A METHOD TO ESTIMATE THE SHADING OF SOLAR RADIATION THEORY AND IMPLEMENTATION IN A COMPUTER PROGRAM

Kurt Källblad, Ph.D.
Department of Building Science
Lund Institute of Technology, Lund University
P.O.Box 118, SE-221 01 Lund - Sweden

ABSTRACT

A general method to determine the shading of direct and diffuse radiation is presented. This can handle an almost infinite number building surfaces and screens of any shape. Furthermore, the method can be used to determine view factors in rooms of arbitrarily shape.

A formula to determine the non-shaded area of a surface sunlit by direct radiation at a certain incidence angle is described. This formula is then used in an algorithm to determine view factors used to estimate the shading of diffuse, isotropic radiation.

The implementation in a computer program is described and the algorithms used as well as some results are discussed

INTRODUCTION

Solar radiation is one of the major heat sources in a building and is thus important to treat in a proper way in computer programs that predict heating and cooling loads of buildings. Different methods to treat shading of direct and diffuse solar radiation exist but they are often restricted in one way or another.

A common technique for the modelling of direct radiation is to divide building surfaces into patches following a regular grid and then trace the ray from the sun to the centre of each patch. If any screen shades a ray, the whole patch is assumed to be shaded. This method can result in large errors in some cases not obvious for the program users.

View factors are often used when shading of diffuse radiation is determined. However, in many cases the number and shape of shading screens are limited. Most of the time only one overhang and two side fins can be taken into account, see for example TRNSYS (1983) and Delsante & Spencer (1994).

Niewienda and Heidt (1996) describe a method similar to the method presented here and implemented in the program SOMBRERO. The main difference between the methods is that a graphical method is used in SOMBRERO while the method presented here uses an analytical solution.

A recent literature review, Dubois (1997) conclude that the development of calculation methods to assess energy use and comfort in buildings equipped with shading devices still needs to be carried out. This resulted in the development of the method presented here.

SHADING OF DIRECT RADIATION

The situation is illustrated in Figure 1 where a surface and some shadows are shown.

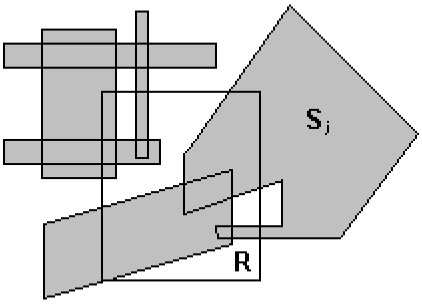

Figure 1. A shaded surface R

Figure 1 shows a surface R upon which different regions of shade containing intersections and unions are formed. It is obvious that these regions depend on the radiation direction as well as on the shape and position of the shading screens.

The shaded part of R is the intersection between R and the union of the shadows. The non-shaded area A_{ns} of R with N shadows can thus be expressed as:

$$A_{ns}(R, S_N) = A(R) - A(R \cap \bigcup_{j=1}^{N} S_j) \qquad (1)$$

where A = Area of a region
N = Number of shadows
S_j = Shadow j

The union of the shadows may be complex to represent. It can consist of several regions of which some may contain holes. In order to avoid unions, the last term is developed.

$$\begin{aligned}A(R\cap\bigcup_{j=1}^{N}S_j)&=A\left[(R\cap S_N)\cup(R\cap\bigcup_{j=1}^{N-1}S_j)\right]\\&=A(R\cap S_N)+A(R\cap\bigcup_{j=1}^{N-1}S_j)-A\left[(R\cap S_N)\cap(R\cap\bigcup_{j=1}^{N-1}S_j)\right]\\&=A_{ns}(R\cap S_N,S_{N-1})+A(R\cap\bigcup_{j=1}^{N-1}S_j)\qquad(2)\end{aligned}$$

Repeated development of the last term gives the final result:

$$A_{ns}(R,S_N)=A(R)-\sum_{J=1}^{N}A_{ns}(R\cap S_j,S_{j-1})\qquad(3)$$

where S_0 is an empty region.

The following useful relations can also be observed:

$$A_{ns}(X\cap S_1,S_0)=A(X\cap S_1)\qquad(4)$$

$$A_{ns}(X\cap S_j,S_{j-1})=0\ \text{if}\ X\cap S_j=0\qquad(5)$$

$$A_{ns}(R,S_N)=0\ \text{if}\ R\cap S_j=R\ \text{for any j}\qquad(6)$$

An intersection between two regions as the one between R and S_j in Figure 1 may also be complex to represent and must be described as two or more regions. In order to avoid problems, the following restriction must be accepted.

All surfaces must be plane and convex.

It is obvious that the shadow from a plane, convex screen is convex and that a non-empty intersection between two convex regions is convex. With this restriction complex intersections thus are avoided.

The restriction is not critical since most building parts are plane and of convex shape. In some cases, a surface may be divided into two or more regions to meet the restriction. In other cases, a surface can have holes, e.g. a wall with windows or doors. However, the holes can be treated as shadows on the wall and the restriction is then meet.

The recursion in the formula is rather easy to simulate if the programming language used does not allow recursion. With N shading screens the recursion can be limited to N levels and each level only requires storage for one polygon, one area and a few pointers.

SHADING OF DIFFUSE RADIATION

The shading of the diffuse sky radiation onto a surface can be estimated by the view factor from the surface to the sky. The ground reflected radiation is normally assumed to be diffuse, thus the view factor to the ground can be used when shading of the ground radiation is estimated. The needed view factors can be obtained in following way.

The diffuse radiation from a tilted surface above a horizontal plane is illustrated in Figure 2. The area of the surface is A. The tilt angle τ is zero for a horizontal, upward facing surface, π/2 for a vertical surface and π for a horizontal, downward facing surface. The surface and its normal vector define a spherical co-ordinate system.

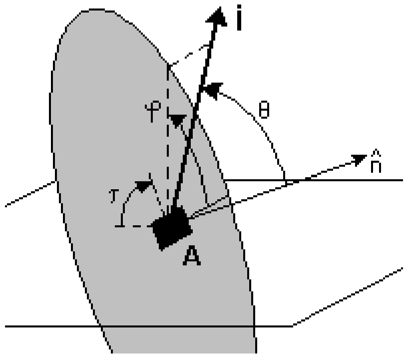

Figure 2. A tilted building surface

As the radiation is assumed to be diffuse (isotropic) the intensity is independent of the direction and the total radiation from the surface given by

$$Q=ipA\qquad(7)$$

Other building surfaces or shading screens may block some of the radiation. Thus, in a specific direction (θ,φ) only radiation from a part of the surface may reach the sky or the ground. This part is obviously equal to the radiation from the non-shaded area for direct radiation in the opposite direction. The radiation to the sky Q_{s-sky} can thus be written as

$$Q_{s-sky}=\int_{j=0}^{2p}\int_{q=0}^{p/2}iA_{ns,sky}(q,j)\cos q\sin q\,dq\,dj\qquad(8)$$

with $A_{ns,sky}(q,j)=A_{ns}(q,j)$ for directions towards the sky, else 0.

The view factor $F_{s\text{-}sky}$ between the surface and the sky can now be written as

$$F_{s-sky} = \frac{Q_{s-sky}}{i\pi A} = \frac{1}{\pi A}\int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi/2} A_{ns,sky}(\theta,\varphi)\cos\theta\sin\theta\, d\theta\, d\varphi \tag{9}$$

In a similar way, the view factor $F_{s\text{-}grd}$ from the surface to the ground can be written as

$$F_{s-grd} = \frac{1}{\pi A}\int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi/2} A_{ns,grd}(\theta,\varphi)\cos\theta\sin\theta\, d\theta\, d\varphi \tag{10}$$

with $A_{ns,grd}(\theta,\varphi) = A_{ns}(\theta,\varphi)$ for directions towards the ground, else 0.

For a non-shaded surface, the integrals above can be solved analytically leading to the following solutions:

$$F_{s-sky} = 0.5(1+\cos\tau) \tag{11}$$

$$F_{s-grd} = 0.5(1-\cos\tau) \tag{12}$$

VIEW FACTORS BETWEEN SURFACES

The radiation reflected from surfaces must be included in the calculation since it can affect the result significantly. A typical example is that of a window with a shading device. The reflected radiation can be assumed diffuse and the distribution can thus be determined by the view factors between the surfaces.

View factors are also commonly used when distribution of diffuse radiation within rooms is treated and when long wave radiation (IR-radiation) between surfaces is determined.

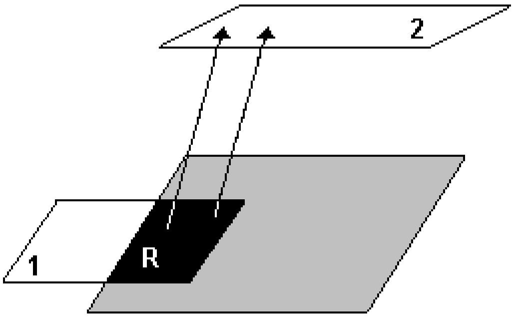

Figure 3. Diffuse radiation between two surfaces

Figure 3 illustrates the radiation in a specific direction from one surface to another. The second surface is projected, in the opposite direction, onto the plane of the first one. The intersection between this projection and the first surface is in the figure represented by the black area. Only the radiation from this intersection will reach the second surface.

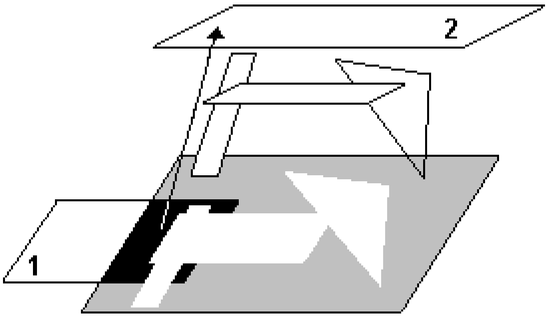

Figure 4. Shaded diffuse radiation between two surfaces

In Figure 4, other surfaces shade some of the radiation between surfaces 1 and 2. These surfaces are also projected onto the plane of the first and these projections partly intersect with R. The remaining black area is now the only part from which radiation can reach the second surface in the given direction. This area is obviously equal to the non-shaded area of R described earlier.

With a spherical co-ordinate system defined by the surface plane and its normal vector the view factor $F_{1\text{-}2}$ from surface 1 to surface 2 can now be determined as

$$F_{1,2} = \frac{1}{\pi A_1}\int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi/2} A_{ns,R}(\theta,\varphi)\cos\phi\sin\theta\, d\theta\, d\varphi \tag{13}$$

with $A_{ns,R}(\theta,\varphi) = 0$ if R is empty, else equal the non-shaded area of R.

IMPLEMENTATION

The method described for shading of diffuse radiation on outer surfaces has been implemented in the program DEROB-LTH (heating and cooling loads of buildings). This implementation and the algorithms used are presented in this section.

DEROB-LTH, which is an acronym for Dynamic Energy Response of Buildings, Lund Institute of Technology, is a flexible simulation tool using a RC-network for thermal model design. The load calculations are performed in a dynamic way for each hour during a specified period of simulation. The program was originally developed at the Numerical Simulation Laboratory at the School of Architecture of the University of Texas at Austin and has been

further developed at Lund University’s Department of Building Science in Sweden.
The program uses view factors from the outer surfaces to the sky and ground to determine primary incident diffuse sky and ground reflected radiation to these surfaces. The view factors are also used when incident long wave radiation from the sky and the ground is estimated. Furthermore, view factors between surfaces are used to calculate the radiation reflected from one surface onto another.

The program is run on PCs and the implementation is written in Fortran 95. With a few exceptions, single precision is used for floating point numbers.

The calculation of the view factors is only carried out once and the results from this calculation are then used at each time step during a whole simulation.

Non- shaded surfaces
If an outer surface is not shaded (11) and (12) are used to determine the view factors to the sky and the ground.

Shaded surfaces
For each shaded outer surface, the calculation is carried out as described below.

A co-ordinate system is defined with its x- and y-axis in the target’s (the current surface) plane and its z-axis as normal vector to the plane. All other surfaces totally or partly in front of the target are treated as screens and the polygons for these are transformed to the new co-ordinate system. The new polygons are cut at the x-y-plane as only parts in front of a surface can cast shadows.

The numerical integration of (9), (10) and (13) is then carried out in finite steps $\Delta\theta$ and $\Delta\varphi$. For each direction, all the selected screens are checked. Those facing the target and casting a shadow on it in the current direction are included when the non-shaded area is determined. Depending on the current direction, a contribution to the view factor from the target to the sky or to the ground is calculated by

$$A_{ns}(\theta,\varphi)\cos\theta\,\sin\theta\,\Delta\theta\Delta\varphi \qquad (14)$$

The contributions in the current direction to the view factor between the target and each screen are then determined. For each screen, the intersection between the target and the shadow from that screen is then used as a new target. Other screens placed between the first target and the current screen are then treated as shading screens. The non-shaded area then becomes a contribution to the view factor between the first target and the selected screen.

Sorting
In DEROB-LTH a surface can be described as a main part with a hole described as another surface. In order to determine the position of the screens in a given direction, the following procedure is used.

a. A surface described as a hole in the target is always assumed closest to the target.
b. The screen’s main parts are sorted according to their minimum z-values.
c. Main parts with overlapping z-values are checked. If the intersection between shadows from two screens with overlapping z-values is not empty the centre of this intersection is determined. The distances from this centre in the current direction to the screens are then used to determine the order between them.
d. Surfaces described as holes are finally placed just before their main part.

Cyclic overlapping surfaces or penetrating surfaces cannot be treated with the procedure described above. However, these cases do not exist in DEROB-LTH and thus, need not to be treated in this implementation.

Intersections
In order to determine the intersection between two polygons following algorithm is used step by step. If the intersection is not determined by one step, the next step is carried out.

a. If the maximum x-value (y-value) for the vertices in one polygon is less than or equal to the minimum x-value (y-value) for the vertices in the other polygon, the intersection is empty.
b. If all the vertices of one polygon are on or outside the same side, or its extension, of the other polygon, the intersection is empty.
c. If all the vertices of one polygon are inside or at the boundary of the other polygon, the intersection is equal to the first polygon.
d. If any vertex in one polygon is an internal or boundary point of the other polygon, this vertex is chosen as the first vertex of the intersection. Otherwise, the sides of one polygon are checked one by one until an intersection between that side and the boundary of the other polygon is found, thus giving the first vertex. Then the innermost way along the polygons is followed until the intersection is closed.

Due to the numerical precision in a computer program, some problems may occur when the algorithm described above is used to determine an intersection. These problems can to be avoided using following steps.

When the distance between points, the distance between a point and a line or when the intersection between two lines are needed, the most critical parts of the numerical analysis are carried out in double precision. Furthermore, the following approximations are accepted:

- Two vertices in a polygon closer to each other than 0.0025 m are replaced with a single vertex.
- Polygons with an area of less than 0.1 % of the current target are neglected.
- A point is a boundary point if the distance to a side of a polygon is less than 0.001 m.
- Vertices in different polygons are considered coincident when the distance between them is less than 0.001m.

Normalisation

With the numerical integration, the sum of the view factors from a surface varies approximately between 0.99 and 1.01. As the sum should be 1, the last step is to normalise the view factors. During this normalisation all the view factors determined with the analytical formulas (11) and (12) are assumed to be correct while those achieved by the numerical integration are adjusted.

SOME EXAMPLES

The first example is a simple room where the view factors between inner surfaces are determined.

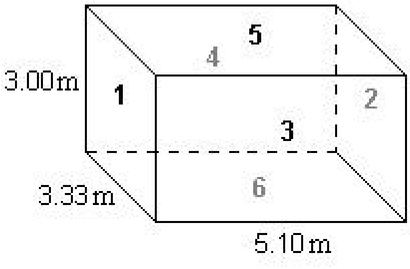

Figure 5. A simple room

The results are shown in Table 1. The formula (13) is used with three different values for Δθ and Δφ (5/5°, 10/10° respective 15/20°) corresponding to 1296, 324 respective 108 directions. The “exact” values are obtained using the analytical formulas given by Siegel & Howell (1981). The results are in this example satisfactory for steps of both 5° and 10°. A step of 15/20° may also be accurate enough in many situations.

Table 1. View factors for surfaces in Figure 5

	Formula (13)			“Exact”
	5°	10°	15/20°	
F_{1-2}	0.098	0.100	0.102	0.098
F_{1-3}	0.214	0.214	0.215	0.214
F_{1-5}	0.237	0.236	0.234	0.237
F_{3-1}	0.140	0.140	0.135	0.140
F_{3-4}	0.237	0.238	0.239	0.236
F_{3-5}	0.242	0.241	0.240	0.242
F_{5-1}	0.139	0.139	0.136	0.139
F_{5-3}	0.218	0.217	0.216	0.218
F_{5-6}	0.286	0.288	0.296	0.286

The next example is a window shaded by an overhang and two side fins, a configuration covered by most methods.

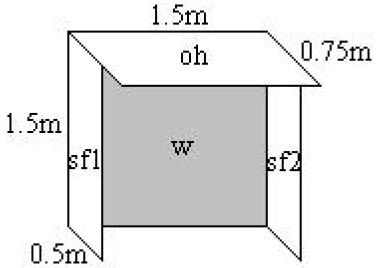

Figure 6. A window with overhang and side fins

The results for this configuration are shown in Table 2. The first three “exact” values in this table are determined as above while the last ones are determined with the subroutine Type 34 in TRNSYS (1983). This subroutine is able to handle a case as the one illustrated in Figure 6. Also here the agreement is as in the first example.

Table 2. View factors for the window in Figure 6

	Formulas (11) - (13)			“Exact”
	5°	10°	15/20°	
F_{w-oh}	0.146	0.146	0.146	0.146
F_{w-sf1}	0.113	0.113	0.109	0.113
F_{w-sf2}	0.113	0.113	0.109	0.113
F_{w-sky}	0.241	0.241	0.246	0.241
F_{w-grd}	0.387	0.387	0.389	0.387

In the last example a 1.17 × 1.17 m window, is shaded by venetian blinds consisting of 18 horizontal slats. Each slat is modelled as two 1.47 × 0.04 m planes. Each plane has two surfaces, there are thus a total of 72 surfaces on the slats.

The view factors from the window to the sky and ground are shown in Table 3. Four view factors from the window to the slats are equal to zero while the remaining vary between 0.0003 and 0.015 depending on their position.

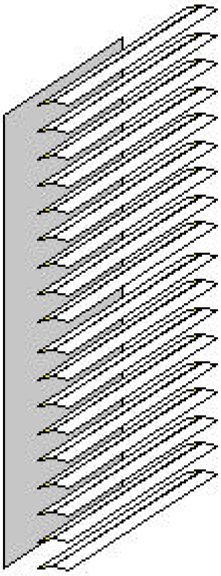

Figure 7. A window with exterior venetian blinds

Table 3. View factors for the window in Figure 7

	Formulas (11) - (13)		
	5°	10°	15/20°
$F_{w\text{ - }sky}$	0.235	0.237	0.229
$F_{w\text{ - }grd}$	0.254	0.256	0.249

Also in this example the results are rather independent of the used step.

This example is taken from an ongoing research at Lund University's Department of Building Science where measurements on various shading devices are carried out, Wallentén & Wall (1999). In this project, the radiation is measured outside the venetian blinds and inside the window. The results of the measurements are compared with calculations carried out with DEROB-LTH where a step of 5° is used. Good agreement has been found between the measurements and the calculations that are performed using the view factors method presented.

The above examples indicate that rather great integration steps may be used. However, with a too great step the view factor between two surfaces may be estimated to be zero in cases where this obvious is not correct, e.g. between surface 1 and 2 in Figure 8.

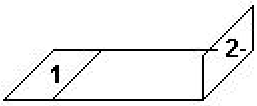

Figure 8. Two surfaces perpendicular to each other

ACKNOWLEDGEMENTS

I express my gratitude to my research fellows at our department for all the help provided and The Swedish Council of Building Research for the financial support provided.

CONCLUSIONS

A general method to estimate the shading of direct and diffuse radiation was described and its implementation in the program DEROB-LTH was discussed. The implementation has been used in several research projects during the last year and works satisfactorily.

The main advantages of the method are that:

- Both direct and diffuse radiation can be treated with the same routines.
- The method can handle almost any number and shape of building surfaces and screens.
- The method can also be used to determine view factors inside rooms of arbitrary shape.
- The accuracy is appropriate for use in programs dealing with heating and/or cooling loads of buildings.

Remaining problems

- In some cases, the calculation is time consuming since all the surfaces not necessary to deal with are treated in the recursive part of the inner loops of the calculations. Current work is focused on solving this problem.
- The numerical considerations described are appropriate for normal building elements but some problems have been noticed when very small elements are used. This problem will also be studied in a close future.
- The influence of the size of the integration steps will also be further studied.

REFERENCES

Delsante, A. E. & Spencer J. W., *Calculation of the proportion of the sky seen by windows with finite horizontal and vertical projections,* Solar Energy, Vol. 52, No 6, pp. 547-551, 1994.

Dobois, Marie-Claude, *Solar Shading and Building Energy Use*, Department of Buildin Science, Lund Institute of Technology, Lund University, Sweden, Report TABK—97/3049, 1997.

Siegel, Robert & Howell, John R., *Thermal Radiation Heat Transfer*, Hemisphere Publishing Corporation, ISBN 0-07-057316-6, 1981.

Niewienda, A. & Heidt F. D., *SOMRBERO: A PC-tool to calculate shadows on arbitrarily orientated surfaces*, Solar Energy, Vol. 58, No 4-6, pp. 253-263, 1996.

TRNSYS, *A Transient System Simulation Program*, Solar Energy Laboratory, University of Wisconsin, Madison, Wisconsin, US, 1983.

Wallentén, Petter & Wall, Maria, *Performance of shading devices in buildings – A collaboration between Lund University and Nordic manufacturers*, 5th Symposium on Building Physics in the Nordic Countries, Gothenburg, Sweden, August 24-26, 1999.

NOMENCLATURE

A	Area (m^2)	Indices:	
F	View factor	grd	Ground
Q	Energy flow (W)	ns	Non-shaded
R	Receiving surface	oh	Overhang
S	Shadow	s	Surface
X	Region	sf	Side fin
i	Intensity (W/m^2,str)	sky	Sky
θ, φ	Spherical co-ordinates	w	Window
τ	Tilt angle		