# ASSESSING UNCERTAINTY IN BUILDING SIMULATION

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## ABSTRACT

The premise underlying this work is that introducing uncertainty considerations into simulation will facilitate risk assessment and that this, in turn, will help to improve designer confidence in simulation.

Sources of uncertainty abound in building simulation and must be factored into the solution process. These sources have been identified and useful techniques for quantifying the effects of uncertainties are presented.

Two approaches are described: the use of traditional statistical methods, and the use of alternative arithmetical methods. The theory behind these methods and implementation of appropriate methods into an existing simulation program is described.

Uncertainty analysis, risk analysis, sensitivity analysis.

## INTRODUCTION

There are many sources of uncertainty when using modelling to assess the thermal performance of a proposed building or refurbishment project. These potential sources have been categorised for building simulation; in Figure 1 four categories of uncertainty have been identified:

- **Abstraction**: In transferring the design to a computer representation certain simplifications or concessions have to be made to accommodate the design e.g. averaging the occupancy of a zone.
- **Databases**: The information contained in databases may not match the element to be modelled - an assumption has to be made as to the properties or measurements made.
- **Modelled phenomena**: When developing simulation software, decisions are made regarding the level of detail with which physical processes are modelled e.g. 1, 2 or 3-D heat transfer. The abstraction phase also impinges on these uncertainties i.e. the user may decide not to model 3-D heat conduction despite the availability of that option.
- **Solution methods**: There are various solution techniques available - in resorting to numerical discretisation techniques a discretisation error is introduced to the solution. This category of uncertainty is generally outwith the control of the user.

The main sources of uncertainty for the user are the first two categories and the effects of these uncertainties can be estimated by using the techniques described here. Of particular interest are the uncertainties of category *Databases* where more detailed information can be found at the expense of time and other resources. It would be useful to ascertain whether the extra effort is necessary to achieve better predictions.

## SENSITIVITY ANALYSIS

The traditional and most widely used methods for assessing uncertainty are borrowed from sensitivity analysis. Sensitivity analysis is used to assess the relationship between variations in input parameters to variations in output (predicted) parameters. The parameters which have the greatest influence are termed the *sensitive parameters* in the model. For an uncertainty analysis a distinction must be drawn between a sensitive parameter and an *important parameter*. It may be that a sensitive parameter is known to within a close tolerance, in which case uncertainty in this parameter will not lead to significant uncertainty in the predictions (Hamby 1994). It is therefore necessary to consider in detail realistic levels of uncertainty in the input parameters.

Sensitivity analysis techniques can however be used to address the following issues.

- **Model realism**: How well (and to what resolution) does the model represent reality?
- **Input parameters**: What values should be used in the absence of measured data?
- **Stochastic processes**: To what extent do the assumptions made regarding future weather, occupancy and operational factors affect the predictions?

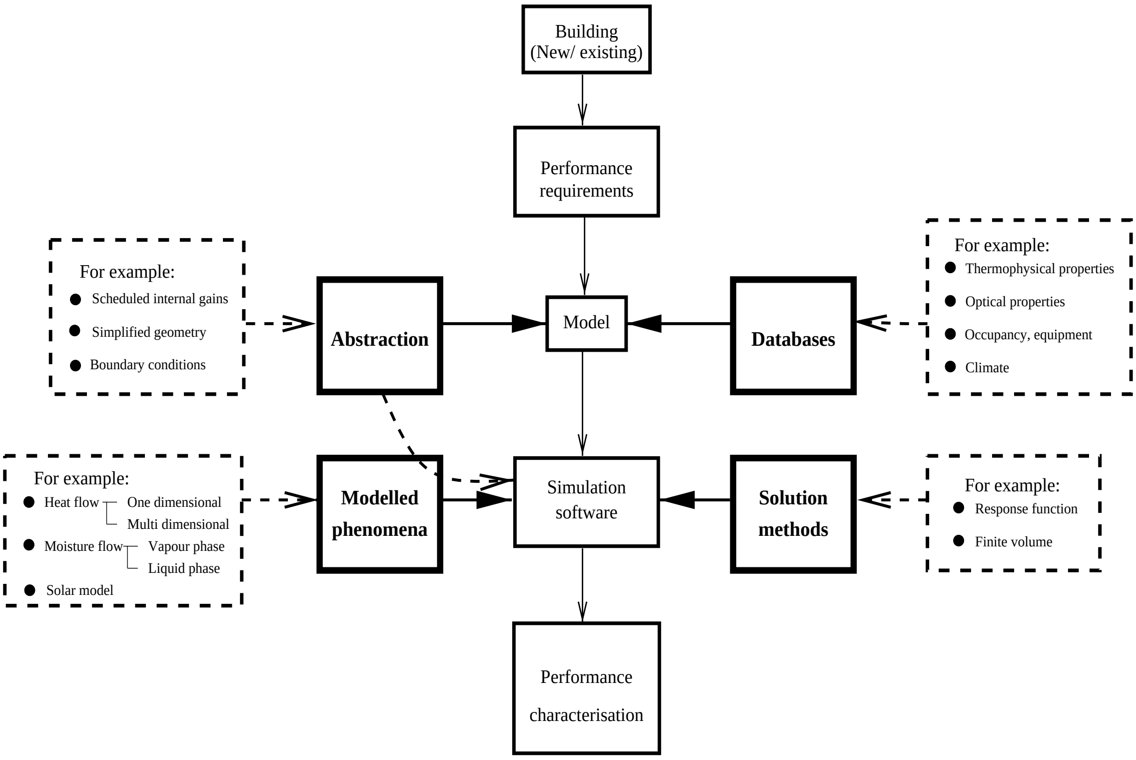


Figure 1: Sources of uncertainties.

- **Simulation program capabilities**: What uncertainties are associated with the particular choice of algorithms for the various heat and mass transfer processes?
- **Design variations**: What will be the effect of changing one aspect of the design?

These issues are related to the categories of uncertainty sources identified in Figure 1. The use of an assessment technique can range from the simple e.g. variation of a few parameters which are deemed to be important to the predictions through to a comprehensive analysis as was the case during the PASSYS project (Jensen 1993). However, due to the difficulties of implementation and managing the analysis of the results, comprehensive analyses have remained in the research domain.

Furthermore, as no integrated techniques are available, sensitivity studies are generally used in an *ad hoc* manner, with specific scripts being written to perform individual studies. However, there is clearly a need for better simulation support to allow the user to assess uncertainty, and indeed to present predictions and their associated uncertainties as a matter of routine (CIBSE 1998).

To achieve better simulation support several issues have to be addressed. The sources of uncertainty affecting a model have to be identified and quantified. Then suitable techniques have to be identified before a structure to assess these variations can be implemented within a simulation package.

## ANALYSIS TECHNIQUES

Several statistical uncertainty analysis techniques have been developed (Hamby 1995, Kleijnen 1996). They can be categorised as structured and non-structured methods. Structured methods are derived from experimental techniques, whereby a series of experiments would be designed to analyse the outcome for predetermined models. Non-structured methods are stochastic in nature. In the former category, the most popular method for application to building thermal simulation is Differential Sensitivity Analysis (DSA); in the latter category Monte Carlo Analysis (MCA) has been used.

It is also possible with deterministic solution techniques to carry the uncertainty information through the calculation procedure. These techniques rely on altering the underlying arithmetical functions as all operations are carried out on ranges rather than individual numbers. These techniques have not been applied to integrated building thermal simulation to date.

Statistical and arithmetical techniques are now described.

*Statistical techniques*

Both DSA and MCA have the advantage of being relatively easy to apply to existing software because the simulation is treated as a black box, and the only parameters which can be influenced are contained in the data model describing the problem. The basis of the methods is described in Lomas and Eppel (1992). The main problem with the use of structured techniques such as DSA is that they were devised for practical experiments with relatively few (typically less than a dozen) measured (and controllable) inputs. When applied to simulations where there are typically hundreds of input parameters the technique becomes cumbersome and time consuming.

The DSA method "is the backbone of nearly all other sensitivity analysis techniques" (Hamby 1994) and requires a base case simulation in which input parameters are set with the best estimates of the parameters under consideration. Then the simulation is repeated with one input parameter changed from $P$ to $P+\delta P$ and the effect on the output parameter(s) of interest noted. This is done for each parameter in turn, giving a total of $N+1$ simulations to analyse the effects of $N$ uncertain parameters. An underlying assumption of this analysis is that the effect of an uncertainty is linear over the perturbance. This assumption can be tested to a limited degree by carrying out further simulations with the parameter values set to $P-\delta P$. If the effect on output parameters is the same (but in opposite directions) then linearity is assumed. The DSA method is not optimised for the number of simulations required and does not identify any parameter interactions. However, it does inform the user which of the analysed parameters have most influence on the output parameter(s).

To measure the interactions between parameters factorial designs can be used. In these designs any number of parameters can be altered at one time (a structured design is created and followed so as the results can be decomposed after the simulations have been carried out). The number of simulations required for a full factorial analysis is $2^N$ and is only practical for a few parameters. However it is possible to only analyse a fraction of all the combinations by assuming that certain interactions will have negligible effect. Fractional factorial designs for up to 12 parameters have been published (Box et al 1978). The main advantage of these methods is that the interactions between parameters can be quantified directly.

MCA is the most commonly used non-structured method. It relies on the central limit theorem to provide an overall assessment of the uncertainty in the predictions being made. The Monte Carlo technique generates an estimate of the overall uncertainty in the predictions due to all the uncertainties in the input parameters, regardless of interactions and quantity of parameters. In operation, a probability distribution is firstly assigned to each input parameter under consideration. For all parameters, values from within their probability distribution are randomly selected and a simulation undertaken. Simulations are undertaken repeatedly with new values randomly selected. Given a large number of simulations, the uncertainty in the output parameter of interest will have a Gaussian distribution, irrespective of the input parameter probability distributions. It has been shown (Lomas and Eppel 1992) that the number of simulations required by this technique is 60-80, after which only marginal gains in accuracy are obtained. Figure 2 shows the 95% confidence bands in the calculated standard deviation normalised by the standard deviation as a function of the number of simulations.

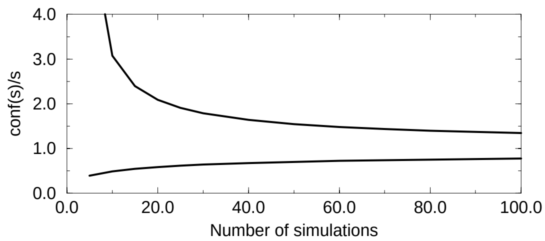


Figure 2: Confidence in Monte Carlo results.

The main difficulty in employing this method is in identifying the distributions that the input parameters are likely to have. In practice it is usual to assume that most parameters will have a Gaussian distribution although any distribution is possible. A major disadvantage is that the method does not distinguish individual parameter sensitivities.

*Arithmetical techniques*

When using deterministic solution techniques it is possible to carry the information relating to uncertainties through the calculation procedure. To achieve this the fundamental arithmetical operations have to be redefined to act on a function rather than individual numbers. Two such approaches are presented, interval and affine arithmetic.

Interval arithmetic is a branch of mathematics which redefines the basic arithmetical operations so that the resulting range is guaranteed to contain all the possible results of the operation. In this process, numbers are defined by their lower and upper bounds e.g. $\pi$ could be represented by the interval number $[3..4]$. In general an interval number $\alpha$ represents the range $[\alpha_{-}..\alpha_{+}]$.

All arithmetical functions have to be redefined e.g. addition:

$$\alpha + \beta = [\alpha_{-}..\alpha_{+}] + [\beta_{-}..\beta_{+}] = [\alpha_{-} + \beta_{-}..\alpha_{+} + \beta_{+}]$$

In general the result of any function is defined by the minimum and maximum values of the possible outcomes. For further details see Neumaier 1990.

This technique allows the implicit calculation of the effects of uncertainties during the calculation procedure i.e. only one simulation would be required to assess the effects of the defined uncertainties.

When applying this technique to the numerical solution of the Fourier field equation an exponential growth is exhibited by the predicted temperature bounds. This is well documented for similar problems (e.g. Rao 1996) and is due to the technique having no *memory* of previous operations. For example in many algorithms a variable is used to calculate a value which is subsequently used in a calculation using the same variable again e.g. $x + y = z$ followed later by $z - y$ in standard arithmetic would result in the value $x$. In interval arithmetic $[4..5] + [1..2] = [5..7]$ and $[5..7] - [1..2] = [3..6] \neq [4..5]$. In this example although the result $[3..6]$ bounds the solution $[4..5]$ there is significant overestimation of the size of the interval.

There are, however, techniques which use the benefits of interval arithmetic without the problems of overestimation. One of these is using an affine transformation to represent the uncertain parameters (Andrade et al 1994). Using a centered linear transformation the uncertainty is now represented as $A + \varepsilon_1$ where $A$ is a real number and the uncertain term $\varepsilon$ is the interval $[-1..1]$. Generalising the above example:

$$(A + \varepsilon_1) + (B + n\varepsilon_1) = A + B + (1 + n)\varepsilon_1$$

and subtraction of $B + n\varepsilon_1$ from the sum:

$$(A + B + (1 + n)\varepsilon_1) - (B + n\varepsilon_1) = A + \varepsilon_1$$

our initial parameter.

## IMPLEMENTATION

ESP-r (ESRU, 1999), a building/plant performance simulation program originating from the University of Strathclyde in Glasgow, was chosen as the simulation program for implementation of routines for uncertainty analysis. ESP-r is a detailed simulation program, and for this reason has a large number of input parameters. Within the PASSYS programme (Jensen 1993), sensitivity analyses (DSA) were conducted by writing a series of Unix scripts specifically targeted at the test cells that were being modelled. These scripts edited the ESP-r input files, ran simulations, extracted the required output data, calculated various statistical measures and prepared the data for graphical display. The test cells were modelled in some detail, with over 50 parameters modified (including geometrical data, constructional data, air flow, climate etc). The exercise was useful for 2 reasons - identifying the most important parameters that were tested and estimating the overall error in the predictions.

However, it was clear that for routine use, the analyses needed to be integrated more fully into ESP-r: an analysis of the time taken on a script-based sensitivity analysis revealed that to create and validate the scripts would require about a week's work; the simulations could then safely be left to run overnight (or the weekend) and the data for analysis could then be sent to third party software for analysis. Clearly one of the major barriers to routine use was the effort required to hand craft the scripts and input files for the model - a fully integrated approach was advocated.

The structure for integrating uncertainty analyses is shown in Figure 3. There are three distinct phases: definition of uncertainties in a database, multiple simulations of required (perturbed) models, and analysis of results.

These phases have been implemented within ESP-r and are now described.

### *Uncertainty Definition*

When defining the uncertainties, a database of uncertainty distributions is referenced to create a unique description of the uncertainties applicable to the model under investigation. There are three distinct sets of information contained in the model uncertainty file, to allow flexibility for the model user: the magnitude of the uncertainty of a parameter, and if necessary, its probability distribution; locations (spatial and temporal) where uncertainties have effect; and the association of the uncertain model parameters with the particular locations.

As an example, the reception area of a building may have an average occupancy of 5 people, with an 80% uncertainty (i.e. the true occupancy is between one and nine people) and a typical open-plan office in the same model may have an occupancy of 50 people, with only a 10% uncertainty, e.g. Figure 4. In this case, there would be two entries in the first part of the database (occupancy uncertainty of 80% and 10%, both with normal distributions). The second part of the database would identify the spatial locations where the uncertainties were applicable, and the last part

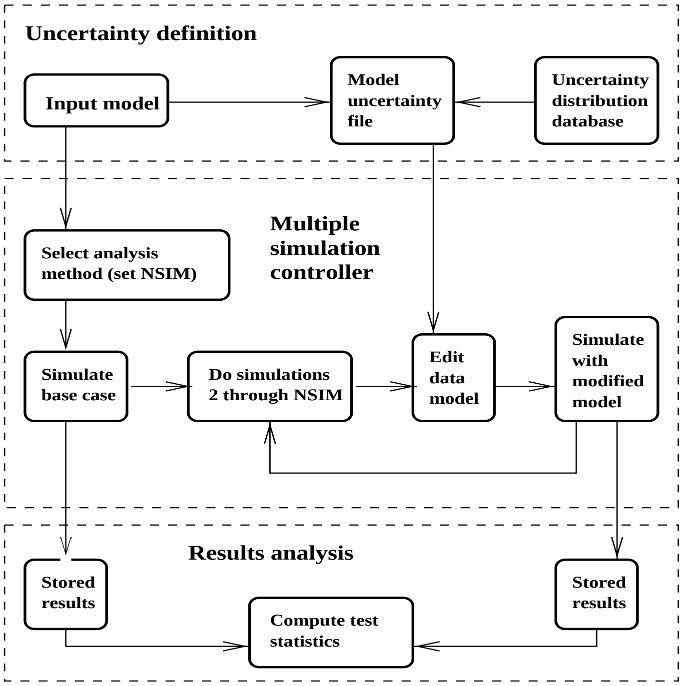


Figure 3: Simulating with uncertainties.

of the database would define the association of uncertain occupancies with the appropriate locations.

One of the major tasks of the research is to populate the database. Some aspects are relatively easy. For example, thermophysical properties can be specified, with the range of uncertainty estimated from the range of experimental results (Clarke et al 1991). On the other hand, inputs connected with occupancy can have high uncertainty. For example, the uncertainty in metabolic rate is estimated as being up to 100%, and typically around 50% (Parsons and Hamley 1989).

*Simulation*

To simulate with the defined uncertainties has required the creation of a multiple simulation controller, which effects the necessary changes in the data model for each parametric variation and initiates the simulations. The controller reads the input data model into memory and all subsequent changes to the model are made there. This is to avoid corruption of the input data model, which can be referenced between each simulation, prior to data manipulation.

Before the simulations are commissioned the total number required is calculated. This is straightforward after the analysis method has been chosen. In the case of DSA there are $2N+1$ simulations for a two-sided test, where $N$ is the number of parametric variations. For MCA, 80 simulations are set by default (although the user can increase or decrease this if necessary). After running the base case simulation the simulation controller references the model uncertainty file and, using information held there, changes one or more parameters in the model (depending on the analysis method chosen), as shown in Figure 5.

*Analysis*

The results database created from the multiple simulations contains full result sets for each of the simulations, thus allowing detailed examination of each set and the differences between sets. Although this requires a lot of disk space, it is considered that at this stage of the development it allows more flexibility. In the future, it may be possible for the user to be selective in choosing which data to store.

When comparing different result sets three measures of uncertainty were identified, their use being applicable to different uncertainty analyses. These measures are:

- range of predictions;
- standard deviation/ variance of Gaussian distribution;
- dimensionless metrics, e.g. sensitivity coefficient, curvature test.

The first measure is the most simple and readily understood as all possible outcomes are represented by the range between the upper and lower curve. This gives data similar to that shown for the predicted data in Figure 6. This measure clearly

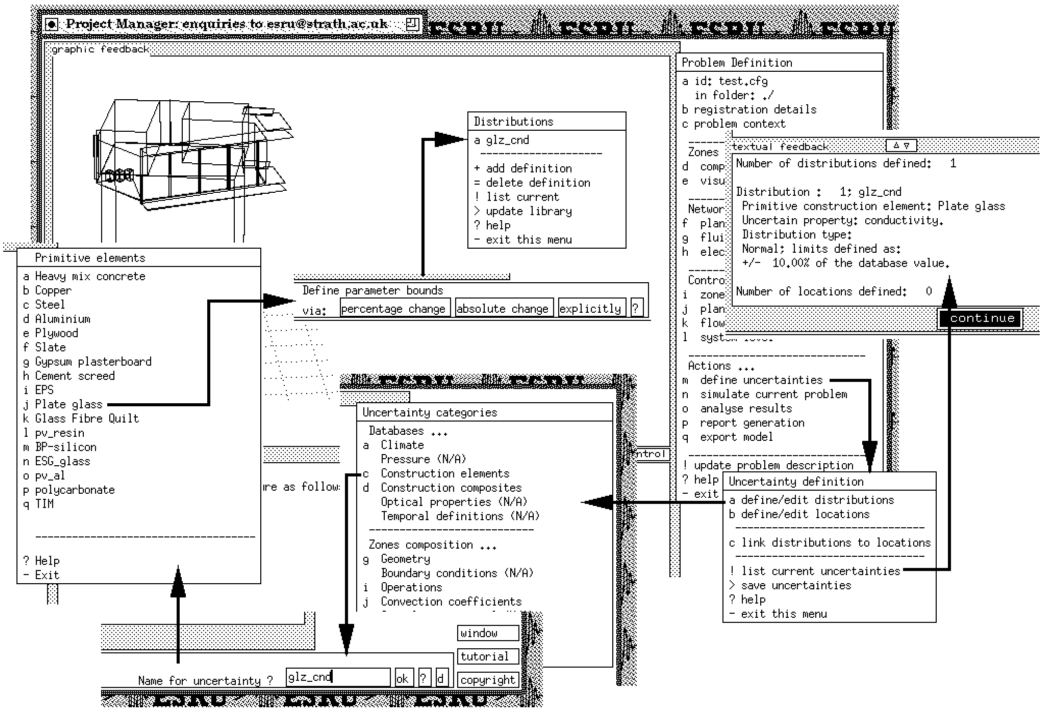


Figure 4: Uncertainty definition.

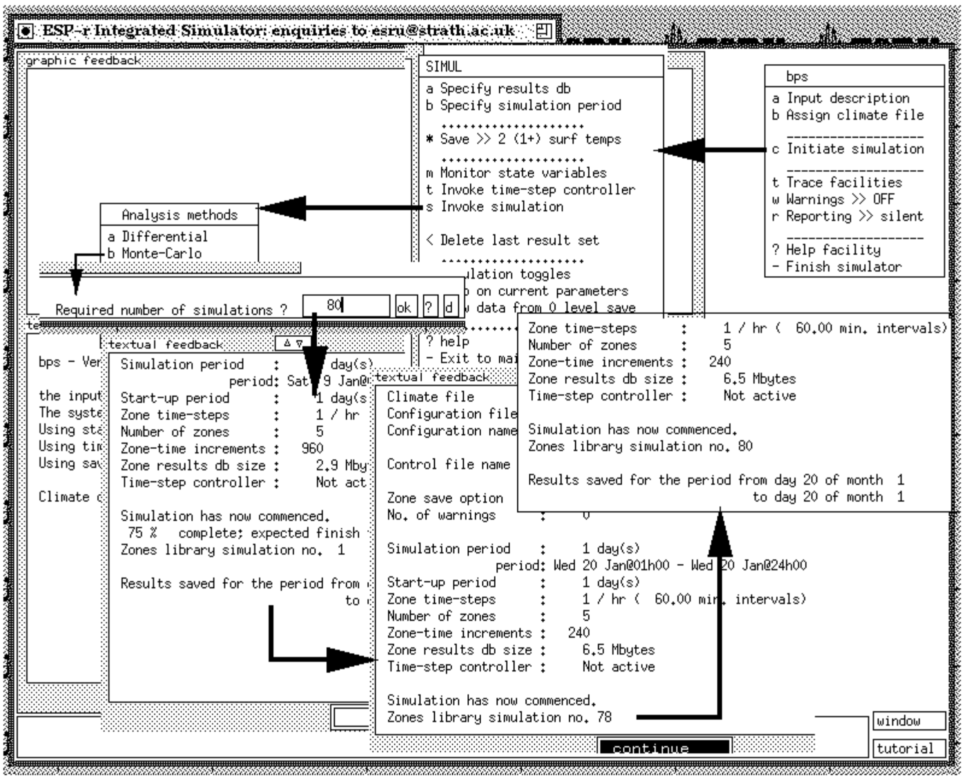


Figure 5: Uncertainty simulation.

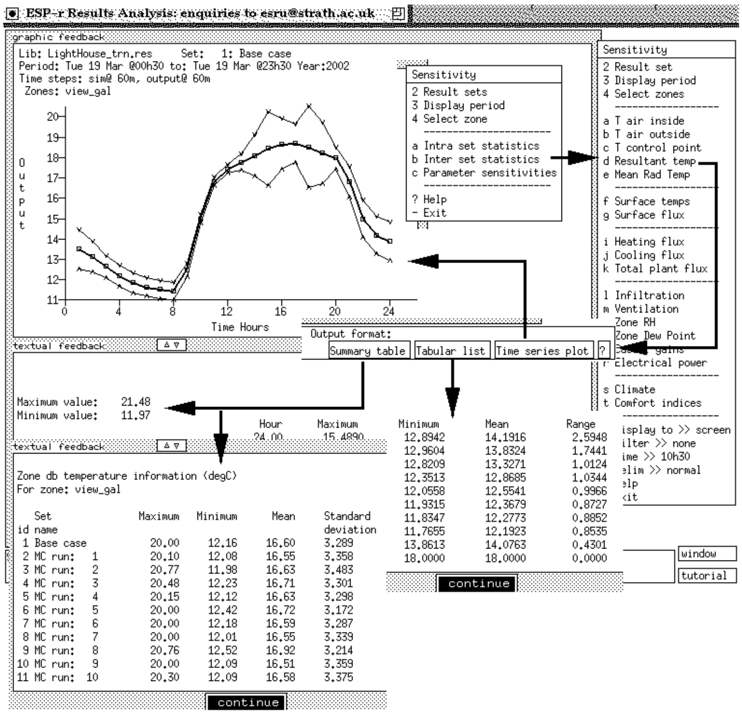


Figure 6: Uncertainty results analysis.

displays the maximum and minimum values and the range at all timesteps and can be used for both DSA and MCA techniques.

Although uncertainties from a differential analysis can be combined to give an overall range this is not recommended - a Monte Carlo analysis should be carried out in its place. This is due to the generally unpredictable nature of parameter interactions in a complex model.

The second measure creates a more terse description of the analysed data and provides a basis for creating a range of predictions from MCA. It can also be used to examine how the size of the range varies over time (as may result from uncertainties in thermal capacity). Figure 6 shows the resulting error bands from an MCA.

Dimensionless metrics, such as sensitivity coefficients, allow the comparison of the relative effects of separate differential analyses and rank ordering of the parameters. Using the normalised standard deviation (i.e. the standard deviation divided by the mean value) in output divided by the normalised standard deviation in input, the effects of individual uncertainties can be compared directly. This measure should be used with care as in most building applications the temperature, heating and cooling flux (common assessment metrics) are close to or equal to zero for large periods causing spurious results e.g. an absolute effect of 1°C is generally not important but if the temperature is close to 0°C the normalised difference will be excessively large.

## DISCUSSION

The use of an integrated approach has resulted in numerous efficiencies:

- The definition of uncertain data has been made more straightforward by removing one of the major barriers to analysing the effects of uncertainties.
- By altering the uncertain values within memory a further improvement in simulation time is achieved. A quick analysis shows that the normalised simulation time (i.e. time for *N* simulations/ time for one simulation) is less than the number of simulations; this will never be the case for a script based approach.

It is recognised by the authors that the differential method is not optimised for number of runs and that the Monte Carlo method is also simulation intensive. However, even on a Sun Sparc 1 an eighty run Monte Carlo analysis of a three zone building (seven day simulation) takes in the order of 1.25 hours; this is not seen as an excessive simulation time.

The results of a sensitivity analysis can be complicated for the casual user to understand. This has been taken into account here through the use of the DSA and MCA methods, but for other methods (e.g. factorial) the user will require knowledge of the method and how to interpret the results.

## FUTURE DEVELOPMENTS

Now that the structure exists for carrying out sensitivity analysis it is possible to implement more esoteric, run-optimised methods e.g. the fractional factorial method and its successors.

The arithmetical techniques have been tested on the core numerical solver of ESP-r (Macdonald 1999). The initial indications are similar to DSA for single instances of an uncertainty. This implementation is being expanded so as more complex models can be tested and compared against other methods e.g. MCA. The potential of integrating this method with ESP-r is also being analysed.

The definition of uncertainties has to become more natural for the user. This requires two issues to be resolved: the user must be aware that all the data that is used in simulation is subject to uncertainty; and the interface to the tool must collect this information without undue burden on the user. The collection of uncertainty information can be likened to working in a greyscale world and then being asked for colour information: the user needs to understand what colour is and why it is needed; and the interface should collect this new information in the same manner as existing data.

## CONCLUSIONS

The premise underlying this work is that introducing uncertainty considerations into simulation will facilitate risk assessment and that this, in turn, will help to improve designer confidence in simulation.

At present the uncertainty database has been populated with occupancy and material thermophysical property uncertainties. The simulation and analysis sections contain the capability for differential sensitivity and Monte Carlo analysis. Although the structure was developed initially for input parameter uncertainty, it is also possible to use it for other sensitivity analyses: to investigate the effect of alternative algorithms within the code, design changes etc. The results analysis allows simple statistical comparisons within and between sets in order to facilitate the generation of measurement statistics due to the different analysis techniques available. These capabilities are being expanded to provide a more comprehensive facility. Finally arithmetical techniques are being investigated further.

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