing inputs and outputs to HVACSIM+. The I/O design of the HVACSIM+ (J) cast off simple physical concept and focuses on the features of simulation. Our improved design has the module of each duct/pipe component designed to ensure that it receives information from all the downstream of components that are joined with it. The new TYPE (Three-way Fitting) of merge/split has input/output information at all ports (Fig.4). During simulation, the judgment of which port is the

2

entrance and which is the exit is made within the module from a positive/negative state of the flow. The fluid that entered is mixed completely inside and flows out in a mixed state. The new I/O design allows determination of entrance and exit according to flow, rather than physically designating the entrance and the exit as in Ver. 7. Therefore, the new HVACSIM+ restored the split and merge module to a single three-way fitting module, allowing construction of simulation without defining the entrance and the exit, even in straight ducts/pipes.

To judge the flow direction of the fluid, it is necessary to define a port as the upstream and a port as the downstream, at the PARAMETER of TYPE. The flowrate/pressure network calculation module mentioned later defines the current that flows from upstream to downstream as 'positive'. Therefore, when the current at the upstream port is positive, or the current at the downstream port is negative, it is judged that fluid enters the three-way fitting at this port. Similarly, when the current at the upstream port is negative, or the current at the downstream port is positive, it is judged that fluid flows out of the three-way fitting at this port. Each port of the three-way fitting exchanges information on temperature, humidity, density, etc. with the connected module. This requires designing of all input/output on a one-to-one basis. However, each duct/pipe component has only one output. There is no need to define the output variable separately for each port. A single output is sufficient for constructing a simulation. For visual indication, the output is illustrated with its starting point located at the center of duct/pipe component image.

As the number of I/O variables differs for air and water, TYPE should be provided separately for ducts and pipes. Equations (5) and (6) are formulas for new TYPE three-way pipe.

$$T_{out} = \Sigma(w_i \cdot D_i \cdot T_i)/\Sigma(w_i \cdot D_i)$$
$$((w_i \cdot D_i) > 0, i = 1,2,3) \qquad (5)$$
$$T_{out} = \Sigma T_i / n$$
$$((w_i \cdot D_i) > 0, \Sigma(w_i \cdot D_i) \leq \varepsilon, i = 1,2,3) \qquad (6)$$

where,

n – number of ports. n=3 in TYPE 522.

ε – decimal fraction to avoid overflow in simulation calculation

w_i – flowrate at port i, i=1,2,3

T_i – connected component outflow fluid's

temperature of port i, i=1,2,3

D_i – upstream/downstream definition code of port i; D_i=+1 at upstream port, D_i=−1 at downstream port

T_{out} – temperature of outflow

Calculation of flowrate and pressure is left to TYPE501 (Fluid Flowrate and Pressure Calculation), so the new type Three-way Pipe (TYPE522) covers calculation of temperature alone. Equation (5) calculates the weighted average temperature of fluid flowing into Three-way Pipe and the outflow ($w_i \cdot D_i$ <0) is not included in it. When inflow to the Three-way Pipe is zero or close to zero, the arithmetic average temperature of outflow temperature of components connected to each port is calculated by equation (6).

For the TYPE three-way duct, thermal variables include temperature, humidity, enthalpy and so on.

Fig. 5 clearly illustrates the effect that resulted from changing the unidirectional input/ output design to respond to bidirectional flow in the HVACSIM+(J). Comparison of calculation will be omitted.
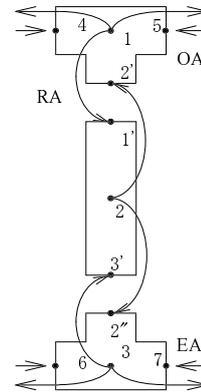

Fig.5 Connection of new TYPE

Correspondence to bidirectional flow not only improved the physical structure of simulation, but also improved the convergence of the simulation. HVACSIM+ solves nonlinear simultaneous equations using the quasi-Newton method, and when flowrate is close to zero, flowrate changes between plus and minus through trial and error during calculation. Calculation that cannot respond to backflow looses its continuity during simulation, and prevents simulation from adhering convergence.

TRANSPORT DELAY

Regarding the calculation of the dynamic

characteristics of temperature and humidity, Two-way Duct or Pipe module is different from other air-conditioning system components in that it considers transport-delay besides thermal delay. In simulations that contain automatic control, the calculation accuracy of transport delay greatly affects the simulation results. Moreover, the error of transport delay calculation may give rise to a situation where simulation does not follow the law of conservation of energy and the law of conservation of mass.

For Two-way duct/pipe, the old TYPE (TYPE2: Conduit) of HVACSIM+ approximated temperature distribution in a duct/pipe by fifth-order polynomial expression to save calculation memory. That is, when inlet was $x = 0$ and outlet was $x=1$, the temperature distribution $T(x,t)$ of fluid was approximated by equation (7).

$$T(x,t) = \sum_{i=0}^{5} a_i(t) \cdot x^i, \ (0 \le x \le 1) \ (7)$$

The dimensionless shifted distance Δx of the fluid in the duct/pipe was calculated as follows:

$$\Delta x = \frac{\Delta t}{(\frac{\rho V}{w})} \qquad (8)$$

where,
Δt – time step
V – volume of conduit
w – mass flow of fluid
ρ – density of fluid

Old TYPE (TYPE2: Conduit) derives the temperature distribution coefficient a_i of equation (7) by picking six points along the length of a conduit and solving simultaneous algebraic equations. By storing the temperature distribution coefficient a_i to memory, it memorizes the temperature distribution of the conduit. When $\Delta x > 0.5$, temperature distribution of the existing fluid part becomes a linear approximation.

Therefore, old TYPE (TYPE2: Conduit) has the following problems:
1) It is limited in response to backflow by the method of memorizing polynomial expression coefficients.
2) Error accumulates in linear approximation when $\Delta x > 0.5$.
3) Convergence is difficult to achieve when $\Delta x < 0.1$.

In Tables 1 and 2, the data of old TYPE are results obtained from the test on absolute transport-delay caused by step-changes and sine-wave changes, against different approach distances (presuming no thermal loss) when using the above mentioned method. Calculation timestep was 1 second.

In the step input of Table 1, the calculation result of old TYPE completely matched the theoretical value when $\Delta x = 0.2$. When $\Delta x = 0.4$, 0.6, 0.8, there were slight delays from the theoretical value. When $\Delta x < 0.1$, it did not converge.

In the sine-wave input of Table 2, it did not converge when $\Delta x < 0.1$, but was correspondent to the theoretical value when $\Delta x = 0.4$. When $\Delta x = 0.6$, 0.8, there as a slight lead from the theoretical value.

With the old TYPE, significant error arose when $\Delta x < 0.2$ for step changes and $\Delta x < 0.4$ for sine wave changes. Errors also resulted in approach distances other than fixed calculation points ($\Delta x = 0.4$, 0.6, 0.8) such as $\Delta x = 0.5$. This owes to the method used to solve the temperature distribution coefficient a_i by simultaneous algebraic equation of six sampling point values.

Calculation of transport delay of Two-way duct/pipe TYPE must be applicable to low flowrate ($\Delta x < 0.2$) as well. The method of memorizing the old temperature distribution with temperature distribution coefficient a_i generated serious errors in the calculation of transport delay, and another method was attempted. Instead of using coefficient a_i of temperature distribution polynomial expression, temperatures at 21 points were derived as elements of memory to memorize the temperature distribution of the conduit[1]. As shown in Tables 1 and 2, this method enabled convergence of transport delay calculation even in the range of $\Delta x < 0.1$, and improved the accuracy of calculation in the range of $\Delta x > 0.2$. However, as the method of describing temperature distribution in the duct/pipe was left unchanged in the polynomial expression of equation (7), no significant improvement was made to the accuracy of calculation of transport delay in the range of $\Delta x < 0.2$. The calculation required to respond to backflow became more complicated then ever.

The method of approximating temperature distribution in a duct/pipe using a fifth-order polynomial expression is only applicable to mild and continuous temperature changes. It cannot be applied to the hunting of temperature from automatic

4

control that occurs in actual air-conditioning systems, nor to the step changes that are often used to test the dynamic characteristics of a system.

The method of memorizing the fluid's temperature by segments is considered applicable to calculate the temperature distribution of bidirectional flow. This method was used to calculate the temperature distribution in a water thermal storage tank which has bidirectional flow, and was found applicable to the calculation of backflow and transport delay. TRNSYS uses a similar method too (TRNSYS TYPE31: Pipe or Duct, etc.). The difference is that equally divided segments are used to calculate the water thermal storage tank, while TRNSYS uses variable size segments. Variable size segments method memorizes the length of each segment, which requires twice amount of memory of the equally divided segment method.

If using 20 equally divided segments to calculate transport delay, the calculation accuracy can be achieved $\Delta x \geq 0.05$. The transport delay calculation between $0 < \Delta x < 0.05$ will be lined according to the law of conservation of energy and the law of conservation of mass. If the calculation accuracy is required further more, the segments number has to be increased too.

When calculating transport delay, the calculation accuracy of the segment memorization method is enhanced as the conduit becomes fractionalized. Accuracy depends on the number of segments into which the conduit is divided. To reduce memory load, segment length and temperature can be compressed to a memory. The numeric value range of INTEGER (4) variable of FORTRAN Compiler compliant to ANSI X3.9-1978 FORTRAN Standard is –2,147,483,648 to 2,147,483,648, which means 9 effective digits can be used. If 4 digits are used to memorize segment length and 5 digits are used to memorize fluid temperature, adequate calculation accuracy may be obtained.

COUPLING PROBLEM

The duct system of the air-conditioning system of Fig. 1 forms a flow/pressure calculation network as in Fig. 6. The flow at three branches are respectively w_1, w_2, and w_3. The pressure at two nodes (connection point of branches) are P_1 and P_2. The flowrate and pressure of this network are solved by the simultaneous algebraic equation of (9).
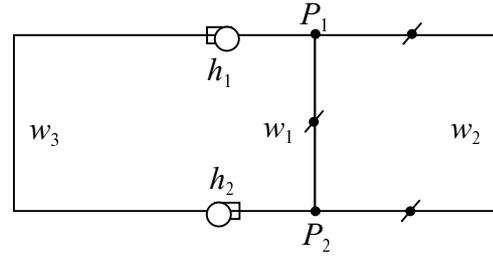

Fig. 6 Fluid network calculation diagram

$$\begin{cases} P_2 - P_1 = K_1 \cdot w_1^2 \\ P_2 - P_1 = K_2 \cdot w_2^2 \\ P_2 - P_1 = K_3 \cdot w_3^2 - (h_1 + h_2) \qquad (9) \\ P_2 = c \\ w_3 = w_1 + w_2 \end{cases}$$

where,

P_1, P_2 – pressure at node 1 and node 2

w_1, w_2, w_3 – flowrate at branch 1, branch 2, and branch 3

K_1, K_2, K_3 – resistance at branch 1, branch 2, and branch 3

h_1, h_2 – head of pump (fan) 1 and pump (fan) 2

c – reference pressure

As equation (9) is a quadratic algebra equation, there are, theoretically, two solutions. Ver. 7 attempted to prevent hunting of simulation caused by two solutions from the quadratic algebra equation by simplifying equation (9). ASHRAE 825-RP attempted to solve a calculation problem to switch turbulent flow to laminar flow at a low flowrate by linear approximation[2].

$$\left\| d_k (\mathbf{x}^{k+1} - \mathbf{x}^k) \right\| \leq e_t \left\| d_t \mathbf{x}^k \right\| \qquad (10)$$

However, as HVACSIM+ tests the convergence of solution for a superblock by equation (10)[4], Ver. 7 places pressure and flowrate of fluid network of air-conditioning system on the same variable vector as all other variables (temperature, control signal, rotational speed, power, and humidity). Also, in HVACSIM+, the unit for pressure is kPa, flowrate is kg/s, fan head is generally 0.392266kPa (40kgf/m^2) at most, and mass flow is only 0.72kg/s even when airflow of 15m/s flows into a duct of 200×200 mm. Air temperature is 10 to 40°C at normal temperature, so if the simulation is solved with flowrate and pressure placed on the same variable vector as temperature, the relative error of flowrate and

pressure becomes 100 times larger than variables such as temperature. As the relative error of flowrate calculation affects the accuracy of calculation of thermal balance, simulation accuracy of Ver. 7 was poor.

In addition, as all variables were placed on a single vector in Ver. 7, flowrate from the quasi-Newton method changed between plus or minus in the convergence calculation when flowrate was close to zero. As in Ver.7, the simultaneous equation became discontinuous and simulation calculation could not converge. For this reason, Ver. 7 could not have zero set as the initial value.

In fact, the flowrate and pressure change in a HVAC system mainly depends on the fluid system operation rather than thermal parameters as temperature or humidity. Though dehumidifying of cooling coil will change air resistance of a coil, but effect few on the flowrate and pressure distribution of the system. Instead, the changes in flowrate and pressure influence temperature and humidity. Flowrate and pressure are correlated, and when either one changes, it affects the other, so they are called 'coupling variables'. Therefore, in the calculation of fluid network by HVACSIM+, the solution for coupling variables, pressure and flowrate, must be solved before other variables such as temperature and humidity to maintain continuance of simulation, even if each duct/pipe component does not respond to backflow.

Decoupling a part of variables from the other part and obtaining solution in advance, is very effective in stabilizing simulation and reducing the time required for simulation.

HVACSIM+ gathered the calculation of heat transfer for an entire building, in a single superblock. If flowrate and pressure calculation of the fluid network follows this and places it in a single superblock, the only result is an increase in the error of the simulation. HVACSIM+ has no simultaneous equations between superblocks. Therefore, the calculation of flowrate and pressure of the fluid network was placed into a single module in order to perform convergence calculation of simultaneous equations with variables such as temperature and humidity, in one superblock.

Flowrate and pressure calculation module has its own way of solving simultaneous algebraic equations, and its own criteria for judging errors. In order to

eliminate insignificant decimal fractions and accelerate the simulation calculation, this flowrate and pressure calculation module introduced a logic which would set the flowrate of a branch to zero when the resistance coefficient of this branch is greater than the threshold resistance coefficient (set by user).

By introducing a flowrate and pressure calculation module, the flowrate and pressure variables are separated from other variables, to obtain the solution of simulation prior to others, and shielding pressure and flowrate variables against influenced by the process of solving nonlinear simultaneous equations by the quasi-Newton method. Pressure and flowrate variables are frozen immediately during this process, and are eliminated from the x^k vector of the equation (10). The calculation of variables such as temperature and humidity thus becomes continuous, allowing stable simulation, and significantly reducing simulation time. The most important achievement here is that the problem of Ver. 7 (which could not have zero as the initial value of pressure and flowrate) has been solved.

CONCLUSIONS

This study on the resolution of problems of fluid network backflow, calculation of transport-delay, and coupling variables that resided in HVACSIM+, has enhanced the simulation functions and convergence of HVACSIM+ and reduced the required simulation time. As a result, it has become possible to simulate a fluid network or water thermal storage tank that could not be simulated by HVACSIM+ Ver. 7. In particular, the solution of coupling problem was found that enhances the simulation of air-conditioning systems. We hope that our study will contribute to the broad diffusion of simulation.

REFERENCES
1. Chen, X. & Nakahara, N.: Simulation on Dynamic Performance of HVAC System - Part5. Thermal Moist Network Calculation, SHASE (The Society of Heating, Air-Conditioning & Sanitary Engineers of Japan) Transactions, 1994 D-28, pp773-776
2. ASHRAE: A Standard Simulation Testbed for the Evaluation of Control Algorithms and Strategies, ASHRAE 825-RP, 1997
3. Zhu, Y. & Jiang, Y.: A New Procedure for Solving

Coupling Problem of Building and System, Proceedings of Tsinghua-HVAC'91, 1991, pp529-540

4. Park, C. & Clark D. R., Kelly G. E.: HVACSIM+ Building Systems and Equipment Simulation Program: Building Loads Calculation, NBSIR 86-3331, 1986, pp16

APPENDIX

Flowrate Network Calculation Module Document

1. General Description

The flowrate and pressure calculation module is designed for calculating the flowrate and pressure in a closed fluid network.

The reference pressure node is specified for calculating the pressure of other nodes.

The relationships of branches and nodes are specified by the parameters of this TYPE.

The flow resistance coefficient, the head characteristics of fan/pump and altitude difference of all branches are designed as the inputs through COMMON /FLOW/.

For forming a network, each branch and node must be numbered, and the index number of branches should be correspondent to the index number of flowrates while that of nodes correspondent to the index number of pressures.

2. Nomenclature

n – number of branches

m – number of nodes

$\mathbf{G}$ – vector of flowrate, $\mathbf{G}=(G_1,G_2,G_3,\cdots,G_n)$

G_i – flowrate flowing through a branch

$\Delta \mathbf{P}$ – vector of pressure difference, $\Delta\mathbf{P}=(\Delta P_1,\Delta P_2,\Delta P_3,\cdots,\Delta P_n)$

ΔP_i – pressure difference between the two ends of a branch

$\mathbf{S}$ – vector of resistance coefficient, $\mathbf{S}=(S_1,S_2,S_3,\cdots,S_n)$

S_i – resistance coefficient of a branch

$\mathbf{H}$ – vector of head of fan/pump, $\mathbf{H}=(H_1,H_2,H_3,\cdots,H_n)$

H_i – head of fan/pump in a branch

$\Delta \mathbf{Z}$ – vector of altitude pressure difference, $\Delta\mathbf{Z}=(\Delta Z_1,\Delta Z_2,\Delta Z_3,\cdots,\Delta Z_n)$

ΔZ_i – altitude pressure difference between the two ends of a branch

$\mathbf{A}$ – network relationship matrix

a_{ij}=1: jth branch is connected to ith node and flow comes in from it;

a_{ij}= −1: jth branch is connected to ith node and flow goes out of it;

a_{ij}=0: jth branch is removed from network;

$1 \leq i \leq m-1$, $1 \leq j \leq n$

$\mathbf{B}$ – basic circuit matrix

b_{ij}=1: jth branch is in ith circuit and direction is same as it;

b_{ij}= −1: jth branch is in ith circuit and direction is opposite to it;

b_{ij}=0: jth branch is removed from network;

$1 \leq j \leq n$, $1 \leq i \leq n-m+1$

k – iteration step

3. Mathematical Description

In a branch, the pressure difference can be described as:

$$\Delta P_i = S_i G_i |G_i| - H_i + \Delta Z_i \qquad (A1)$$

And at a node, the sum of flow rates is:

$$\sum_{i=1}^{n} G_i = 0 \qquad (A2)$$

So the network is described as:

$$\mathbf{A}\mathbf{G} = 0 \qquad (A3)$$
$$\mathbf{B}\Delta\mathbf{P} = 0 \qquad (A4)$$

The matrix $\mathbf{A}$ can be divided into two parts:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{\mathrm{I}} & \mathbf{A}_{\mathrm{II}} \end{bmatrix}$$

where $\mathbf{A}_{\mathrm{I}}$ is a $(m\text{-}1)\times(m\text{-}1)$ square matrix of which the branches belong to the tree of the network.

The basic circuit matrix is described as:

$$\mathbf{B} = \begin{bmatrix} -(\mathbf{A}_{\mathrm{I}}^{-1}\mathbf{A}_{\mathrm{II}})^{\mathrm{T}} & \mathbf{I} \end{bmatrix} \qquad (A5)$$

where $\mathbf{I}$ is a unit matrix.

Let $\mathbf{G} = \begin{pmatrix} \mathbf{G}_{\mathrm{I}} & \mathbf{G}_{\mathrm{II}} \end{pmatrix}^{\mathrm{T}}$, following is derived from equation (A3):

$$\mathbf{G}_{\mathrm{I}} = -(\mathbf{A}_{\mathrm{I}}^{-1}\mathbf{A}_{\mathrm{II}})\mathbf{G}_{\mathrm{II}} \qquad (A6)$$

and $$\mathbf{G} = \mathbf{B}^{\mathrm{T}}\mathbf{G}_{\mathrm{II}} \qquad (A7)$$

Newton iteration is used to solve equation (A4).

Let $f(\mathbf{G}) = \mathbf{B}\Delta\mathbf{P} = 0$, then

$$\Delta \mathbf{G}_{\mathrm{II}}^{k+1} = -\left[\frac{\partial f(\mathbf{G}^k)}{\partial \mathbf{G}_{\mathrm{II}}^k}\right]^{-1} f(\mathbf{G}^k) \qquad (A8)$$

$$\frac{\partial f(\mathbf{G})}{\partial \mathbf{G}_{\mathrm{II}}} = \frac{df(\mathbf{G})}{d\mathbf{G}} \cdot \frac{\partial \mathbf{G}}{\partial \mathbf{G}_{\mathrm{II}}} = \frac{df(\mathbf{G})}{d\mathbf{G}} \mathbf{B}^{\mathrm{T}} \quad (A9)$$

Combining equation (A8) and (A9), following is obtained:

$$\Delta \mathbf{G}_{\mathrm{II}}^{k+1} = -\left[\frac{df(\mathbf{G})}{d\mathbf{G}} \mathbf{B}^{\mathrm{T}}\right]^{-1} f(\mathbf{G}^k)$$

$$= \left[-\mathbf{B}\frac{d\Delta \mathbf{P}^k}{d\mathbf{G}^k}\mathbf{B}^{\mathrm{T}}\right]^{-1} f(\mathbf{G}^k) \qquad (A10)$$

or $\Delta \mathbf{G}_{\mathrm{II}}^{k+1} = \mathbf{M}^{-1} f(\mathbf{G}^k)$ (A11)

here $\mathbf{M}$ is a symmetric, positively definite matrix. It is inverted by the square-root method and computing time can be saved.

The convergence depends on the construction of $\mathbf{B}$ matrix - construction of the tree. Branches of part II are the branches of the tree. The purpose of choosing the tree is to make matrix $\mathbf{M}$ diagonal, dominant. The branch flow resistance coefficient S_i is used as the weight for constructing the smallest tree, i.e. the branches with the smaller S_i values are chosen as the branches of the tree.

In order to decrease the scale of the network as much as possible, a flow resistance criterion for blockage is specified by the user as a parameter. When the resistance coefficient of a branch is higher than this criterion, the branch is considered as blocked and is deleted from the network, and the flowrate is set to zero. This criterion can be set very high if the user prefers to avoid the state of blocking.

The flow resistance coefficient, head characteristics of fan/pump and altitude difference of all branches are built as inputs through the COMMON block, named /FLOW/. If there is any construction change of inputs, such as head characteristics of fan/pump or flow resistance coefficient, in a TYPE, the COMMON /FLOW/ and all TYPEs that are related to this change must change their input construction, too. Also, two parameters must be the TYPEs related to flow-network, (1) flow-network index which this branch belongs to; (2) order index of branch in the flow-network.

Table 1. Test results of transport delay from step changes

Conduit length (m)	50	25	12.5	10	8.33	6.25
Approach distance Δx	0.1	0.2	0.4	0.5	0.6	0.8
Theoretical delay (s)	10	5	2.5	2	1.67	1.25
Old TYPE (s)	3	5	3	4	2	2
21 points by P.E.C.[*] (s)	9	5	3	2	2	2

* P.E.C.: Polynomial Expression Coefficient

Table 2. Test results of transport delay from sine wave changes

Conduit length (m)	50	25	12.5	10	8.33	6.25
Approach distance Δx	0.1	0.2	0.4	0.5	0.6	0.8
Theoretical delay (s)	10	5	2.5	2	1.67	1.25
Old TYPE (s)	3	2	2.5	1	1	1
21 points by P.E.C.[*] (s)	3	2	2.5	2	1	1

* P.E.C.: Polynomial Expression Coefficient