# OPTIMAL CONTROL FOR CENTRAL COOLING PLANTS

B.C. Ahn, Ph. D.
Kyungwon University, Kyungi-Do, 461-701, Korea
J.W. Mitchell, Ph.D., P.E.
University of Wisconsin, Madison, Wisconsin, U.S.A.

## ABSTRACT

The optimal supervisory control strategy for a central chilled water facility has been determined. A quadratic linear regression relation for the total system power in terms of the controlled and uncontrolled variables was developed using simulated data. The supply air, chilled water, and condenser water temperatures that minimize energy consumption are determined as functions of load, ambient wet bulb temperature, and sensible heat ratio. The use of the quadratic expression to determine optimal control is demonstrated.

## INTRODUCTION

In a cooling plant, There are many control variables that may be controlled to minimize building energy consumption. Global optimal control methods for the cooling plants have been studied by Sud[1984], Lau[1985], and Johnson[1985]. Their studies showed significant savings associated with the use of optimal control method as compared with traditional control one. However, they did not develop general algorithms suitable for on-line control.

Braun[1988] developed a methodology for determining the optimal control strategy for an HVAC system and showed that saving of 10 to 20 % are possible by applying optimal control to the system. He showed that the power consumption of a cooling plant may adequately be represented as a quadratic function of continuous control variable and uncontrolled variables. In addition, he showed that the power consumption of a chiller may also be adequately represented as a quadratic function of the load and the temperature difference between leaving condenser and evaporator water temperatures. He also demonstrated that the power of pumps and fans may be represented with a quadratic function of control variables and flow rates. His study provides the basis for the method used in this paper.

Pape et al.[1991] developed an overall empirical cost function of the optimal control for the total power consumption of the cooling plant. In his paper, the cost function with respect to two control variables such as supply air temperature and chilled water temperature, and uncontrolled variables. However, The optimal strategy for cooling tower associated with the condenser water set temperature is not considered. Ulleberg[1993] developed a building energy management system (BEMS) controller. The controller, which learns the behavior of the air conditioning system with the passage of time and attempts to control the system based on an optimal set point control such as supply air and chilled water temperatures, was tested on an emulator.

The object of this paper is to illustrate the optimal control in which, for a given building load and ambient conditions, the overall system energy consumption was minimized while maintaining comfort conditions in the building. The optimal set temperatures for supply air, chilled water, and condenser water, are selected such that energy consumption is minimized under a range of uncontrolled variable, which are load, ambient wet bulb temperature, and sensible heat ratio. The chilled water loop pump and cooling tower fan speeds were controlled by the PID controller such that the supply air and condenser water set temperatures reach the set points designated by the optimal supervisory controller. Then, the effects of the controlled variables on the total system and component power consumption were compared.

## OPTIMAL CONTROL ALGORITHM

An optimization process is able to find the minimum of the sum of the operating costs $J$ as a function of the forcing functions $f$ with respect to the discrete control variables $M$ and the continuous control variables $u$. Expressed mathematically

$$J = J(f, u, M) \qquad (1)$$

The forcing function $f$ is a vector containing all uncontrolled variables, such as load, ambient wet bulb temperature, and sensible heat ratio. The continuous control variables $u$ are those such as supply air temperature, chilled water temperature, and condenser water temperature that can varied continuously over the control range. The discrete control variables $M$ are those controls that have discrete settings, such as high and low fan speed, the number of operating chillers, and the number of operating fans or pumps.

A simple function for which an optimum exists that can be determined analytically is a quadratic function. Braun et al[1988] have shown that power requirements for chillers, fans, and pumps can be adequately expressed as quadratic relationships. Similarly, they have shown that the cost of energy for an entire cooling plant can also be represented as a quadratic function. Their process for optimization will be reviewed here. Therefore, the cost function represented by equation(1) can be written to represent the entire system

$$J(f, M, u) = u^T A u + b^T u + f^T C f + d^T f + f^T E u + g \qquad (2)$$

where $A$ is a symmetric matrix , $C$, and $E$ are coefficient matrices, $b$ and $d$ are coefficient vector, and $g$ is a scalar. The optimal control is determined by equating the first derivative of the power with respect to each control variable to zero ( $\partial J/\partial u = 0$ ). Solving for the optimal values of continuous control variables yields linear relations between the control variables and the forcing function.

$$u = -\frac{1}{2} A^{-1} b - \frac{1}{2} A^{-1} E f \qquad (3)$$

This formulation and resulting linear control laws are the result of unconstrained optimization. And the control laws results in a minimum power consumption only if the Hermitian of the cost function is a positive definite matrix[Ogata(1967)]. The coefficients in equation (2), which depend on the discrete control variables, i.e., the various operating modes, need to be determined empirically. To complete this task a least square linear regression technique was employed. The total number of possible coefficients of equation (2) in terms of the controlled and uncontrolled variables in the quadratic equation for each feasible set of discrete control modes is

$$N_{coeff} = N_u^2 - \frac{N_u(N_u - 1)}{2} + N_u + N_f^2 - \frac{N_f(N_f - 1)}{2} + N_f + N_f N_u + 1 \qquad (4)$$

where $N_u$ is the number of continuous control variables and $N_f$ is the number of uncontrolled variables. In this study there are three controlled variables and three uncontrolled variables, thus 28 coefficients need to be determined by regression techniques.he exact model of the quadratic formula has the form

$$y = B_0 + B_1 x_1 + B_2 x_2 + \cdots + B_{n+1} x_1^2 + \cdots + B_{2n+1} x_1 x_2 + \cdots + e \qquad (5)$$

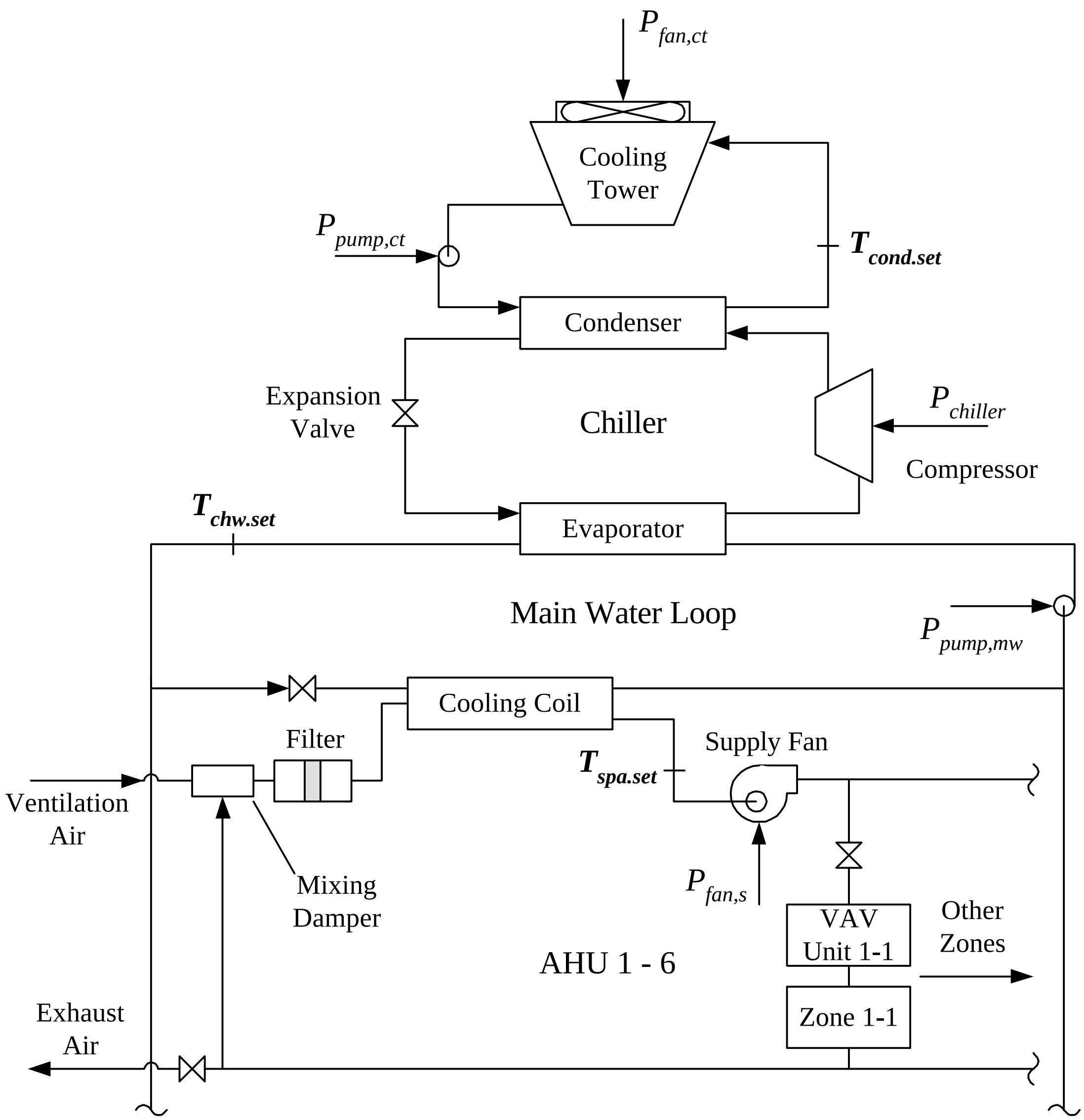
where $y$ are the responses, $x_i$ are the predictors, $B_i$ are the regression coefficients, $n$ is the number of variables in the equation ( $n = 6$ in the considered case), and $e$ is an error term with a normal distribution with a mean of zero and a standard deviation . A second order least squares linear regression estimates the $B_i$ with $b_i$ and predicts

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \cdots + b_{n+1} x_1^2 + \cdots + b_{2n+1} x_1 x_2 + \cdots \qquad (6)$$

where $\hat{y}$ is the fitted or predicted value. The two outputs, standard deviation (S) and the coefficient of determination ($R^2$), from the regression can be used to judge the quality of fit between predicted and observed values. The coefficient of determination is the fraction of the variation in y that is explained by the fitted equation. These two parameters have to be improved as far as possible to obtain a good fit. As it may be anticipated from equation (6), the values of the coefficients depend strongly on the number of data sets (sets of predictors and corresponding response) used in the regression.

## APPLICATION TO THE COOLING PLANT

Figure 1 shows a schematic of the typical variable air volume(VAV) air-conditioning system used in this study. It is a centralized chilled water facility, with chilled water distributed to a set of air-handling units. The HVAC system meets the air conditioning requirements of the building zones through control of the supply fan speed, cooling tower fan speed, chilled water and condenser water pump speed, supply air temperature setting, chilled water temperature setting, and condenser water temperature setting. The system has three control variables: the chilled water set temperature, the supply air set temperature, and condenser water set temperature. The significant




$$P_{tot} = P_{chiller} + P_{fan,ct} + P_{pump,ct} + P_{pump,mw} + P_{fan,s}$$

Figure 1 Schematic of a conventional air conditioning system

uncontrolled variables are the cooling load, the ambient wet bulb temperature, and sensible heat ratio. The ambient dry bulb temperature as a ambient condition has almost no effect on the total power consumption of cooling system (Braun 1988).

The cooling coil requires feedback control to provide the supply air temperature set by supervisory control system. The control signal for the chilled water main loop pump modulating cooling coil water flow rate is updated until the control variable, which is the supply air set temperature, reaches the set point designated by the supervisory controller. The cooling tower fan speed is also controlled by the feedback control such that the condenser water set temperature reaches the set point designated by the supervisory controller. The feedback controller uses the typical PID control algorithm to return the control variable to set point. The PID controller for the pump speed was designed to control the supply air set temperature. The controller in the frequency domain has a transfer function

$$\frac{M(s)}{E(s)} = G_c(s) = K_p + \frac{K_i}{s} + K_d s \tag{7}$$

Where $M(s), E(s), G_c(s), K_p, K_i, K_d$ and $s$ are continuous data controller output, error, controller transfer functions, proportional, integral, derivative control gains, and Laplace variable. The integral $K_i / s$ can be approximated by the z-transform on the polygonal integration (Kuo [1977]).

$$\frac{M(z)}{E(z)} = D(z) = K_p + \frac{K_i T(z+1)}{2(z-1)} + \frac{K_d (z-1)}{Tz}$$

$$\begin{aligned} K_1 &= K_p + \frac{K_i T}{2} + \frac{K_d}{T} \\ K_2 &= -K_p + \frac{K_i T}{2} - \frac{2K_d}{T} \\ K_3 &= \frac{K_d}{T} \end{aligned} \tag{8}$$

where $T$ is sampling time. The following difference equation was obtained from equation (8), which is easy to implement in a digital controller.

$$\begin{aligned} m(k) &= m(k-1) + K_1 e(k) + K_2 e(k-1) + K_3 e(k-2) \\ e(k) &= r - c(k) \end{aligned} \tag{9}$$

where $m(k), e(k)$, $r$ ,and $c(k)$ are the $kth$ controller output, the $kth$ error, required set-point, and measured control value, respectively.

The supply air flow rate was evaluated using the room and supply air set temperatures and the sensible load. The cooling tower fan speed was controlled by the feedback control such that the condenser water set temperature reached the set point designated by the supervisory controller. A PID speed control algorithm was also used as the feedback controller. The water flow rate in cooling coil or air flow rate in cooling tower under a PID controller was determined by the following control law

$$\begin{aligned} &\dot{m} = \dot{m}_{MAX} - m(k) \\ &\text{if} \quad \dot{m} > \dot{m}_{MAX} \quad \text{then} \quad \dot{m} = \dot{m}_{MAX} \end{aligned} \tag{10}$$

where $\dot{m}, \dot{m}_{MAX}$ , and $m(k)$ are water or air flow rate, maximum water or air flow rate, and PID controller output, respectively. The pump in the cooling tower was assumed to operate at constant speed. Optimal set point temperatures for condenser water temperature, chilled water temperature, and supply air temperature were such that energy consumption was minimized under variation of load, ambient wet bulb temperature, and sensible heat ratio as uncontrolled variables.

The simulation study was done by using the TRNSYS Ver. 14.2 (Klein et al. 1996) program. The system model studied in the paper is based on the system used by Pape[1992] and the models of Braun[1988] included in TRNSYS program were used as system components. Modular programs of the TRNSYS were modified for the control of each component and additionally some control programs such as supervisory control were developed.

The total system power is not exactly a quadratic function of the controlled and uncontrolled variables. However, near the optimum, the behavior of the system power can be very well approximated with a quadratic function. Therefore, to obtain an accurate equation, data near the optimum is needed. Thus, it is important that the region in which the data is taken includes the optimal conditions. After collecting enough data near optimal conditions, a regression was performed to obtain the quadratic correlation for the power. In this study, the MINITAB regression program package release 11 was used to obtain the coefficients of the quadratic equation.

The quadratic cost function for the total system of cooling plant can be presented as

$$\begin{aligned} P = P(&Tchw.set, Tspa.set, Tcond.set\ , \\ &Load, Twb, SHR) \end{aligned} \tag{11}$$

All the coefficients of equation (2) were determined empirically using linear least squares regression techniques. In the system which was considered in this paper, only one set of discrete control variables exits, and thus one formula for system power was developed.

The Hermitian matrix A in the quadratic cost function of equation (2) must be positive definite in order to obtain the minimum power. Since A is a real symmetric matrix, the Sylvester criterion (a necessary and sufficient condition in order that the quadratic form, where A is real symmetric matrix, be positive definite is that the determinant of A be positive and the successive principal minors of the determinant of A be positive) can be used to check if Hermitian matrix A is positive definite or not. After the regression technique was applied to the example system, matrix manipulation and rearranging yielded the coefficients, controlled variables, and uncontrolled variables for its quadratic cost function (equation (2)) as follows:

$$\begin{aligned} A &= \begin{bmatrix} 1.124 & -1.145 & 0.013 \\ -1.145 & 2.025 & -0.028 \\ 0.013 & -0.028 & 0.511 \end{bmatrix} \\ C &= \begin{bmatrix} 0.002 & -0.035 & -0.795 \\ -0.035 & 0.039 & -13.259 \\ -0.795 & -13.259 & 70.58 \end{bmatrix} \\ E &= \begin{bmatrix} 0.007 & 0.011 & -0.006 \\ 0.168 & -0.111 & -0.665 \\ 1.246 & 21.695 & -0.732 \end{bmatrix} \\ b &= \begin{bmatrix} -1.800 & -42.264 & -9.373 \end{bmatrix}^T \\ d &= \begin{bmatrix} -0.469 & 10.998 & -192.670 \end{bmatrix}^T \\ g &= 317.69 \end{aligned} \tag{12}$$

$$u = [Tchw.set \quad Tspa.set \quad Tcond.set]^T$$
$$f = [Load \quad Twb \quad SHR]^T$$

Then, the successive principal minors of the determinant of A and the determinant of A are solved analytically to determine if the Sylvester criterion was satisfied in the quadratic cost function to obtain the minimum power,:

$$\begin{aligned} &|a_{11}| = 1.124 > 0 \\ &\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 1.147 > 0 \\ &\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0.586 > 0 \end{aligned} \quad (13)$$

Once the empirical coefficients in equation (2) were known, the derivatives with respect to the controlled variables were taken to yield linear control laws. In this paper, derivatives were taken with respect to the supply air temperature $(Tspa.set)$, chilled water temperature $(Tchw.set)$, and condenser water temperature $(Tcond.set)$. The following three equations were solved analytically for the optimal set point temperatures:

$$\begin{aligned} &\frac{\partial P}{\partial Tchw.set} = 0 \\ &\frac{\partial P}{\partial Tcond.set} = 0 \\ &\frac{\partial P}{\partial Tspa.set} = 0 \end{aligned} \quad (14)$$

According to equation (3), the resulting equations for three set point temperatures are linear relation with respect to uncontrolled variables or ambient conditions such as building load, ambient wet bulb temperature, and sensible heat ratio:

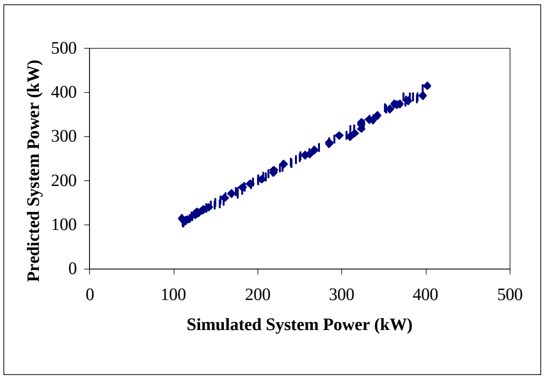
$$\begin{aligned} &Tchw.set.optimal = G_1(Load, Twb, SHR) \\ &Tspa.set.optimal = G_2(Load, Twb, SHR, \\ &\qquad Tchw.set.optimal) \\ &Tcond.set.optimal = G_3(Load, Twb, SHR, \\ &\qquad Tchw.set.optimal, Tspa.set.optimal) \end{aligned} \quad (15)$$

These three unbounded optimal set point temperatures are linear functions of the forcing functions due to the assumption of the quadratic dependence of power on these variables.

In equation (15), the optimal supply air set point temperature is affected by the chilled water set temperature selected from optimal control algorithm, while the optimal condenser set temperature is also affected by the optimal supply air and chilled water set temperatures.

The predicted power obtained from the quadratic regression equation is presented as a function of the simulated power in Figure 2. The standard deviation was S = 3.166 (about 2%) and the coefficient of determination was $R^2$ = 99.4%. The quality of fit between predicted and observed system power is a straight line occurs which characterizes a good fit. Only very few points are off the line, and these points represent conditions which are far away from the optimal control.

In this study, the three set points temperatures are constrained to avoid problems with components operation. The chilled water temperature is constrained between 11°C and 3°C to provide sufficient dehumidification and to prevent freezing in




**F**igure 2 Predicted versus simulated total system powers

the evaporator tubes. The supply air temperature is constrained between 15°C and 8°C to avoid overcooling or becoming too humid in the conditioned spaces. The condenser water temperature is also constrained to be greater than 19°C to avoid the possibility of compressor lubrication problem and surging.

The set point temperatures are linear functions of the forcing functions due to the assumption of quadratic dependence of power to the variables. The linear control laws are shown in Figure 3 to 5.

The optimal set point temperatures (Tspa.set.optimal, Tchw.set.optimal, and Tcond.set.optimal) and the total system power are presented as functions of the building load in Figure 3, as functions of the ambient wet bulb temperature in Figure 4, and as functions of sensible heat ratio in Figure 5, respectively. In Figure 3, the ambient wet bulb temperature (18°C) and sensible heat ratio (0.8) are fixed. The optimal supply air and chilled water set temperatures decrease with increasing load, while the optimal condenser water set temperature increases with increasing load. The temperature difference between the optimal supply air and chilled water set point temperatures increases with load. In the predicted total system power consumption, the increase of the power at higher loads is greater than at lower loads. If the load is less than 100 ton or greater than 450 ton, the optimal set temperatures may reach the constraints. However, it is really not important and the figure does not show it.

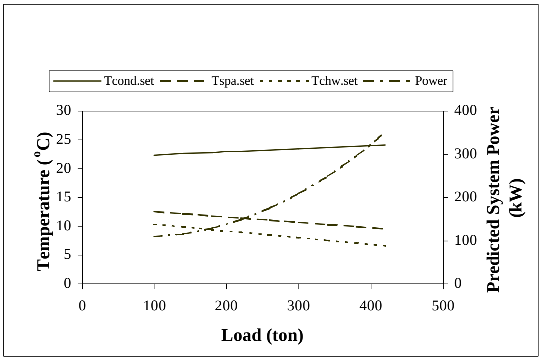



Figure 3 Optimal setpoint temperatures and predicted system power for varying load

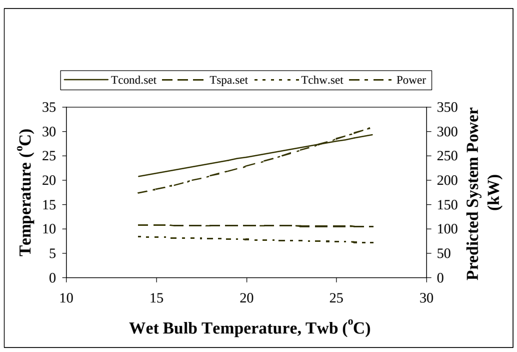
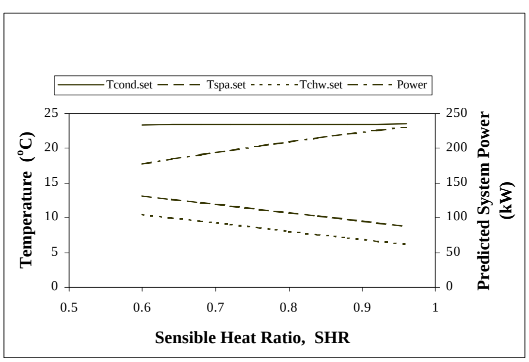



Figure 4 Optimal setpoint temperatures and predicted system for varying ambient wet bulb temperature




**F**igure 5 Optimal setpoint temperatures and predicted system power for varying ambient wet bulb temperature

In Figure 4, the load (300 ton) and sensible heat ratio (0.8) are fixed. The optimal chilled water set temperature decreases a little with increasing ambient wet bulb temperature, while the ambient wet bulb temperature has very a little influence on the optimal supply air set temperature because it influences on the very small amount of ventilation load. Thus, the temperature difference between the optimal supply air and chilled water set point temperatures increases with ambient wet bulb temperature more than with load. The optimal condenser water set temperature increases with ambient wet bulb temperature. The influence of the ambient wet bulb temperature on the optimal condenser water set temperature is relatively very high. The predicted total system power increases linearly with increasing ambient wet bulb temperature.

In Figure 5, the load (300 ton) and ambient wet bulb temperature (18°C) are fixed. The optimal chilled water and supply air set temperatures decrease with increasing sensible heat ratio. The temperature difference between the optimal supply air and chilled water set point temperatures is held constant. However, the sensible heat ratio has very a little effect on the optimal condenser water set temperature. The predicted total system power increases linearly with sensible heat ratio.

In Figure 6 to 8, the simulated and predicted total system power, the chiller power, the cooling tower fan power, the main water pump power, and air handling unit fan power are presented as functions of the supply air set temperatures (Figure 6), as functions of the chilled water set temperature (Figure 7), and as functions of the condenser water set temperature (Figure 8), respectively. The power consumption of the cooling tower pump are constant for all conditions and are therefore not shown. In this air conditioning system, there are 6 supply fans (AHU 1-6). The figures show the power for one supply fan. However, the total power included 6 supply fan powers. The figures are valid for a fixed set of forcing function (Load = 300 ton, Twb = 18°C,

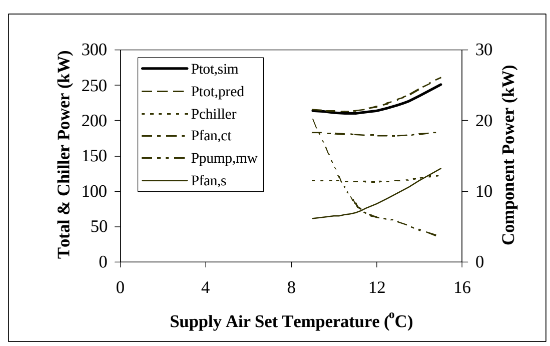



Figure 6 Predicted, simulated total system, and component powers for varying supply air set temperature

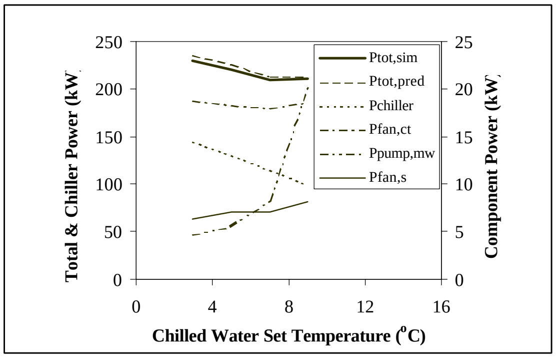



Figure 7 Predicted, simulated total system, and component powers for varying chilled water set temperature

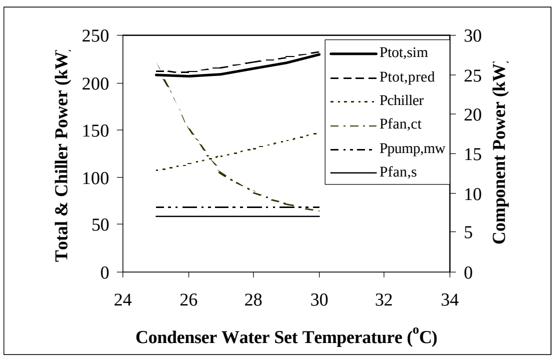



Figure 8 Predicted, simulated total system, and component powers for varying condenser water set temperature

SHR = 0.8). The total system and chiller power are shown on left y axis and other component powers are on right y axis.

Figure 6 shows that the total system and component power consumption vary with changing supply air set temperature. The chilled water and condenser water set temperatures are fixed at 7°C and 26°C, respectively. The predicted total system power is a good fit to that simulated. The supply air fan power increases with the supply air set temperature because the flow rate has to be increased to meet the load, while the main water loop pump power decrease fast with increasing supply air set temperature because less water has to be provided for a fixed chilled water temperature. Through the trade-off between supply air fan power and the main water loop pump power, the total system power use in both simulated and predicted system are minimized at a supply air set temperature of 11°C. The change of the supply air set temperature does not have an effect on the chiller and cooling tower fan powers.

Figure 7 shows that the total system and component power consumption vary with changes in the chilled water set temperature. The supply air and condenser water set temperatures are fixed at 11°C and 26°C, respectively. The predicted total system power consumption is a good fit to the simulated power. As the chilled water set temperature increases, the chiller power decreases while the main water loop pump power increases. The influences of the chilled water set temperature on the cooling tower fan and supply air fan power are relatively very small. Through the trade-off between the main water loop pump power and chiller power, the total system power use in both simulated and predicted system are minimized at a supply air set temperature of 7°C. However, the only slight difference of the total system powers between 7°C and 9°C in higher chilled water set temperatures is occurred.

Figure 8 shows that the total system and component power consumption vary with changes in the condenser water set temperature. The supply air and chilled water set temperatures are fixed at 11°C and 7°C, respectively. As the condenser water set temperature increase, the cooling tower fan power decreases while the chiller power increases. The changes of the condenser water set temperature does not have effects on the main water loop pump and supply air fan powers. Through the trade-off between cooling tower fan power and chiller power, the total system power use in both simulated and predicted system are minimized at a supply air set temperature of 26°C. However, the difference of the total system powers between 25°C and 26°C in lower condenser water set temperatures is very small.

## CONCLUSION

The optimal control strategy in which, for a given building load and ambient condition, the total system operation cost is minimized is applied to the cooling plant. A quadratic least square regression technique is used for predicting the total system power in terms of the forcing function and control variables. The optimal values of set temperatures such as supply air, chilled water, and condenser water temperature, are selected that energy consumption is minimized with varying load, ambient wet bulb temperature, and sensible heat ratio as uncontrolled variables.

A quadratic regression equation for predicting the total cooling system power in terms of the forcing function and control variables is fit to the data collected under different values of controlled and uncontrolled variables. The standard deviation and the coefficient of determination shows that the fit is good. As the Hermitian matrix of the system quadratic cost function turned out to be positive definite, the optimal minimum power was able to be obtained. The optimal control is determined by equating the first derivative of the power with respect to each control variable to zero. The resulting set of equations determine the optimal values of the controlled variables.

The influence of the load and sensible heat ratio on the optimal supply air and chilled water set temperatures are relatively high, while ambient wet bulb temperature have slight effects on them. In contrast to that result, the ambient wet bulb temperature have much influences on the optimal condenser water set temperature, while the load has less and the sensible heat ratio has no influence on it.

The trade-off among the component power consumption results in that the total system power use in both simulated and predicted system are minimized at lower supply air, higher chilled water, and lower condenser water set temperature conditions. However, the total system powers varies slightly in near that set temperature conditions.

The quadratic cost function approach is good for optimal control and is a algorithm suitable for on-line control. This study will be a basis for fault detection and diagnosis researched in the next time.

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## NOMENCLATURE

| | |
|---|---|
| $A^{-1}$ | inverse matrix |
| $b$ | coefficient vector |
| $b_i$ | regression coefficients |
| $C$ | coefficient matrix |
| $c(k)$ | the measured control variable |
| $d$ | coefficient vector |
| $D(z)$ | controller z-transfer function |
| $e$ | error with normal distribution |
| $E$ | coefficient matrix |
| $e(k)$ | the $kth$ control error |
| $e(k-1)$ | the $(k-1)th$ control error |
| $e(k-2)$ | the $(k-2)th$ control error |
| $E(s)$ | error s-transfer function |
| $E(z)$ | error z-transfer function |
| $f$ | forcing function (uncontrolled variables) |
| $g$ | scalar |
| $G_1, G_2, and G_3$ | functions |
| $J$ | operating cost function |
| $K_p$ | proportional control gain |
| $K_i$ | integral control gain |
| $K_d$ | derivative control gain |
| $Load$ | cooling load |
| $M$ | discrete control variables |
| $m_{MAX}$ | maximum mass flow rate |
| $m(k)$ | the $kth$ controller output |
| $m(k-1)$ | the $(k-1)th$ controller output |
| $M(s)$ | controller output s-transfer function |
| $M(z)$ | controller output z-transfer function |
| $n$ | the number of variables |
| $N_{coeff}$ | total number of coefficients |
| $N_f$ | the number of continuous uncontrolled variables |
| $N_u$ | the number of continuous control variables |
| $P$ | power |
| $r$ | the required set point |
| $R^2$ | the coefficient of determinition |
| S | standard deviation |
| $s$ | Laplace variable |
| $SHR$ | sensible heat ratio |
| $T$ | sampling time |
| $Twb$ | ambient wet bulb temperature |
| $u$ | continuous control variables |
| $x_i$ | predictors |
| $z$ | variable $(z = e^{Ts})$ |