# BUILDING INTEGRATED HEATING SYSTEMS

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## ABSTRACT

This paper presents the preliminary modelling perspectives of an ongoing project where a flexible simulation tool for component and system analysis of district heating consumer installations is developed. The simulation tool makes it possible to simulate district heating consumer installations containing water based central heating systems, domestic hot tap water systems, buildings as well as load predictions of the systems. The main purpose of the project is to improve the inter action of the system components, decrease the energy consumption and analyze the performance of the heating system and the relation between the building and the heating system, Building Integrated Heating System. This is done with emphasis on increasing the system performance of the district heating system. The paper presents the modelling approach with main emphasis on lumped parameter modelling using statistical methods. Some prior results are discussed and illustrated through examples, including analysis of the heat dynamics of a building and a new approach on handling discontinuities exemplified in the modelling of the transient flows pipes.

**Keywords:** Building Integrated Heating Systems, District Heating Systems, Lumped Parameter Modelling, Simulation studies.

## INTRODUCTION

Since the first energy price crisis in the early seventies the energy policy in Denmark has been focused on how to reduce the consumption and the dependency of a single fuel. In recent years the energy demand in new buildings in Denmark has been reduced considerably due to new building regulations. The reduced heat demand and the growing focus on the indoor climate increases the requirements of the performance of the heating system and the interaction between building and the heating system, Building Integrated Heating System. More than 60 % of the heat consumption is based on district heating, DH, due to the national energy plans since the second energy crisis in the late seventies. Today the systems are characterized by pooled operation with combined heat and power plants mainly based on incineration, coal and natural gas. The suppliers have made a great effort on optimizing the system performance in order to minimize the costs; during the last fifteen years the optimization process has focused on the production plants and the distribution systems. Integration of the consumer installations in the process started around five years ago, as the natural last step in optimizing the overall efficiency. The design and operation of the consumer installations are improved with respect to the system performance, i.e. the installations must operate at a low temperature level with a high cooling and few peaks. The manufacturers of the components are very important in this process and the product development has changed from focusing on the component (sub optimization) to focusing on the inter action of the components (system optimization). Developing new products or improving existing is traditionally a complex and expensive task including numerous tests in the lab followed by final tests of a prototype in situ. Today a large part of these tests can be replaced by computer simulation studies. The use of dynamic models in computer simulation studies has proven to be a perceptive and practical method to analyze the performance and the control strategies in heating systems. Thence, the dynamics of the heating system can be analyzed second by second with varying heat supply from persons, machines and the sun in order to minimize the heat demand in the buildings and optimize the thermal comfort. However, the prospects from applying simulation studies in development of system components depends strongly on the accuracy and flexibility of the models used for the simulation.

The paper presents the modelling approach and the idea behind a project concerning the development of a model library for simulation purposes.

## THE MODELLING OBJECTIVE

The project is a joint project with collaborators from some of the largest manufacturers of DH components in Denmark and public research institutes. The modelling objective of the project is to establish a model library of the components in typical DH systems, sketched in Fig. 1, as a foundation for system analysis, simulation studies and product development. An essential aspect of the project, is, that the modelling of system components is continuously validated on experimental data to ensure accuracy and reliability in analysis

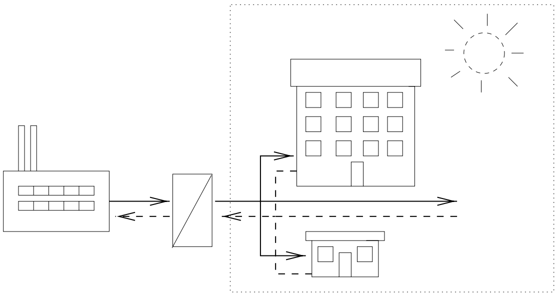

Fig. 1: Sketch of a typical DH system. The combined heat and power plant supplies the consumer via heat exchangers with hot water (solid line) for heating and domestic water. After consumption, the cooled water (dotted line) returns to the power plant.

and simulation studies. For this reasons an experimental setup are used as an empirical base for model validation.

In order to analyze the influence from the heating system on the indoor climate the models are divided into two categories, namely:

(i) Models of system components.

(ii) Models of the total system.

The models of system components are gathered in the model library and may be used for analyzing purposes and simulation studies. The user of the model library can select components from a graphical user interface (GUI) environment. Specific arrangements or setups are created using a drag-and-drop technique of the components into a worksheet, where the components are connected into a desired setup. The model library includes models of:

- Plate heat exchangers.
- Radiators.
- Thermostatic and pressure relief valves.
- Pipes and fittings.
- Pumps.
- Thermal zones and buildings.

By descending order, each class of the components cf. above, contain different models of each component. The models may vary in complexity, control strategies and user interface. Furthermore, each component may be adjusted to a specific product, i.e. physical properties such as heat capacities, resistances, sizes etc. may be controlled by the user.

### The model interface

As sketched in Fig. 1 the total DH system besides the dynamics of the power plant is considered. The system is affected by climate variables as well as the load from the power plant, interaction between components, human activity etc. In order to achieve a well-defined user interface, only the models of the total system and entire rooms and buildings may be affected directly by the climate variables, such as the outdoor temperature, solar radiation etc. System components, such as radiators and valves, are only indirectly affected by the climate variables, i.e. via the building in which they are placed. The interface of the system components are indoor temperatures, supply- and return water temperatures as well as flow and pressure drop in pipes.

The models of system components used in the simulation tool are mainly lumped parameter models. There are several arguments for preferring and using lumped parameter models for system components compared to black box models and models in terms of partial differential equations. Firstly, lumped parameter models based on the laws of nature are directly physically interpretable, a key issue where the black box models fail. Secondly, lumped parameter models in terms of ordinary differential equations are far simpler to establish and for simulation purposes compared to distributed parameter models, i.e. models in terms of partial differential equations. The lumped parameter models may be used for 'stand alone' simulation studies or together with other models of components in simulation studies of component interactions. The models of a total system may in principle be a system of lumped parameter models, corresponding to a setup of lumped parameters models of the components in the system. However, for large scale simulations, this may not be necessary or even to complex a task. When only the relation between the input and the output signals of the system is of interest, the transfer function representation is regarded as a sufficient description of the system. The user interface is still physically interpretable, e.g. flow rates, supply temperatures and pressure drops, whereas the transfer function from input to output is not physically interpretable.

## THE EXPERIMENTAL SETUP

In order to get an empirical base for the dynamical models of the components in the consumer installation the input-output relationship and dynamical characteristics of each component as well as the interaction between the system components are investigated in an experimental setup. This section presents a designed test rig for detailed testing of DH system components.

The experimental setup, a 100 kW test rig, corresponding to 20-30 apartments, represents a typically consumer installation in Western European countries. The heating system is separated from the DH system by a heat exchanger. The radiator system is divided into three individually zones, where hot water tanks emulate the dynamics of each zone. The load in the domestic hot water system is emulated by predictions, i.e. tap programs from a computer. The domestic hot water is produced instantaneous in a heat exchanger. In the main setup of the heating system only the load from the radiator system is of importance, not the performance of the individual radiator. Therefor, the empirical data of the radiator is assembled in a separate set up, where thermo-vision is used to verify the heat transfer from the radiator. The heat exchangers, radiators and hot water tanks are connected through pipes and fittings where pumps and valves controlled by a computer may generate respecified operating conditions. Furthermore the setup has the possibility for hardware-in-the-loop simulations, where new products, such as controllers and pumps, can be analyzed. The test rig are used for a variety of experiments and applications, such as model analysis and hardware-in-the-loop experiments, diagnosis and failure detection as well as data acquisition for modelling and simulation purposes.

## THE MODELLING METHOD

The modelling of system components are based on the physical characteristics of the components and empirical data from the experimental setup. This section presents briefly the modelling method for establishing physically interpretable lumped parameter models of system components using state-of-the-art modelling techniques. The emphasis is made on the total modelling process and the modelling method is illustrated through examples.

### The grey box modelling method

The lumped parameter models of the system components included in the model library are designed wrt. two main criteria:

∗ Accurate static and dynamic simulation performance.

∗ The model interface.

To comply with the design criteria, (i) and (ii), a novel modelling method, the *grey box modelling method*, see Melgaard [1], is applied. The method has proven adequate for modelling components in DH systems, see e.g. Ref. [2–4]. The total modelling process may be described by a flowchart, consisting of the three stages, sketched in Fig. 2. The modelling process is characterized by that both physically insight, statistical methods and experimental data are used to identify the model structure, estimate model parameters, estimate the model uncertainty and validate the model.

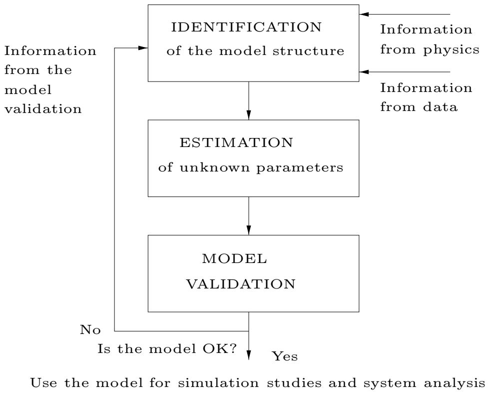

Fig. 2: The modelling procedure

### Identification

Using experimental data from the test rig and physically insight, such as well known hydraulic and thermodynamic relationships, a model structure may be identified, i.e. the first step in Fig. 2, in terms of stochastic differential equations:

$$d\mathbf{X}_t = \mathbf{f}\left(\mathbf{X}_t, \mathbf{U}_t, \theta, t\right) dt + \mathbf{G}(\theta, t) d\mathbf{W}_t, \quad (1)$$

where $\mathbf{X}_t$ is a vector of system states and the vector $\mathbf{U}_t$ contains the known inputs. $\mathbf{f}$ is a known function describing the evolution of the system states. Finally, $\theta$ is a vector of parameters, $\mathbf{W}_t$ is a Wiener process and $\mathbf{G}(\theta, t)$ is a function describing how the disturbance is entering the system. There are several reasons for applying a stochastic term, i.e. the system dynamics is described in terms of stochastic differential equations, referring to Ref. [5]:

- Modelling approximations. The evolution of the system states, described by the function $\mathbf{f}$, might be an approximation to the true system.
- Unrecognized and unmodelled inputs may affect the evolution of the states.
- Measurements of the input are noise-corrupted.

Thus, a model formulation in terms of stochastic differential equations accounts for that the function $\mathbf{f}$ only is an approximation to the true evolution of the system states. The continuous time model formulation based on laws of nature ensures that the the model structure is directly physical interpretable.

### Parameter estimation

Having measured some of the state variables, a state space representation can be formulated

$$d\mathbf{X}_t = \mathbf{f}\left(\mathbf{X}_t, \mathbf{U}_t, \theta, t\right) dt + \mathbf{G}(\theta, t) d\mathbf{W}_t, \tag{2}$$
$$\mathbf{Y}_t = \mathbf{h}\left(\mathbf{X}_t, \mathbf{U}_t, \theta, t\right) + \mathbf{e}_t, \tag{3}$$

where $\mathbf{Y}_t$ is a vector of the actually observed state variables. Eq. (2) is a continuous-time system equation and Eq. (3) is the discrete time observation equation. The function $\mathbf{h}$ describes the relationship between the state variables $\mathbf{X}_t$ and the measurements $\mathbf{Y}_t$. $\mathbf{e}_t$ is a vector describing the measurement noise which is assumed to be Gaussian distributed. The parameter vector $\theta$ contains the equivalent thermal and hydraulic components, i.e. capacitances, resistances etc., and is estimated by a Maximum Likelihood (ML) method. Descriptions of the ML method can be found in Ref. [1].

## Model validation

The last stage in the modelling process concerns the model validation. The models are validated using both experimental data, statistical validation techniques and physically interpretation. The latter by comparing the model parameters, such as time constants and the estimated model parameters, $\hat{\theta}$, with the equivalent physical characteristics of the system. As a supplement to the physically interpretation of the model, statistical methods are applied. By using stochastic independent data from the test rig and statistical methods, the models are cross validated. Hereby, it is possible to determine the goodness-of-fit using statistical criteria such as residual analysis, test for model order, test for parameter significance etc. If the model validation indicates an inadequate model performance, information from the model validation can indicate how to improve the model structure, as indicated in Fig. 2.

# MODELLING EXAMPLES

This section gives two examples of models included in the model library. The first example illustrates the modelling of a total system, namely an approach on the modelling of the heat dynamics of a building. It is argued how to extend the analysis by applying dynamic models of system components such as radiators and thus achieve a more detailed model. The second example concerns the analysis and modelling of a single component, namely the transient flow rates in pipes. Emphasis is made on a method for handling model discontinuities.

## Heat dynamics of a building

The dynamical characteristics of a building depend strongly on the building parameters, i.e. how thermal heavy the building is, the orientation towards the sun, type of insulation etc. The model library includes three different types of models of rooms or thermal zones where the parameters may be adjusted in order to model a thermal differently rooms or zones:

1. Models containing only one time constant. Only the long term variations are modelled.
2. Models containing two time constants. May be applied for analysis where both the short- and the long term dynamics are important. Both the heat capacity of the heavy building materials as well as the heat capacity of the indoor air are considered.
3. Models containing three time constants. May be applied for analysis where both the short- and the long term dynamics are important. The heat capacity of the building materials are separated, e.g. the masses of the floor and the walls.

In Fig. 3 a model of a thermal zone, cf. type (2) above, is sketched. The model contains two time constants, one describing the dynamics of the indoor air, the other describing the dynamics of the thermal heavy building materials. A simulation study may be performed e.g. by connecting a model of a room, with a model of a radiator and using climate variables as input variables. The model of the room is of type (2), since both short term and long term variations are important when analyzing the inter actions with the radiator. The model describing the heat dynamics of a single room heated by a radiator can be formulated by considering the heat balance, Eq. (4):

$$C\frac{dT}{dt} = \sum \Phi_{in} - \sum \Phi_{out}, \tag{4}$$

where $C$ (J/°C) denotes heat capacity, $T$ denotes temperature (°C) and $\Phi$ (W) the heat transfer that is appraised to influence on the heat dynamics. To derive a parameterization for $\Phi$, well known

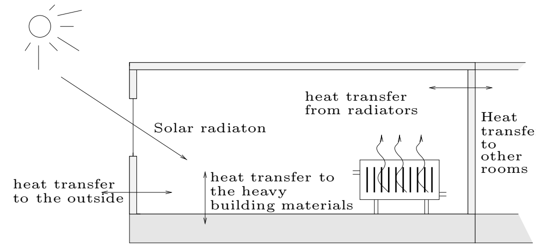

Fig. 3: Physically interpretation of a model of containing two thermal zones. The air is considered as a thermal light zone, whereas the thermal heavy masses are located in the floor. The zones are affected by the climate variables and interaction between system components such as heat input from radiators and heat transfer between building materials.

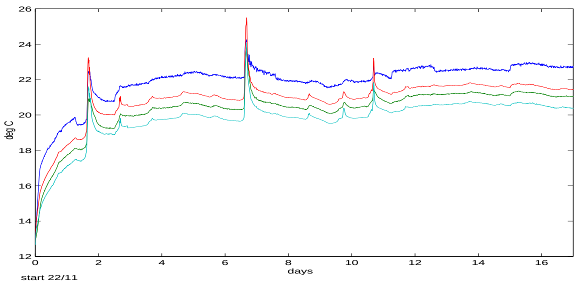

Fig. 4: Indoor air temperatures.

deterministic expressions for convective, conductive and radiate heat transfer are applied. Now, Eq. (4) can be used to describe the heat dynamics for the temperature of the air and floor in a single test room, respectively:

$$C_a \frac{dT_a}{dt} = \sum_w B_w \left(T_w - T_a\right) + (1-p)\Phi_s + \Phi_r \tag{5}$$

$$C_f \frac{dT_f}{dt} = B_f \left(T_a - T_f\right) + p\Phi_s \tag{6}$$

where $B_w$ (W/°C) is the thermal conductivity for a specific wall or floor $w$ with temperature $T_w$ (°C). $T_a$ (°C) is the indoor air temperature. Index $a$ denotes the indoor air and index $f$ denotes the floor. $\Phi_r$ and $\Phi_s$ denotes the heat input from the sun and radiator, respectively. $p$ is the fraction of solar radiation transmitted directly to the floor. Note that only the heat capacity of the floor and the air are modelled. Hence, the heat capacities of the walls are included in the 'heat capacity of the air', and only the thermal resistance of the walls are modelled. It is clear, that the model Eqs. (5-6) is a simplification of the system. However, the model has proven adequate for buildings where the walls are light. A simple dynamic model of a radiator may be formulated in terms of a single ordinary differential equation by considering the heat balance, Eq. (4):

$$C_r \frac{dT_r}{dt} = c_p \rho q \left(T_i - T_o\right) - B_r \left(T_r - T_a\right)^n \tag{7}$$

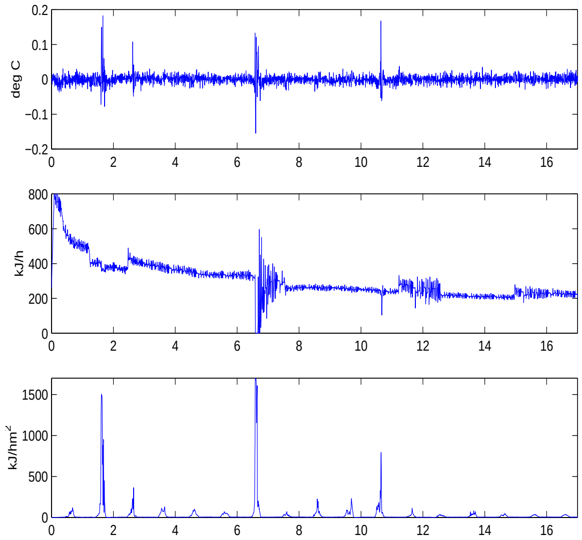

Fig. 5: Residuals, heat transfer from radiator and solar radiation.

where $T_r$ is the temperature of radiator surface and $n$ is the radiator exponent. $C_r$ is the heat capacity of the radiator, where $B_r$ (W/°C) is the thermal conductivity and $T_i$ and $T_o$ is the inlet and outlet water temperature to the radiator, respectively. $c_p$ is the specific heat capacity, $\rho$ is the mass density and $q$ is the flow. The model of the radiator, Eq. (7), may be used in calculating $\Phi_r$. Hereby, the room is modelled in terms of three differential equations. Thus, by applying models of system components the simulation becomes more detailed. The model of the room as well as the radiator are validated on experimental data. Although the model Eqs. (5-7) are very simple, the model formulation is verified by applying statistical tests. Using data from an experiment in a residential-like test building, Ref. [4], a south facing test room including a radiator, may be modelled. The flow to the radiator is controlled by a thermostatic valve. The indoor air temperature, $T_a$ was measured four different places in the test room, plotted in Fig. 4. Applying the modelling method, discussed in Section , it is found that using the first principal component of these 4 temperatures gives the most accurate results in predicting the indoor air temperature. It is found that the first principal components explains 99 % of the variations in the four signals. In Fig. 5 the model residuals, solar radiation and the heat transfer from the radiator is plotted. An increase in the residuals is seen when the solar radiation is high and the heat transfer is low. The indoor air temperature signal and a comparison with the simulated signal is shown in Fig. 6. A rapid increase in the indoor air temperature is seen when the solar radiation is high. A cross validation of

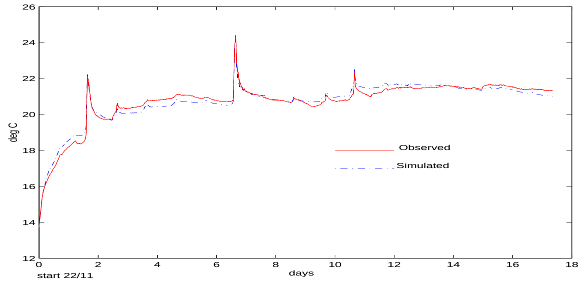

Fig. 6: Comparison between measured (solid line) and simulated indoor air temperature (dotted line).

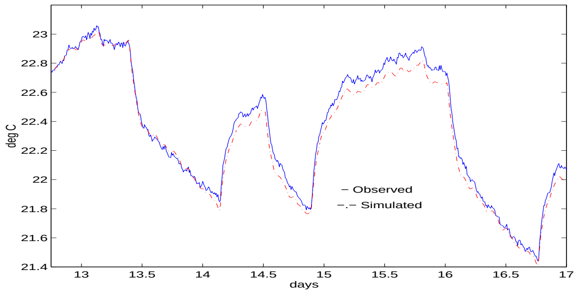

Fig. 7: Cross validation of the model in a northern faced test room. Measured (solid line) and simulated indoor air temperature (dotted line).

the model, using data from a northern faced test room, is shown in Fig. 7. It is concluded that the model, Eqs. (5-6), gives a very accurate description of the dynamics of the indoor air temperature. The calculated time constant of the radiator is 12 min. while the calculated time constant of the test room is 42 min. and 26 hours, respectively. The estimates of the model parameters are found very reasonable compared to a priori estimates. Only the heat capacity of the indoor air is very large compared to the expected value. This is due to the lumped parameterization, i.e. the heat capacities of the walls, furnitures etc. are aggregated in the heat capacity of the air. The largest deviation between the measured and simulated temperatures are only 0.4 °C. However, there is reason to believe that the model performance may be improved by extending the model with a more detailed dynamic model of the radiator as well as a dynamic model of the thermostatic valve. Furthermore, different thermal zones may be connected in order to model the heat dynamics of a total building. A model of a single room is presented in Ref. [6], while a model of a total building is presented in Ref. [7].

## Transient flow in pipes

This example illustrates the modelling of a single component, namely the fluid flow in pipes. The accuracy of the pipe model is important since it is a part of nearly any simulation study in linking different models of components together. However, a difficulty in the modelling of DH system components is the handling of discontinuities at some operating point. Examples of such discontinuities are valves shutting down, phase transition in heat exchangers and the transition from laminar to turbulent flow in a pipe. In this example, a new method for handling such discontinuities is described, exemplified by modelling the transient fluid flow in a pipe. The flow in a pipe may be described in terms of partial differential equations. However, the modelling and simulation in terms of partial differential equations is a difficult and extremely computer intense task. By assuming that the flow is one dimensional, a simple lumped parameter model may be formulated by applying Newtons second law. Hereby it easily follows that the flow, $q$, of an incompressible fluid out a pipe with the pressure drop, $\Delta p_p$, can be determined by the first order non-linear differential equation:

$$\frac{dq}{dt} = \frac{A}{l}\left(\frac{\Delta p_p}{\rho} - (\psi + \frac{\lambda l}{d})\frac{q^2}{2A^2}\right), \tag{8}$$

where $A$ is the cross-section area of the pipe with diameter $d$ and length $l$, $\rho$ is the density of the water, $\psi$ is the minor loss coefficient and $\lambda$ is the friction factor. However, Eq. (8) is not valid for both laminar and turbulent flow, since the friction factor depends on the actual flow. For laminar flow the friction factor can be calculated using the formula:

$$\lambda = \frac{64}{\mathrm{Re}}, \quad \mathrm{Re} = \frac{ud}{v}, \tag{9}$$

where Re is the Reynolds number, $v$ is the kinematic viscosity of the fluid and $u$ is the average speed of flow. For fully developed turbulent flow, the Colebrook formula Eq. (10) holds:

$$\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{\gamma}{3.7d} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right), \tag{10}$$

where $\gamma$ is the absolute pipe roughness. The turbulent region can be divided into smaller regions. For example, when the Reynolds number is less than $10^5$ and the pipe is smooth, the friction factor can be described by the Blasius equation:

$$\lambda = \frac{0.316}{\mathrm{Re}^{0.25}} \tag{11}$$

Depending on the physical properties of the actual pipe, such as the roughness of the pipe etc., the flow in a pipe is known to be laminar for $\mathrm{Re} \leq 2100$ and turbulent for $\mathrm{Re} \geq 2500$. In between, $2100 \leq \mathrm{Re} \leq 2500$, the flow is neither purely laminar nor purely turbulent, but is assumed to be a combination of both. In this interval, no expression for the friction factor, $\lambda$, exist. In Fig. 8 the friction factor, $\lambda$ as a function of the Reynolds number, $Re$, is sketched. In order to handle the discontinuity the hyperbolic function, $tanh(x)$, $x \in \mathbb{R}$, is applied as the foundation of a smooth threshold function. The derivative, $\frac{d}{dt}(tanh(x)) = 1 - tanh(x)^2$, is continuous $\forall x \in \mathbb{R}$. By transforming the threshold function, the weight function, $w(x) = (tanh(a(Re - b)) + 1)/2$, is obtained. Hereby, the threshold has its center at point $b$ while the parameter $a$ determines the shape (sharpness) of the threshold. The weight function as a function of the Reynolds number is

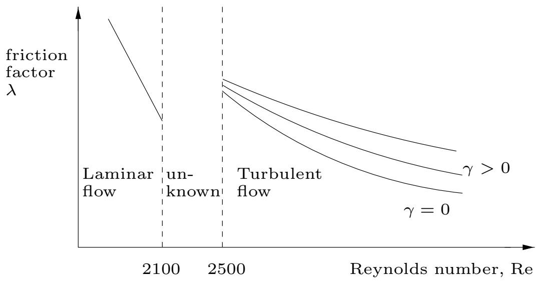

Fig. 8: The friction factor as a function of the Reynolds number. For turbulent flow the Reynolds number is a function of the absolute pipe roughness, $\gamma$.

sketched in Fig. 9. The weight function, $w(Re) \in [0\ 1]$, is applied in the model for the transient flow in a pipe. Let $\mathbf{f_1}(\mathbf{x},\mathbf{u},\theta,\mathbf{t})$ and $\mathbf{f_2}(\mathbf{x},\mathbf{u},\theta,\mathbf{t})$ be functions that describes the evolution of the flow for laminar and turbulent flow, respectively, both on the form Eq. (8) but using different expressions for the friction factor, $\lambda$. For laminar flow Eq. (9) is applied and for turbulent flow Eq. (11) is applied, respectively. Applying the smooth threshold gives the total model:

$$\begin{aligned} \mathbf{f}(\mathbf{x},\mathbf{u},\theta,t) = & (1-w(Re))\,\mathbf{f}_1(\mathbf{x},\mathbf{u},\theta,t) \\ & + (w(Re))\,\mathbf{f}_2(\mathbf{x},\mathbf{u},\theta,t) \qquad (12) \end{aligned}$$

It is seen that when the Re is small only the model describing the laminar flow is active, whereas only the model describing the turbulent flow is active when Re is large. When Re is in the transient region the flow is modelled as a weighted mean between the two models, corresponding to the weights sketched in Fig. 9. Applying the ML method, the model parameters are estimated. In Fig. 10 the estimated threshold is shown. The center of the threshold is found to be at a flow equivalent to Re=1700. This value is smaller than expected from the preliminary analysis of the system and can be explained by the fact that the rapid changes in the flow causes the flow to become turbulent at a a smaller flow than under steady state conditions. The proposed model has been cross validated using stochastic independent data. A comparison between the measured and simulated flow is shown in Fig. 11. It is seen that the model agrees nicely with the measured data.

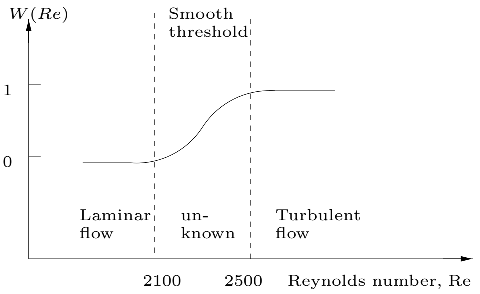

Fig. 9: The weight function as a function of the Reynolds number.

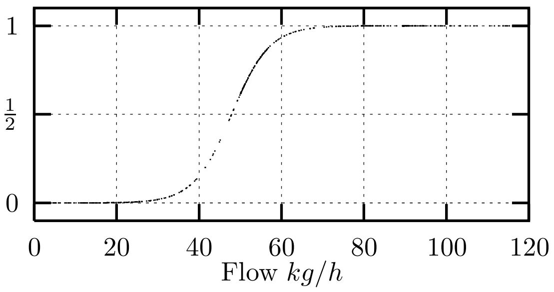

Fig. 10: The estimated threshold function. The center of the threshold corresponds to Re=1700.

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## CONCLUSION

The paper described the aim and preliminary analysis of an ongoing project. The aim of the project is to develop a simulation tool for simulation purposes of DH systems in order to improve on the interaction between system components and between the building and the heating system. The modelling method has been described with emphasis on the identification process of lumped parameter models where statistical methods are applied in estimation and model validation. The simulation tool may be used for simulation of system components or for simulation of total systems. The latter may be done by creating a specific setup using models of system components. The modelling method was illustrated through two exam-

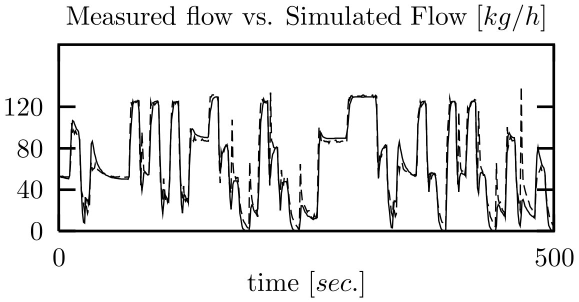

Fig. 11: Cross validation of the smooth threshold pipe model. Measured flow (solid line) and simulated flow (dotted line).

ples. The first example described the modelling of the heat dynamics in buildings. The model parameterization was found adequate using statistical methods. It was discussed how a simulation study could be more detailed by applying dynamic models of system components. The second example illustrated a new method for handeling system discontinuities. The method was illustrated in the modelling of single component, namely a model of the transient flow in pipes.

## NOMENCLATURE

| | |
|---|---|
| $T_{(\cdot)}$ | Temperature |
| $C_{(\cdot)}$ | Heat capacity |
| $\Phi_{(\cdot)}$ | Power |
| $B_{(\cdot)}$ | Thermal conductivity |
| $p$ | Fraction |
| $n$ | Radiator exponent |
| $c_p$ | Specific heat capacity |
| $\rho$ | Mass density |
| $q$ | Flow |
| $\Delta P$ | Pressure drop |
| $\psi$ | Loss coeff. |
| $\lambda$ | Friction factor |
| $A, d, l$ | Pipe area, diameter and length |
| $\gamma$ | Pipe roughness |
| $v$ | Kinematic viscosity |
| $u$ | Average speed of flow |

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